



Stable indirect fuzzy adaptive control

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Abstract

This paper investigates new fuzzy model-based observer adaptive control for multi-input multi-output continuous-time nonlinear systems. The proposed adaptive scheme uses Takagi–Seguno (TS) fuzzy models to estimate the plant states and dynamics. Using stability arguments, it is shown that the proposed scheme is globally asymptotically stable. The observation and tracking errors are shown to converge asymptotically to zero, despite the presence of external disturbances and approximation errors. The performance of the developed approach is illustrated, by simulation, on two-link robot model.

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1. Introduction

Fuzzy adaptive control is generally applicable to plants that are mathematically poorly modeled and where heuristics do not provide enough information to specify all the parameters of the control problem. Conceptually, adaptive fuzzy systems combine linguistic information from experts with numerical information from sensors [9,17]. Despite their learning capabilities and practical implementations, the earlier fuzzy adaptive systems suffer from the lack of stability analysis, i.e. the stability of closed-loop system is not guaranteed and the learning process do not lead to a well-defined dynamic [9]. Recently, an important class of a fuzzy adaptive systems have been developed, and their stability is guaranteed using the Lyapunov theory (see [3,4,12,7,14,15,8,17] and references therein). Early works were concerned with the single-input single-output (SISO) nonlinear systems case with full information on the state Variables. Lastly, the multi-input multi-output (MIMO) nonlinear systems problem was also investigated, and some schemes were proposed [7,14,8], and an observer-based

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fuzzy adaptive control scheme was proposed in [15]. Apart from few works [9,8], most of the them used the Mamdani model to approximate the plant or the controller dynamics. Both of Mamdani and TS fuzzy models [13] are proven to be universal approximators [5,9,10,17,18], however, the major advantage of TS fuzzy model is its representation power, i.e. it is capable of describing a highly nonlinear plants using few rules. Moreover, since the output of the model has an explicit functional form, analytical knowledge about the plant dynamic can be incorporated in the rules, and it is possible to analyze TS fuzzy system behavior using conventional tools.

This investigation proposes a stable indirect fuzzy adaptive control for MIMO continuous-time nonlinear systems. A TS fuzzy model-based observer is developed to approximate the nonlinear plant dynamic and estimate its state variables. This adaptive scheme presents the advantages that qualitative and analytic information about the plant operating can be used to design the fuzzy model rules, and few rules (i.e., parameters) are to be tuned which allows fast control update, which is a limiting factor for some applications. The proposed adaptive scheme achieves asymptotic tracking of a stable reference model, and the tracking and observation errors are shown to converge asymptotically to zero. The performance of this approach is evaluated on a two-link robot model.

2. Problem statement

We consider the MIMO nonlinear systems class given by

$$\dot{x}_i = A_i x_i + \underline{b}_i [f_i(x, u) + d_i], \tag{1}$$

$$y_i = \underline{c}_i^T x_i, \quad i = 1 \dots p, \tag{2}$$

where $x_i \in R^{n_i}$ is the i th subsystem state vector, $x^T = [x_1^T \dots x_p^T] \in R^n$ is the state vector, with $n = n_1 + n_2 + \dots + n_p$, $u^T = [u_1 \dots u_p] \in R^p$ is the input vector, and y_i is the i th measured output, f_i is a smooth unknown nonlinear function, d_i is bounded external disturbance, and $A_i, \underline{b}_i, \underline{c}_i$ are defined as

$$A_i = \begin{bmatrix} 0 & I_{n_i-1} \\ 0 & 0 \end{bmatrix}_{n_i \times n_i}, \quad \underline{b}_i^T = [0 \dots 0 \ 1]_{1 \times n_i}, \quad \underline{c}_i^T = [1 \ 0 \dots 0]_{1 \times n_i}.$$

For system (1) is it assumed that $\partial f_i / \partial u_i > 0$ in the relevant control region, this is equivalent to the assumption of known input gain sign. The case of $\partial f_i / \partial u_i < 0$ can also be handled. The nonlinear affine systems can be viewed as special case of the more general class (1).

The stable, LTI and controllable reference models are defined by the following state equations:

$$\dot{x}_{m_i} = A_{m_i} x_{m_i} + \underline{b}_{m_i} r_i, \quad i = 1 \dots p, \tag{3}$$

where $x_{m_i} \in R^{n_i}$ is the state vector, r_i is a bounded reference input, and $A_{m_i}, \underline{b}_{m_i}$ are given by

$$A_{m_i} = \begin{bmatrix} 0 & I_{n_i-1} \\ -\underline{a}_{m_i} & 0 \end{bmatrix}_{n_i \times n_i}, \quad \underline{b}_{m_i}^T = [0 \dots 0 \ b_{n_i,m}]_{1 \times n_i}$$

with $\underline{a}_{m_i} \in R^{n_i}$.

The control problem can be stated as that of designing the control inputs u such that the states of the plant (1) follow those of the reference model (3), under the condition that all involved signals

in the closed-loop remain bounded. Since the nonlinear function is not known, the system states are not available, and the input do not appear explicitly in (1), a TS fuzzy model-based observer will be used to realize the control objective.

3. Fuzzy adaptive approach

3.1. Fuzzy modelling

The fuzzy models to be considered here, are multi-input single-output TS fuzzy system characterized by a set of If–Then fuzzy rules of the form

$$R_i^k: \text{ If } \underline{z}_i \text{ is } Z_k^i \text{ Then } \hat{f}_i = \underline{a}_k^i \underline{x} + \underline{b}_k^i \underline{u}, \quad k = 1 \dots m_i, \tag{4}$$

where m_i is the i th fuzzy model rules number, $\underline{a}_k^i \in R^n$ and $\underline{b}_k^i \in R^p$ are the k th rule consequence parameter vectors, and $\underline{z}_i \in R^{q_i}$ is the fuzzy model input vector, assumed here to be composed of measured signals only. The fuzzy sets Z_k^i operate a fuzzy partition of the fuzzy model input space.

The final output of the fuzzy model (4) is inferred as follows:

$$\hat{f}_i = \frac{\sum_{k=1}^{m_i} \mu_k^i(\underline{z}_i)(\underline{a}_k^i \underline{x} + \underline{b}_k^i \underline{u})}{\sum_{k=1}^{m_i} \mu_k^i(\underline{z}_i)}, \tag{5}$$

where $\mu_k^i(\underline{z}_i)$ is the grade of membership of \underline{z}_i in Z_k^i . In this paper, it assumed that there exist always at least one active rule, i.e. $\sum_{k=1}^{m_i} \mu_k^i(\underline{z}_i) > 0$.

The fuzzy model output (5) can also be written in following matrix form:

$$\hat{f}_i = \underline{\phi}_i(\Psi_i \underline{x} + \Theta_i \underline{u}), \tag{6}$$

where

$$\Psi_i = \begin{bmatrix} \underline{a}_1^i \\ \underline{a}_2^i \\ \vdots \\ \underline{a}_{m_i}^i \end{bmatrix}_{m_i \times n}, \quad \Theta_i = \begin{bmatrix} \underline{b}_1^i \\ \underline{b}_2^i \\ \vdots \\ \underline{b}_{m_i}^i \end{bmatrix}_{m_i \times p}$$

and $\underline{\phi}_i$ is the normalized firing strengths vector given by

$$\underline{\phi}_i = \frac{1}{\sum_{k=1}^{m_i} \mu_k^i} [\mu_1^i \quad \mu_2^i \quad \dots \quad \mu_{m_i}^i].$$

Following the universal approximation results [2,5,10], the fuzzy model (6) is rich and able to approximate the nonlinear function $f_i(\cdot)$ on a compact operating space to any degree of accuracy. Next, we define the optimal fuzzy model parameters Θ_i^* and Ψ_i^* be such that

$$\sup_{\underline{x}, \underline{u} \in X_c} |f_i(\underline{x}, \underline{u}) - \hat{f}_i(\underline{x}, \underline{u}, \Psi_i^*, \Theta_i^*)| < \varepsilon_i, \tag{7}$$

where ε_i is an arbitrary positive constant, and X_c is a compact region of the system space. It follows that, the nonlinear system (1) can be represented, using the optimal fuzzy model as

$$\dot{\underline{x}}_i = A_i \underline{x}_i + \underline{b}_i [\underline{\phi}_i (\Psi_i^* \underline{x} + \Theta_i^* \underline{u}) + d_i + \omega_i], \tag{8}$$

where ω_i is the minimum approximation error achieved by the optimal fuzzy model.

Using the actual parameters estimate, (8) can be rewritten as

$$\dot{\underline{x}}_i = A_i \underline{x}_i + \underline{b}_i [\underline{\phi}_i (\Psi_i \underline{x} + \Theta_i \underline{u}) + \underline{\phi}_i (\tilde{\Psi}_i \underline{x} + \tilde{\Theta}_i \underline{u}) + \eta_i], \tag{9}$$

where $\eta_i = d_i + \omega_i$, and $\tilde{\Psi}_i = \Psi_i^* - \Psi_i$, $\tilde{\Theta}_i = \Theta_i^* - \Theta_i$ are the parameters estimation errors.

3.2. Observer-based fuzzy control

To estimate the state vector of the nonlinear system (1), we define, based on the actual estimated fuzzy model, the following state observers:

$$\begin{aligned} \dot{\hat{\underline{x}}}_i &= A_i \hat{\underline{x}}_i + \underline{b}_i [\underline{\phi}_i (\Psi_i \hat{\underline{x}} + \Theta_i \underline{u}) + u_{s_i}] + \underline{l}_i \tilde{y}_i, \\ \hat{y}_i &= \underline{c}_i^T \hat{\underline{x}}_i, \quad i = 1 \dots p, \end{aligned} \tag{10}$$

where $\hat{\underline{x}}_i$ is the estimated state vector and $\tilde{y}_i = (y_i - \hat{y}_i)$ is the i th output estimation error. The vector $\underline{l}_i \in R^{n_i}$ specifies the i th observer dynamic, it is chosen such that $[A_i - \underline{l}_i \underline{c}_i^T]$ is a Hurwitz matrix [1,6]. The additional term u_{s_i} is introduced to reflect the uncertainty term in (9).

Subtracting (10) from (9) yields the following observation error dynamic:

$$\dot{\tilde{\underline{x}}}_i = [A_i - \underline{l}_i \underline{c}_i^T] \tilde{\underline{x}}_i + \underline{b}_i [\underline{\phi}_i \Psi_i \tilde{\underline{x}} + \underline{\phi}_i (\tilde{\Psi}_i \underline{x} + \tilde{\Theta}_i \underline{u}) - u_{s_i} + \eta_i], \tag{11}$$

where $\tilde{\underline{x}}_i = \underline{x}_i - \hat{\underline{x}}_i$ is the state estimation error.

To eliminate the unavailable state vector \underline{x} from (11), we use the fact that

$$\tilde{\Psi}_i \underline{x} = \tilde{\Psi}_i \hat{\underline{x}} + \Psi_i^* \tilde{\underline{x}} - \Psi_i \tilde{\underline{x}}. \tag{12}$$

Then, introducing (12) in (11) yields

$$\dot{\tilde{\underline{x}}}_i = [A_i - \underline{l}_i \underline{c}_i^T] \tilde{\underline{x}}_i + \underline{b}_i [\underline{\phi}_i \Psi_i^* \tilde{\underline{x}} + \underline{\phi}_i (\tilde{\Psi}_i \hat{\underline{x}} + \tilde{\Theta}_i \underline{u}) - u_{s_i} + \eta_i]. \tag{13}$$

The term in (13) involving the optimal parameters reflects the uncertainty resulting from the combination of the errors on the states and the parameters.

Using the fact that $\tilde{y}_i = \underline{c}_i^T \tilde{\underline{x}}_i$ and (13), the transfer function is given by

$$\tilde{y}_i = \underline{c}_i^T [sI_{n_i} - A_{oi}]^{-1} \underline{b}_i [\underline{\phi}_i \Psi_i^* \tilde{\underline{x}} + \underline{\phi}_i (\tilde{\Psi}_i \hat{\underline{x}} + \tilde{\Theta}_i \underline{u}) - u_{s_i} + \eta_i] \tag{14}$$

where s is the Laplace operator, and

$$A_{oi} = A_i - \underline{l}_i \underline{c}_i^T. \tag{15}$$

Let's define the polynomial $H_i(s) = s^{n_i-1} + \lambda_2 s^{n_i-2} + \dots + \lambda_{n_i-1} s + \lambda_{n_i}$, such that $H_i^{-1}(s)$ is proper stable transfer function. Then, devising and multiplying (14) by $H_i(s)$ yields

$$\tilde{y}_i = G_i(s) [\underline{\phi}_i^f \Psi_i^* \tilde{\underline{x}} + \underline{\phi}_i^f (\tilde{\Psi}_i \hat{\underline{x}} + \tilde{\Theta}_i \underline{u}) - u_{s_i}^f + \eta_i^f], \tag{16}$$

where

$$G_i(s) = H_i(s)\underline{c}_i^T [sI_{n_i} - A_{oi}]^{-1} \underline{b}_i \tag{17}$$

and

$$\begin{aligned} \underline{\phi}_i^f &= H_i^{-1}(s)\underline{\phi}_i, \\ u_{s_i}^f &= H_i^{-1}(s)u_{s_i}, \\ \eta_i^f &= H_i^{-1}(s)\eta_i. \end{aligned}$$

Hence, (16) can be rewritten in the following state-space form:

$$\begin{aligned} \dot{\tilde{x}}_i &= A_{oi}\tilde{x}_i + \underline{b}_{oi}[\underline{\phi}_i^f \Psi_i^* \tilde{x} + \underline{\phi}_i^f (\tilde{\Psi}_i \hat{x} + \tilde{\Theta}_i u) - u_{s_i}^f + \eta_i^f], \\ \tilde{y}_i &= \underline{c}_i^T \tilde{x}_i, \end{aligned} \tag{18}$$

where $\underline{b}_{oi}^T = [0 \dots 0 \ 1 \ \lambda_2 \dots \lambda_{n_i}] \in R^{n_i}$.

The estimated tracking error $\hat{e}_i = \underline{x}_{m_i} - \hat{x}_i$, using (3) and (10), is given by

$$\dot{\hat{e}}_i = A_{m_i}\hat{e}_i - \underline{b}_i[\underline{\phi}_i(\Psi_i \hat{x} + \Theta_i u) + \underline{a}_{m_i}\hat{x}_i - b_{n_i m} r_i + u_{s_i}] - \underline{l}_i \underline{c}_i^T \tilde{x}_i. \tag{19}$$

Using the estimated parameters and states, we define the following certainty equivalent control inputs:

$$u_i = \sum_{j=1}^p \beta_{ij} [b_{n_j m} r_j - \underline{\phi}_j \Psi_j \hat{x} - \underline{a}_{m_j} \hat{x}_j - u_{s_j}], \quad i = 1 \dots p, \tag{20}$$

where $[\beta_{ij}] := B^{-1}$ and

$$B = \begin{bmatrix} b_{11} & \dots & b_{1p} \\ \vdots & \ddots & \vdots \\ b_{p1} & \dots & b_{pp} \end{bmatrix} \tag{21}$$

with

$$b_{ij} = \underline{\phi}_i \theta_j^i, \tag{22}$$

where θ_j^i is the j th column of Θ_i . The substitution of (20) in (19) yields the following estimated tracking error dynamics:

$$\dot{\hat{e}}_i = A_{m_i}\hat{e}_i - \underline{l}_i \underline{c}_i^T \tilde{x}_i, \quad i = 1 \dots p. \tag{23}$$

Eq. (23) indicates that, under the control inputs (20), the estimated tracking errors are governed by stable state-space equations whose inputs are the observation errors.

4. Stability analysis

To establish the stability of the proposed adaptive approach, the following lemma and assumptions are used.

Assumption 1. The parameters Ψ_i^* are upper bounded by $\text{tr}[\Psi_i^* \Psi_i^{*\text{T}}] \leq M_{\Psi_i}$, where M_{Ψ_i} are known bounds.

Assumption 2. The external disturbances and the approximation errors are upper bounded by $|d_i| < d_{0i}$ and $|\omega_i| < \omega_{0i}$, respectively, where d_{0i} and ω_{0i} are known constants.

Lemma 1 (Vidyasagar [16]). If the transfer function $G_i(s) = \underline{c}_i^{\text{T}}(sI_{n_i} - A_{oi})^{-1} \underline{b}_{oi}$ is strictly positive real (SPR), then there exists two symmetric positive definite matrices P_i and Q_i such that the following equations are verified:

$$P_i A_{oi} + A_{oi}^{\text{T}} P_i = -Q_i, \tag{24}$$

$$\underline{b}_{oi}^{\text{T}} P_i = \underline{c}_i^{\text{T}}. \tag{25}$$

In our case, since A_{oi} is Hurwitz, and the pairs $(A_{oi}, \underline{b}_{oi})$, $(A_{oi}, \underline{c}_i^{\text{T}})$ are controllable and observable, respectively, $H_i(s)$ can be always chosen such that the SPR condition is verified. In what follows, we consider that this condition is fulfilled.

Consider the following Lyapunov function:

$$V = \sum_{i=1}^p V_i \tag{26}$$

with

$$V_i = \frac{1}{2} \underline{\tilde{x}}_i^{\text{T}} P_i \underline{\tilde{x}}_i + \frac{1}{2\gamma_{i1}} \text{tr}[\tilde{\Psi}_i \tilde{\Psi}_i^{\text{T}}] + \frac{1}{2\gamma_{i2}} \text{tr}[\tilde{\Theta}_i \tilde{\Theta}_i^{\text{T}}], \tag{27}$$

where P_i are symmetric positive definite matrices solution of (24) and (25) for given symmetric definite positive matrices Q_i .

The differentiation of (27) along (18) gives

$$\begin{aligned} \dot{V}_i = & -\frac{1}{2} \underline{\tilde{x}}_i^{\text{T}} Q_i \underline{\tilde{x}}_i + \frac{1}{\gamma_{i1}} \text{tr}[\tilde{\Psi}_i \dot{\tilde{\Psi}}_i^{\text{T}}] + \frac{1}{\gamma_{i2}} \text{tr}[\tilde{\Theta}_i \dot{\tilde{\Theta}}_i^{\text{T}}] \\ & + \underline{\tilde{x}}_i^{\text{T}} P_i \underline{b}_{oi} [\underline{\phi}_i^f \Psi_i^* \underline{\tilde{x}} + \underline{\phi}_i^f (\tilde{\Psi}_i \hat{x} + \tilde{\Theta}_i u) - u_{s_i}^f + \eta_i^f]. \end{aligned} \tag{28}$$

Exploiting (2) and (25), (28) can be arranged as

$$\begin{aligned} \dot{V}_i = & -\frac{1}{2} \underline{\tilde{x}}_i^{\text{T}} Q_i \underline{\tilde{x}}_i + \underline{\tilde{x}}_i^{\text{T}} \underline{c}_i \underline{\phi}_i^f \Theta_i \underline{\tilde{x}} - \tilde{y}_i (u_{s_i}^f - \eta_i^f) \\ & + \frac{1}{\gamma_{i1}} \text{tr}[\tilde{\Psi}_i (\dot{\tilde{\Psi}}_i^{\text{T}} + \gamma_{i1} \hat{x} \underline{\phi}_i^f \tilde{y}_i)] + \frac{1}{\gamma_{i2}} \text{tr}[\tilde{\Theta}_i (\dot{\tilde{\Theta}}_i^{\text{T}} + \gamma_{i2} \varphi_{ik}^f u \tilde{y}_i)]. \end{aligned} \tag{29}$$

New, let us define the following parameters update laws:

$$\dot{\tilde{\Psi}}_i = \gamma_{i1} (\underline{\phi}_i^f)^{\text{T}} \hat{x}^{\text{T}} \tilde{y}_i, \tag{30}$$

$$\dot{\tilde{\Theta}}_i = \gamma_{i2} (\underline{\phi}_i^f)^{\text{T}} u^{\text{T}} \tilde{y}_i. \tag{31}$$

Then, introducing (30) and (31) in (29), and using the fact that $\dot{\tilde{\Psi}}_i = -\dot{\Psi}_i$ and $\dot{\tilde{\Theta}}_i = -\dot{\Theta}_i$ yields

$$\dot{V}_i = -\frac{1}{2} \tilde{x}_i^T Q_i \tilde{x}_i + \tilde{x}_i^T c_i \phi_i^f \Psi_i^* \tilde{x} - \tilde{y}_i (u_{s_i}^f - \eta_i^f). \tag{32}$$

The switching control terms are chosen as

$$u_{s_i} = \eta_{0i} \operatorname{sgn}(\tilde{y}_i), \tag{33}$$

where $\eta_{0i} = \omega_{0i} + d_{0i}$ is the upper bound on the i th uncertainty term. Then, substituting (33) in (32) yields

$$\dot{V}_i \leq -\frac{1}{2} \tilde{x}_i^T Q_i \tilde{x}_i + \tilde{x}_i^T c_i \phi_i^f \Psi_i^* \tilde{x}. \tag{34}$$

Summing (34) over $i = 1 \dots p$, we get

$$\dot{V} \leq -\frac{1}{2} \sum_{i=1}^p \tilde{x}_i^T Q_i \tilde{x}_i + \sum_{i=1}^p \tilde{x}_i^T c_i \phi_i^f \Psi_i^* \tilde{x}, \tag{35}$$

which can be arranged as

$$\dot{V} \leq -\frac{1}{2} \tilde{x}^T Q \tilde{x} + \tilde{x}^T C \Phi \Psi^* \tilde{x}, \tag{36}$$

where $Q = \text{block-diag}[Q_1, \dots, Q_p]$, $\Phi = \text{block-diag}[\phi_1^f, \dots, \phi_p^f]$ and

$$\Psi^* = \begin{bmatrix} \Psi_1^* \\ \vdots \\ \Psi_p^* \end{bmatrix}.$$

Applying norms to (36) yields

$$\dot{V} \leq -\frac{1}{2} \lambda_{\min}(Q) |\tilde{x}|^2 + M_\Psi |\tilde{x}|^2, \tag{37}$$

where we have used the fact that: $|C| = 1$ and $|\Phi| \leq 1$, $\lambda_{\min}(Q) = \min_i \{\lambda_{\min}(Q_i)\}$, and $M_\Psi = \sum_{i=1}^p M_{\Psi_i}$.

If the matrices Q_i are chosen such that

$$\lambda_{\min}(Q) - 2M_\Psi \geq \rho > 0 \tag{38}$$

for some positive constant ρ , we get

$$\dot{V} \leq -\frac{1}{2} \rho |\tilde{x}|^2. \tag{39}$$

The following theorem resumes the stability results for this approach.

Theorem 1. *The feedback system composed of the nonlinear system (1) and (2), the reference models (3), the fuzzy model-based observers (10), the control inputs (20) and the update laws*

(30) and (31), is globally asymptotically stable, and the tracking and observation errors converge to zero.

Proof. From (39) we have that \dot{V} is always negative in the \tilde{x} space if $\tilde{x} \neq 0$, which implies that $\dot{V}_i, \tilde{x}_i, \tilde{Y}_i$ and $\tilde{\Theta}_i \in L_\infty$ for $i = 1 \dots p$. From the boundedness of \tilde{x}_i , and since (23) is stable linear system it follows that $\hat{e}_i \in L_\infty$, which implies that $\hat{x}_i \in L_\infty$ for $i = 1 \dots p$ (since x_{m_i} is bounded). Thus, all variables in the right-hand side of (18) are bounded, then $\dot{\tilde{x}}_i \in L_\infty$, therefore \tilde{x}_i is uniformly continuous. The integral of (39) yields

$$V(\infty) - V(0) \leq -\frac{\rho}{2} \int_0^\infty |\tilde{x}|^2 dt \tag{40}$$

since $V(\infty) \geq 0$, the inequality (40) implies that

$$\int_0^\infty |\tilde{x}|^2 dt \leq \frac{2}{\rho} V(0), \tag{41}$$

hence, $\tilde{x}_i \in L_2$, which with the Barbalat’s lemma [11] yields that $\lim_{t \rightarrow \infty} \dot{V} = 0$ and $\lim_{t \rightarrow \infty} \tilde{x}_i = 0$, i.e. $\lim_{t \rightarrow \infty} \hat{x}_i = x_i$ for $i = 1 \dots p$.

From (23), the convergence of \tilde{x}_i implies the convergence of \hat{e}_i , i.e. $\lim_{t \rightarrow \infty} \hat{e}_i = 0$. Hence, since $e_i = \hat{e}_i - \tilde{x}_i$ it follows that the tracking errors converge to zero, $\lim_{t \rightarrow \infty} \hat{e}_i = 0$ for $i = 1 \dots p$. \square

Remark 1. Since the additional switching terms u_{s_i} are discontinuous, control chattering may occur, which is undesirable in practice because it involves high control activity and may excite high-frequency plant dynamics. To overcome this problem, the switching control terms can be smoothed as

$$u_{s_i} = \eta_{0i} \left(1 + \frac{\sigma_i}{\delta_i} \right) \frac{\tilde{y}_i}{|\tilde{y}_i| + \sigma_i}, \tag{42}$$

where $\sigma_i, \delta_i > 0$ are design parameters. The constant σ_i is selected based on engineering consideration to achieve the admissible tracking error amplitude. The ratio σ_i/δ_i determines the gain amplitude. Replacing (42) in (32) gives

$$\dot{V}_i = -\frac{1}{2} \tilde{x}_i^T Q_i \tilde{x}_i + \tilde{x}_i^T c_i \phi_i^f \Psi_i^* \tilde{x} - |\tilde{y}_i| \left(\eta_{0i} \left(1 + \frac{\sigma_i}{\delta_i} \right) \frac{|\tilde{y}_i|}{|\tilde{y}_i| + \sigma_i} - \eta_i^f \operatorname{sgn}(\tilde{y}_i) \right). \tag{43}$$

If $|\tilde{y}_i| \geq \delta_i$ then the third term in (43) is ≥ 0 , which yields the same result as in (34). On the other hand, if $|\tilde{y}_i| < \delta_i$ (43) becomes

$$\dot{V}_i \leq -\frac{1}{2} \tilde{x}_i^T Q_i \tilde{x}_i + \tilde{x}_i^T c_i \phi_i^f \Psi_i^* \tilde{x} + \sigma_i |\eta_i^f|. \tag{44}$$

Hence summing (44) over $i = 1 \dots p$, and using the same notation as in (36) yields

$$\dot{V} \leq -\frac{1}{2} \tilde{x}^T Q \tilde{x} + \tilde{x}^T C \Phi \Psi^* \tilde{x} + \sum_{i=1}^p \sigma_i |\eta_i^f|. \tag{45}$$

If the condition (38) holds, we get

$$\dot{V} \leq -\frac{\rho}{2} |\tilde{x}|^2 + \sum_{i=1}^p \sigma_i |\eta_i^f|. \tag{46}$$

Then \dot{V} is negative outside the region defined by

$$\Omega(\tilde{x}) = \left\{ \tilde{x} / |\tilde{x}| \leq \sqrt{\frac{2 \sum_{i=1}^p \sigma_i \eta_{0i}}{\rho}} \right\}. \tag{47}$$

This result indicates that the observation errors converge to the bounded region defined by (47). Conducting a similar analysis as above, we can show that \hat{e} and e are also bounded.

Remark 2. The above stability result is achieved under the assumption that the fuzzy models are well designed such that B is always invertible. If the above condition is not fulfilled, the control inputs (20) may be large at certain time, and the internal dynamic of (19) will be instable. More, if the switching control terms (42) is used the parameters boundedness is no longer ensured. To prevent this situation, the update laws (30) and (31) should be modified.

To assure the boundedness of the parameters Ψ_i and Θ_i and the invertibility of the matrix B , the following additional assumption is required.

Assumption 3. The matrix B elements are bounded by $|b_{ij}| < b_{ij}^H$ for $i, j = 1 \dots p$, where b_{ij}^H are known constants.

Using the projection algorithm [17], the update laws (30) and (31) are modified as

$$\dot{\Psi}_i = \begin{cases} \gamma_{i1} (\underline{\phi}_i^f)^T \hat{x}^T \tilde{y}_i & \text{if } (\text{tr}[\Psi_i \Psi_i^T] < M_{\Psi_i}) \\ & \text{or } (\text{tr}[\Psi_i \Psi_i^T] = M_{\Psi_i} \text{ and } \underline{\phi}_i^f \Psi_i \hat{x} \tilde{y}_i \leq 0), \\ \gamma_{i1} (\underline{\phi}_i^f)^T \hat{x}^T \tilde{y}_i - \gamma_{i1} \frac{\underline{\phi}_i^f \Psi_i \hat{x} \tilde{y}_i}{\text{tr}[\Psi_i \Psi_i^T]} \Psi_i & \text{if } \text{tr}[\Psi_i \Psi_i^T] = M_{\Psi_i} \text{ and } \underline{\phi}_i^f \Psi_i \hat{x} \tilde{y}_i > 0. \end{cases} \tag{48}$$

$$\dot{\underline{\theta}}_j^i = \begin{cases} \gamma_{i2} (\underline{\phi}_i^f)^T u_j \tilde{y}_i & \text{if } (|\underline{\theta}_j^i| < b_{ij}^H) \text{ or } (|\underline{\theta}_j^i| = b_{ij}^H \text{ and } \underline{\phi}_i^f \underline{\theta}_j^i u_j \tilde{y}_i \leq 0), \\ \gamma_{i2} (\underline{\phi}_i^f)^T u_j \tilde{y}_i - \gamma_{i2} \frac{\underline{\phi}_i^f \underline{\theta}_j^i u_j \tilde{y}_i}{|\underline{\theta}_j^i|^2} \underline{\theta}_j^i & \text{if } |\underline{\theta}_j^i| = b_{ij}^H \text{ and } \underline{\phi}_i^f \underline{\theta}_j^i u_j \tilde{y}_i > 0. \end{cases} \tag{49}$$

The stability of fuzzy adaptive system, with the modified update law, can be verified using the same analysis. It can be shown that, $\text{tr}[\Psi_i \Psi_i^T] \leq M_{\Psi_i}$ and $|\underline{\theta}_j^i| \leq b_{ij}^H$ for $i, j = 1 \dots p$, if the initial values of $\Psi_i(0)$ and $\underline{\theta}_j^i(0)$ are chosen properly.

5. Design example

The proposed fuzzy adaptive system is applied to the two-link robot model described by

$$M(x_3) \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \end{bmatrix} + C(x) \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} + G(x_1, x_3) = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \tag{50}$$

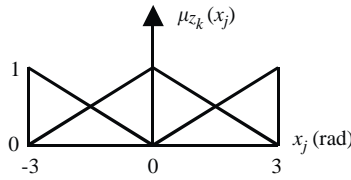


Fig. 1. Fuzzification for the fuzzy models inputs x_1, x_3 .

where

$$M = \begin{bmatrix} \frac{1}{3}m_1 l^2 + \frac{4}{3}m_2 l^2 + m_2 l^2 \cos(x_3) & \frac{1}{3}m_2 l^2 + \frac{1}{2}m_2 l^2 \cos(x_3) \\ \frac{1}{3}m_2 l^2 + \frac{1}{2}m_2 l^2 \cos(x_3) & \frac{1}{3}m_2 l^2 \end{bmatrix},$$

$$C = \begin{bmatrix} -m_2 l^2 \sin(x_3)x_4 & -\frac{1}{2}m_2 l^2 \sin(x_3)x_4 \\ \frac{1}{2}m_2 l^2 \sin(x_3)x_2 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{1}{2}m_1 gl \cos(x_1) + \frac{1}{2}m_2 gl \cos(x_1 + x_3) + m_2 gl \cos(x_1) \\ \frac{1}{2}m_2 gl \cos(x_1 + x_3) \end{bmatrix}.$$

$\underline{x}^T = [x_1 \ x_2 \ x_3 \ x_4]$ is the vector of the angular positions and velocities and \underline{u} is the vector of control torques. The physical parameters values are $l = 1$ m, $m_1 = m_2 = 1$ kg and $g = 9.81$ m/s².

The dynamic of (50) can be reformulated as in (1), with

$$\begin{bmatrix} f_1(\underline{x}, \underline{u}) \\ f_2(\underline{x}, \underline{u}) \end{bmatrix} = M^{-1} \left[-C \begin{bmatrix} x_2 \\ x_4 \end{bmatrix} - G + \underline{u} \right] \tag{51}$$

Taking the angular positions x_1 and x_3 as the measured outputs, we construct the following fuzzy approximations of (51):

$$\begin{aligned} \text{If } \underline{z}_1 \text{ is } Z_k^1 \text{ then } \hat{f}_1 &= \underline{a}_k^1 \hat{x} + \underline{b}_k^1 \underline{u}, \\ \text{If } \underline{z}_2 \text{ is } Z_k^2 \text{ then } \hat{f}_2 &= \underline{a}_k^2 \hat{x} + \underline{b}_k^2 \underline{u}, \end{aligned} \quad k = 1 \dots 9, \tag{52}$$

where Z_k^1 and Z_k^2 are fuzzy sets constructed by combining the fuzzy sets of the fuzzification depicted in Fig. 1, with the input vectors $\underline{z}_1 = \underline{z}_2 = [x_1 \ x_3]$. The outputs of (52) are given by

$$\hat{f}_1 = \underline{\phi}_1 (\Psi_1 \hat{x} + \Theta_1 \underline{u}), \tag{53}$$

$$\hat{f}_2 = \underline{\phi}_2 (\Psi_2 \hat{x} + \Theta_2 \underline{u}), \tag{54}$$

where $\underline{\phi}_1$ ($\underline{\phi}_2$) is the 1×9 vector of the normalized strengths, Ψ_1 (Ψ_2) and Θ_1 (Θ_2) are, respectively, 9×4 and 9×2 consequences parameter matrices. Based on the fuzzy approximations (53) and (54), we construct the state observers as in (10) with $\underline{l}_1^T = \underline{l}_2^T = [20 \ 100]$. To ensure the SPR condition we take $H_1(s) = H_2(s) = (s + 4)$. The reference models are chosen as $a_{m_{1,2}} = [1 \ 2]$, $b_{m_{1,2}} = 1$.

The parameters of both the fuzzy models are updated using the update laws (48) and (49), with the update gains $\gamma_{11} = \gamma_{22} = 50$ and $\gamma_{12} = \gamma_{21} = 5$. Based on physical considerations on the robot model,

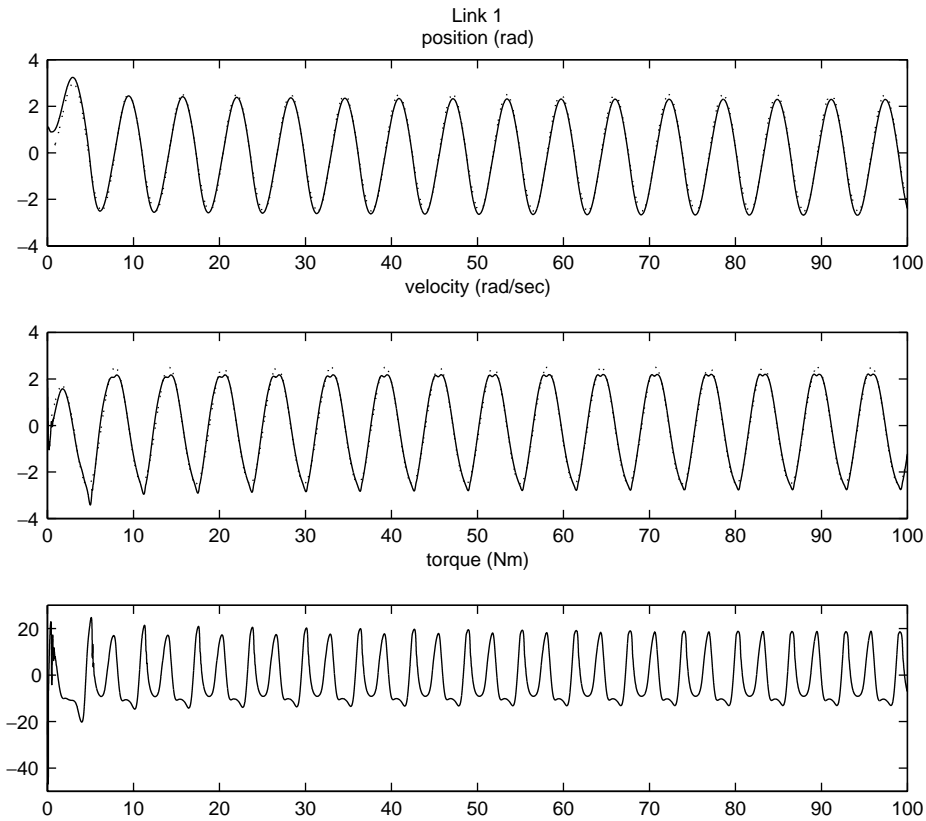


Fig. 2. Link 1 tracking performance: (—) robot; (...) reference.

the upper bounds on the fuzzy models parameters are fixed to $M_{\psi_1} = M_{\psi_2} = 1$, $b_{11}^H = 1.73$, $b_{22}^H = 13.93$, $b_{12}^H = b_{21}^H = 4.38$. The fuzzy model parameters are initialized as $\Psi_1(0) = \Psi_2(0) = 0$, $\theta_1^1(0) = 0.2$, $\theta_2^2(0) = 0.8$ and $\theta_2^1(0) = \theta_1^2(0) = 0$. The upper bounds on the approximation errors are estimated to be $\omega_{01} = \omega_{02} = 1$ and the switching terms are selected as in (42) with $\sigma_1 = \delta_1 = 0.003$ and $\sigma_2 = \delta_2 = 0.006$.

In this simulation, the initial states values are taken as $\underline{x}(0) = [1 \ 2 \ 0.5 \ -1]$, $\underline{x}_m(0) = 0$ and $\hat{\underline{x}}(0) = 0$ for robot model, the fuzzy model-observers and the reference models. Figs. 2 and 3 show the tracking performance for link 1 and link 2, respectively. It can be seen that the tracking error is reduced rapidly without large initial picks. After the initial pick to compensate for the initial errors, the developed torques are seen to be smooth. The observation dynamics for the links 1 and 2, are depicted in Figs. 4 and 5, respectively. It is clear that the observed states converge rapidly to the real ones, with small oscillations at the beginning due to the large initial errors and the switching terms.

6. Conclusion

A new fuzzy indirect adaptive control for MIMO nonlinear continuous-time systems is developed. The proposed TS fuzzy model-based observer was shown to be efficient in estimating the nonlinear

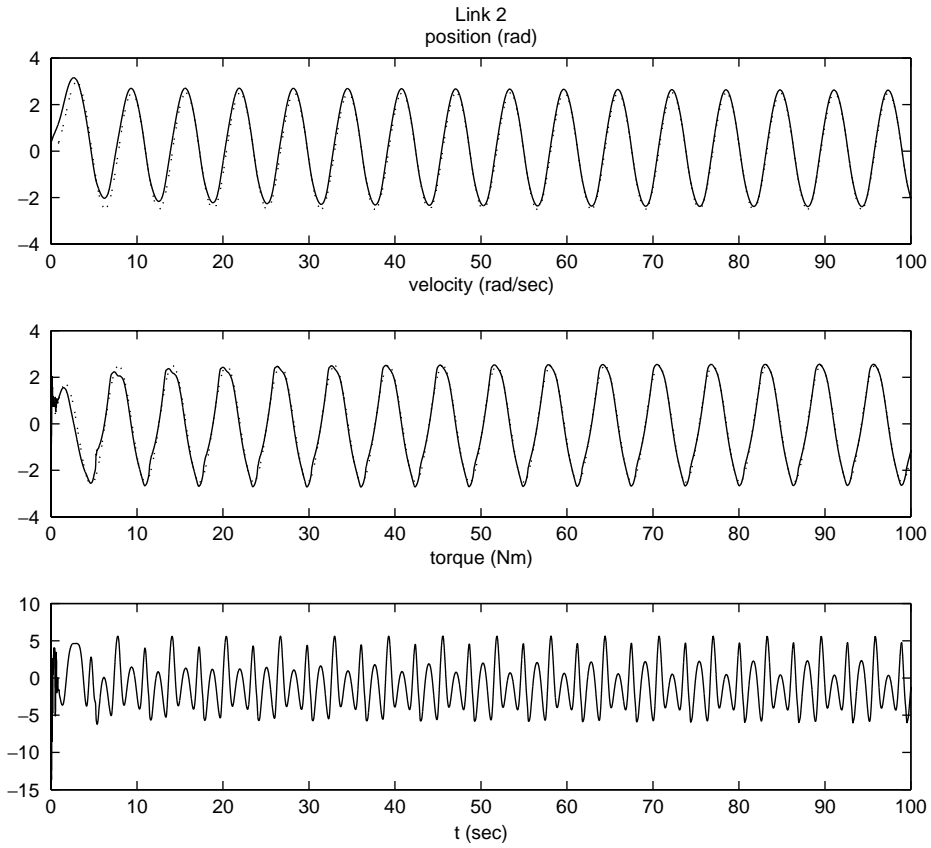


Fig. 3. Link 2 tracking performance: (—) robot; (...) reference.

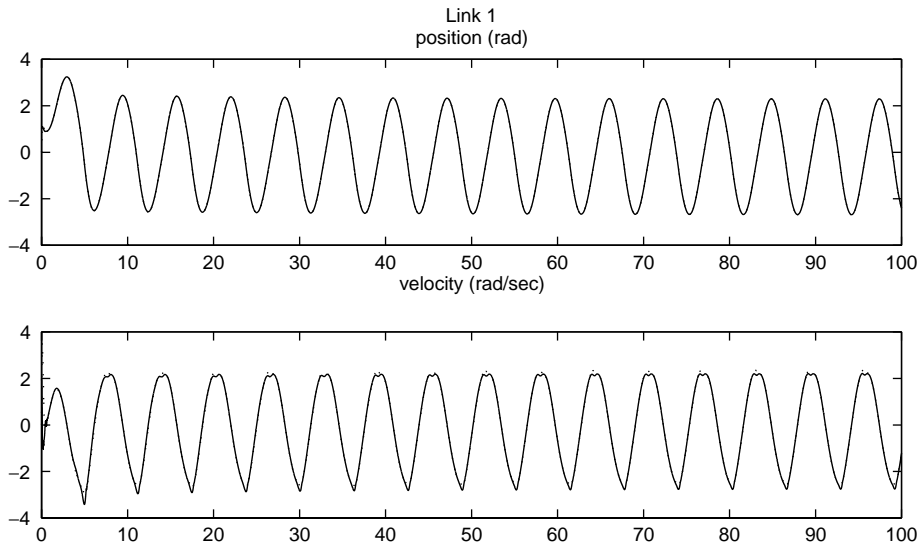


Fig. 4. Link 1 observer performance: (—) robot; (...) observer.

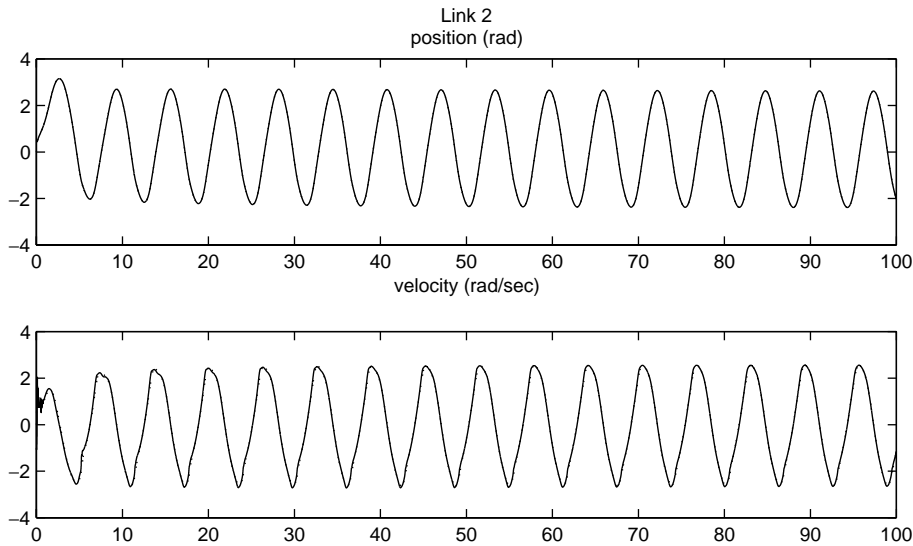


Fig. 5. Link 2 observer performance: (—) robot; (...) observer.

system dynamics and states. The advantage of using TS fuzzy model is that few rules are to be learned and analytic knowledge, if it exists, can be incorporated in the rules consequences. The stability analysis has shown that this adaptive scheme achieves asymptotic tracking of stable reference model and asymptotic state estimation. Furthermore, this algorithm is proved to be robust against external disturbances and approximation errors.

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