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UNIVERSITY LARBI BEN MHIDI OUM EL BOUAGHI

FACULTY OF SCIENCES AND APPLIED SCIENCES

DEPARTEMENT OF SCIENCES AND TECHNOLOGY

COURSE HANDOUT

Physics – 2:

Electricity and Magnetism

Course & Tutorials

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Foreword

This Physics-2 course handout provides a summary of the basic concepts of electricity and magnetism. It is intended for core students (first year LMD) in material sciences (SM), science and technology (ST), and math and computer science (MI).

The objective of this course is to introduce students to basic concepts of electrostatics, conductors in equilibrium, electrokinetics and electromagnetism. Different concepts are covered, ranging from the calculation of electric and magnetic fields and potentials to the application of the laws of electrokinetics/electromagnetism (Gauss, Kirchoff, Faraday, Ampère, etc.).

This course is developed in accordance with the official curriculum set by the Comité Pédagogique National des Domaines CPND (Canevas 2019). According to the second semester curriculum for Physics-2 (electricity and magnetism), the handout has been divided into four chapters. This course aims to provide students with a foundation to understand electricity and magnetism. We hope this course achieves the desired objectives.

Enjoy reading and enjoy your course!

Summary

Chapter 0: Mathematical reminders

Chapter 1: Electrostatics

Chapter 2: Conductors in equilibrium

Chapter 3: Electrokinetics

Chapter 4: Electromagnetism

Tutorials

Chapter 0

Mathematical reminders

Summary

1. Scalar field - Vector field

1.1. Field Concept

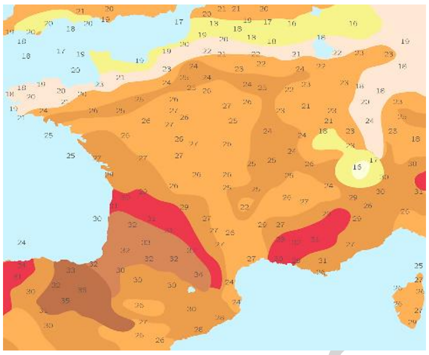
In physics, a field of physical quantity is a set of values which can be attributed at time t to a set of points M in space. It can be scalar or vector. (See **figure.1**)

1.2. Scalar field f: if we can associate a scalar quantity (value) to each point M in space, we thus obtain a scalar field.

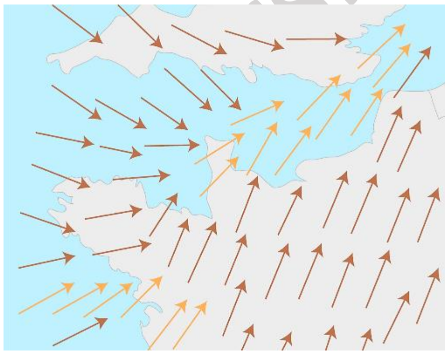
Examples: (Temperature, pressure, charge density and electric potential… etc.).

1.3. Vector field $\vec{A}$: if we can associate a vector quantity (value and direction) to each point M in space, we thus obtain a vector field.

Examples: (vector field of velocity, gravity, electric field, magnetic field… etc.).



Scalar field: temperature



Vector field: force of the wind

Figure 1. Scalar field and vector field

2. Differential operators

A differential operator is a mathematical quantity that is used to perform an operation on a vector field ($\vec{A}$) or on a scalar field (f). In electrostatics and electromagnetism, we will mainly use these four operators:

- **Gradient** ($grad$)
- **Divergence** (div)
- **Rotational** (rot)
- **Laplacian** (Δ)

These operators are defined from a very simple operator, called **Nabla**.

2.1. Operator of "Nabla"

We define a vector quantity called ≪ Nabla operator ≫, denoted $\vec{\nabla}$ by:

In the Cartesian coordinate system:

$$\vec{\nabla} = \frac{\partial}{\partial x}\,\vec{\imath} + \frac{\partial}{\partial y}\,\vec{\jmath} + \frac{\partial}{\partial z}\,\vec{k}$$

In the cylindrical/polar coordinate system (ρ or r) :

$$\vec{\nabla} = \frac{\partial}{\partial \rho}\,\overrightarrow{u_\rho} + \frac{1}{\rho}\frac{\partial}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{\partial}{\partial z}\,\vec{k}$$

In the spherical coordinate system:

$$\vec{\nabla} = \frac{\partial}{\partial r}\,\overrightarrow{u_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\,\overrightarrow{u_\varphi}$$

$\vec{\nabla}$ Can operate on vector fields and scalar fields

2.2. "Gradient" operator

Let f be a scalar function. We call gradient of f denoted: $\overrightarrow{grad}\,f$ or $\vec{\nabla}\,f$ the following vector field:

In the Cartesian coordinate system: $f\ (x, y, z)$

$$\overrightarrow{grad}\,f = \vec{\nabla}f = \frac{\partial f}{\partial x}\,\vec{\imath} + \frac{\partial f}{\partial y}\,\vec{\jmath} + \frac{\partial f}{\partial z}\,\vec{k}$$

In the cylindrical/polar coordinate system: $f\ (\rho, \theta, z)$

$$\overrightarrow{grad}\,f = \frac{\partial f}{\partial \rho}\,\overrightarrow{u_\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{\partial f}{\partial z}\,\vec{k}$$

In the spherical coordinate system: $f\ (r, \theta, \varphi)$

$$\overrightarrow{grad}\,f = \frac{\partial f}{\partial r}\,\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\,\overrightarrow{u_\varphi}$$

Example 1:

Calculate the gradient of the function $f(x, y, z) = x^2yz$

$$\overrightarrow{grad}\,f = \vec{\nabla}f = 2xyz\,\vec{\imath} + x^2z\,\vec{\jmath} + x^2y\,\vec{k}$$

2.3. Divergence operator

Let $\vec{A}$ be a vector field. We call $div\vec{A}$ the scalar product of the operator $\vec{\nabla}$ by the vector field $\vec{A}$.

In the Cartesian coordinate system: $\vec{A}\ (A_x, A_y, A_z)$

$$div\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

In the cylindrical/polar coordinate system: $\vec{A}\ (A_\rho, A_\theta, A_z)$

$$div\vec{A} = \frac{\partial A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

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In the spherical coordinate system: $\vec{A}\,(A_r, A_\theta, A_\varphi)$

$$div\vec{A} = \frac{\partial A_r}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

Example 2:

Calculate the divergence of the vector $\vec{A} = 2xy\vec{\imath} + xy^3\vec{\jmath} - 3yz^2\vec{k}$

$div\vec{A} = \vec{\nabla}.\vec{A} = 2y + 3xy^2 - 6yz$

2.4. Rotational operator

The rotational of a vector field $\vec{A}$ is defined as the vector product between $\vec{\nabla}$ and $\vec{A}$:

In the Cartesian coordinate system: $\vec{A}\,(A_x, A_y, A_z)$

$$\overrightarrow{rot}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\vec{\imath} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\vec{k}$$

In the cylindrical/Polar coordinate system: $\vec{A}\,(A_\rho, A_\theta, A_z)$

$$\overrightarrow{rot}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \overrightarrow{u_\rho} & \overrightarrow{u_\theta} & \vec{k} \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_\rho & A_\theta & A_z \end{vmatrix}$$
$$= \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\overrightarrow{u_\rho} + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\overrightarrow{u_\theta} + \left(\frac{\partial A_\theta}{\partial \rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial \theta}\right)\vec{k}$$

In the spherical coordinate system: $\vec{A}\,(A_r, A_\theta, A_\varphi)$

$$\overrightarrow{rot}\,\vec{A} = \vec{\nabla}\wedge\vec{A}$$
$$= \left(\frac{1}{r}\frac{\partial A_\varphi}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial \varphi}\right)\overrightarrow{u_r} + \left(\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial A_\varphi}{\partial r}\right)\overrightarrow{u_\theta}$$
$$+ \left(\frac{\partial A_\theta}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial \theta}\right)\overrightarrow{u_\varphi}$$

Example 3:

Calculate the rotational of the vector $\vec{A} = 2xy\vec{\imath} - 3yz^2\vec{\jmath}$

$$\overrightarrow{rot}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -3yz^2 & 0 \end{vmatrix} = 6yz\vec{\imath} - 2x\,\vec{k}$$

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2.5. "Laplacian" operator

The scalar product $\vec{\nabla}.\vec{\nabla}$ gives a second order operator called Laplacian. This operates on a vector field or on a scalar field.

In the Cartesian coordinate system:

$$\vec{\nabla}.\vec{\nabla} = \vec{\nabla}^2 = \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta \vec{A} = \frac{\partial^2 \vec{A}}{\partial x^2} + \frac{\partial^2 \vec{A}}{\partial y^2} + \frac{\partial^2 \vec{A}}{\partial z^2} \text{ (vector field)}$$

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \text{ (scalar field)}$$

In the cylindrical/polar coordinate system: $\Delta \vec{A} = \frac{\partial^2 A_\rho}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^2 A_\theta}{\partial \theta^2} + \frac{\partial^2 A_z}{\partial z^2}$

In the spherical coordinate system : $\Delta \vec{A} = \frac{\partial^2 A_r}{\partial r^2} + \frac{1}{\rho^2}\frac{\partial^2 A_\theta}{\partial \theta^2} + \frac{1}{\rho^2 \, sin^2\theta}\frac{\partial^2 A_\varphi}{\partial \varphi^2}$

Properties:

$\overrightarrow{grad}(f+g) = \overrightarrow{grad}\, f + \overrightarrow{grad}\, g$

$\overrightarrow{grad}\,(f.g) = g.\overrightarrow{grad}\, f + f.\overrightarrow{grad}\, g$

$div\left(\overrightarrow{grad}\, f\right) = laplacien\, f = \Delta f$

$div\left(f.\vec{A}\right) = \vec{A}.\overrightarrow{grad}\, f + f.div\,\vec{A}$

$div\left(\overrightarrow{rot}\,\vec{A}\right) = \overrightarrow{rot}\,(\overrightarrow{grad}\, A) = 0$

3. Elements of length, surface and volume in different coordinate systems

To express an element of length, surface or volume in a different coordinate system at a point M, we move this point infinitesimally according to the chosen coordinate system and we note the drawn geometric shape.

3.1. Cartesian coordinate system (x, y, z)

Consider a Cartesian frame $R(O,x,y,z)$ with an orthonormal basis $(\vec{\imath},\vec{\jmath},\vec{k})$. In this frame, a point M is located in space by its x, y and z coordinates. Vectorially, the position of the point M with respect to the origin O is given by the vector $\overrightarrow{OM}$ defined as follows:

$$\overrightarrow{OM} = x\,\vec{\imath} + y\vec{\jmath} + z\,\vec{k}$$

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When the coordinates x, y, z undergo an elementary variation dx, dy and dz, the point M performs an elementary displacement from M to M', as shown in **figure 2**.

a) Length element (Displacement)

We consider the infinitesimal displacement from point $M(x, y, z)$ to point $M'(x + dx, y + dy, z + dz)$. The displacement $\overrightarrow{dl} = \overrightarrow{MM'}$ can then be written:

$\overrightarrow{dl} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k}$

where: $x, y,\ z\ \in R$.

b) Surface element

$dS_z = dxdy; (dz = 0)$

$dS_y = dxdz; (dy = 0)$

$dS_x = dydz. (dx = 0)$

c) Volume element

$dV = dx\ dy\ dz$

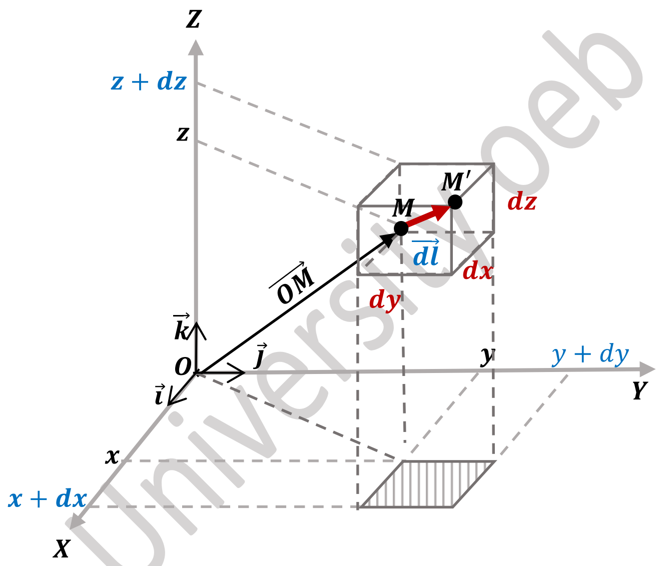

Figure 2. Element of length, surface and volume in Cartesian coordinates

3.2. Cylindrical coordinate system ($\boldsymbol{\rho, \theta, z}$)

Consider a cylindrical frame with a direct orthonormal basis $(\overrightarrow{u_\rho}, \overrightarrow{u_\theta}, \vec{k})$. In this frame, a point M is located in space by its coordinates ρ, θ and z. The position vector $\overrightarrow{OM}$ is defined as follows:

$$\boldsymbol{\overrightarrow{OM} = \rho\,\overrightarrow{u_\rho} + z\,\vec{k}}$$

When the coordinates ρ, θ and z undergo an elementary variation $d\rho, d\theta$ and dz, the point M performs an elementary displacement from M to M', as shown in **figure 3**.

a) Length element

We consider the infinitesimal displacement from point $M(\rho, \theta, z)$ to point $M'(\rho + d\rho, \theta + d\theta, z + dz)$. The displacement $\overrightarrow{MM'}$ or $\overrightarrow{dl}$ can then be written:

$d\overrightarrow{OM} = \overrightarrow{dl} = d\rho\,\overrightarrow{u_\rho} + \rho d\theta\,\overrightarrow{u_\theta} + dz\,\vec{k}$

where: $\rho \geq 0$; $0 \leq \theta \leq 2\pi$ and $z\ \in R$.

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b) Surface element

$dS_\rho = \rho d\theta dz; (\rho = ct)$ (lateral Surface element)

$dS_\theta = d\rho dz; (\theta = ct)$

$dS_z = d\rho \rho d\theta. (z = ct)$ (Surface element of Base)

c) Volume element

$dV = d\rho\ \rho\ d\theta\ dz$

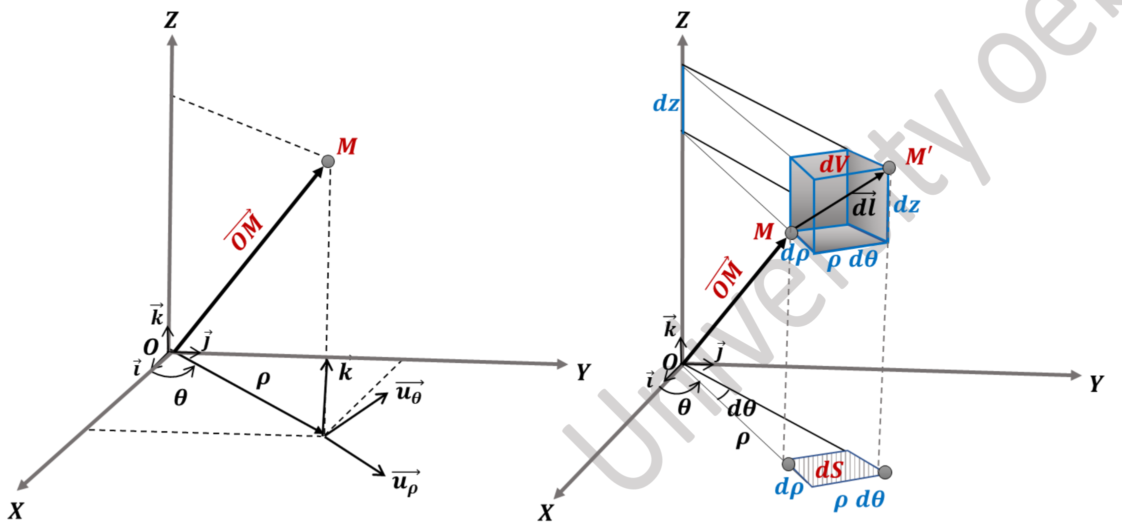

Figure 3. Element of length, surface and volume in cylindrical coordinates

3.3. Polar coordinate system (r, $\boldsymbol{\theta}$)

a) Length element

$$d\overrightarrow{OM} = \overrightarrow{dl} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta},$$

Where: $r \geq 0$ and $0 \leq \theta \leq 2\pi$

b) Surface element

$dS = dr\ r\ d\theta$

3.3. Spherical coordinate system $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$

Consider a spherical frame with a direct orthonormal basis $(\overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{u_\varphi})$. In this frame, a point M is located in space by its coordinates r, θ and φ. The position vector $\overrightarrow{OM}$ is defined as follows:

$$\overrightarrow{\boldsymbol{OM}} = \boldsymbol{r}\,\overrightarrow{\boldsymbol{u_r}}$$

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Similarly, when the coordinates r, θ and φ undergo an elementary variation $dr, d\theta$ and $d\varphi$, the point M performs an elementary displacement from M to M', as shown in **figure 4**.

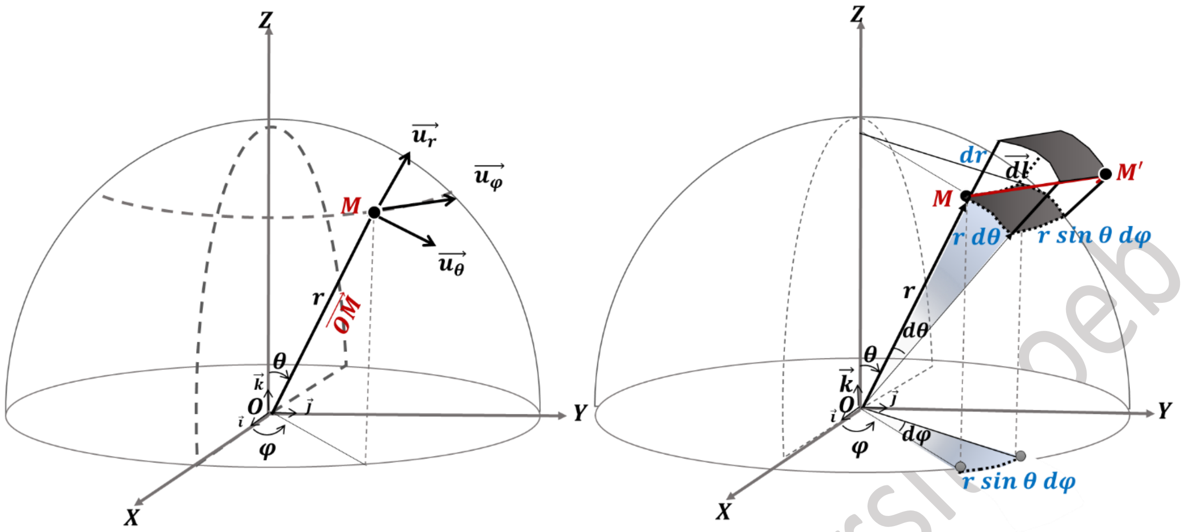


Figure 4. Element of length, surface and volume in spherical coordinates

a) Length element

We consider the infinitesimal displacement from point $M(r, \theta, \varphi)$ to point $M'(r + dr, \theta + d\theta, \varphi + d\varphi)$. The displacement $\overrightarrow{MM'}$ can then be written: $\overrightarrow{MM'} = d\overrightarrow{OM} = d(\boldsymbol{r}\,\overrightarrow{\boldsymbol{u_r}}) = dr\,\overrightarrow{u_\rho} + r\,d\overrightarrow{u_r}$.

$d\overrightarrow{OM} = \overrightarrow{dl} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + r\,\sin\theta\,d\varphi\,\overrightarrow{u_\varphi}$

Where: $r \geq 0$; $0 \leq \theta \leq \pi$ and $0 \leq \varphi \leq 2\pi$.

b) Surface element

$dS_r = r^2 sin\theta d\theta d\varphi; (r = ct)$

$dS_\theta = rdrsin\theta d\varphi; (\theta = ct)$

$dS_\varphi = rdrd\theta. (\varphi = ct)$

c) Volume element:

$dV = r^2\,dr\,\sin\theta\,d\theta\,d\varphi$

Chapter 1

Electrostatics

Summary

1. General concepts

1.1. Definition

Electrostatics is the branch of physics which studies electrical phenomena created by electrical charges in the rest state (static, immobile) and their interactions (forces).

1.2. Electrification phenomenon

The phenomenon of electrification was discovered by the Greek mathematician Thales of Miletus (624-547 BCE) when he observed the attraction of straw twigs to yellow amber.

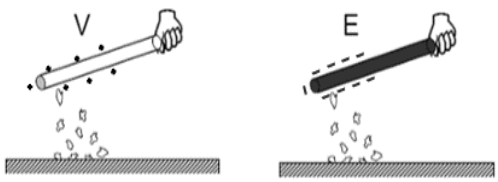
We have all noticed that when the hair is combed, the comb becomes able to attract the small pieces of paper, we therefore say that the comb is electrified.

1.2.1. Electrification modes

- Electrification by friction
- Electrification by contact
- Electrification by influence

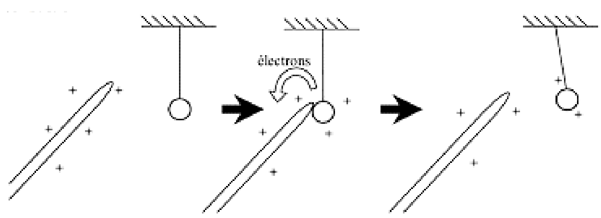
A) Electrification by friction:

We rub a rod of (glass (V), ebonite (E), plastic, amber, etc.) with (wool, silk, cloth, etc.). The rod becomes capable of attracting fine objects (paper, hair, etc.), which means that it is electrified and that it has a charge (+: glass) or (-: ebonite).



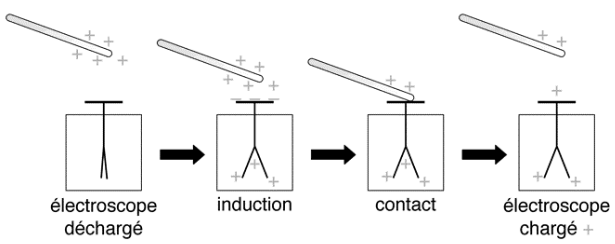
B) Electrification by contact:

The already electrified glass rod is brought into contact with a polystyrene ball of an electrically neutral electrostatic pendulum. When we put the two objects in contact, there will be an attraction. Right after contact, the rod and the ball will repel each other because both are now positively electrified.



C) Electrification by influence:

We approach the already electrified glass rod to the metal disk of the electroscope, we notice that the aluminum sheets repel each other, that is to say they are charged with the same sign.



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Conclusion

- We deduce that these materials have acquired a new property called "electrification"
- This property creates an attraction much more intense than the universal attraction produced by the masses.
- There are two types of electrification, positive and negative.

1.2.2. Explanation of the electrification phenomenon

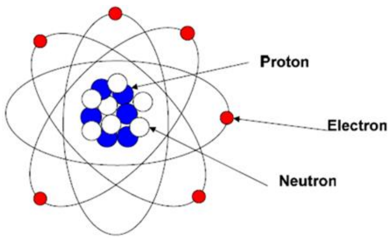
The atoms of materials contain in their natural states an equivalent number of electrons and protons, these materials are therefore electrically neutral. But if this equilibrium is disrupted by an increase or decrease for any reason such as electrification, then matter becomes charged. The phenomenon of electrification is explained by a movement of electrons.

- The body that gains (receives) electrons becomes negatively charged (-).
- The body that loses (gives) electrons becomes positively charged (+).

2. The electric charge

2.1. The elementary electric charge

The electrical properties of matter find their principle at the level of the atom. Each atom is made up of a nucleus, around which a cloud of negatively charged electrons orbits, the nucleus in turn is made up of protons with positive charges (p^+) and neutrons with zero charges (n^0). The electrons (e^-) and the protons (p^+) carry the same electric charge in absolute value which we note by "e". This electric charge is called "the elementary electric charge $''e''$ or quanta of the electric charge; whose value is given by:



$$\boldsymbol{e = 1.602 \times 10^{-19} C}$$

2.2. The punctual electric charge

A punctual electric charge denoted "q" can take any value, it presents an integer multiple of the elementary electric charge "e", we write:

$$\boldsymbol{q = \pm n\, e}$$

The electric charge of a body q is expressed in Coulomb "C".

2.3. Principle of conservation of electric charge

Whatever the phenomena produced by the electric charges q_i, in the isolated system:

$$\sum_{i=1}^{n} qi = C^t$$

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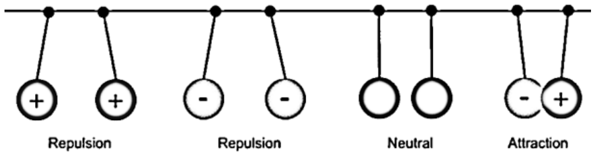
2.4. Attraction and repulsion between charges

- Two bodies carrying a charge of the same sign repel each other
- Two bodies carrying a charge of the opposite sign attract each other



3. Coulomb's law

Let be two particles carrying electric charges q_A and q_B, separated by a distance "r" interact with each other. This interaction is modeled by a force called the electrostatic force $\overrightarrow{F_e}$. This force is attractive if the charges have the opposite sign and is repulsive if the charges have the same sign.

According to the principle of action/reaction (Newton's 3rd law):

- $\vec{F}_{A/B}$ = - $\vec{F}_{B/A}$ (The two forces have opposite directions).
- $\|\vec{F}_{A/B}\| = \|\vec{F}_{B/A}\|$ (The two forces have the same intensity).
- $\vec{u}_{A/B}$ (The forces are radial, carried by the axis which joins the two charges).

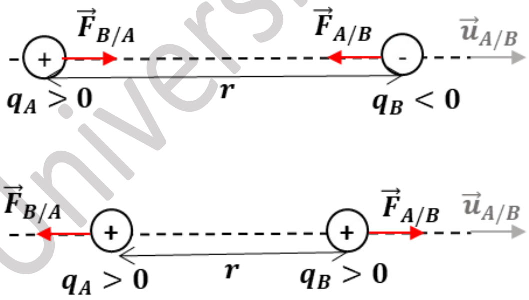


Figure 1. Interaction between two electric charges

According to several experiments, the intensity of the electrostatic force exerted between the two charges q_A and q_B is inversely proportional to the square of the distance separating the two particles and it is proportional to the charge of the two particles.

3.1. The electrostatic force $\overrightarrow{F_e}$

The electrostatic force $\overrightarrow{F_e}$ exerted between two charges is given by the mathematical expression, called Coulomb's law:

$$\overrightarrow{F_{A/B}} = -\overrightarrow{F_{B/A}} = k\,\frac{q_A\,q_B}{r^2}\overrightarrow{u_{AB}}$$

$$F_{A/B} = F_{B/A} = k\,\frac{|q_A|\,|q_B|}{r^2}$$

- $\overrightarrow{u_{AB}}$: the unit vector of $\overrightarrow{AB}$ which indicates the direction of the force.
- $\boldsymbol{k}$: Coulomb's constant or the constant of proportionality. Defined by:

$$k = \frac{1}{4\,\pi\,\varepsilon_0}\ ; \quad \text{where :} \quad k = 9 \times 10^9\ N.m^2.C^{-2}$$

- **ε_0**, is the vacuum permittivity equal to $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$

Exercise-1

1. Calculate the electrostatic force exerted on the electron of the hydrogen atom ${}_1^1H$
2. Compare this force to the force of gravitational attraction.

We give: $e = 1.602 \times 10^{-19} C$; $me = 9 \times 10^{-31} kg$; $mp = 1850\ me$; $r_{atom} = 5 \times 10^{-11} m$

$$G = 6.67 \times 10^{-11}\ N.m^2.kg^{-2};\quad k = 9 \times 10^9\ N.m^2.C^{-2}$$

Solution-1

1- The electrostatic force F_e

$$F_e = k\frac{|q_e|\,|q_p|}{r^2} = k\frac{e^2}{r^2} = 0.92 \times 10^{-7} N$$

2- Comparison

$$F_G = G\frac{m_e m_p}{r^2} = G\frac{1850\ m_e^2}{r^2} = 4 \times 10^{-47} N$$

By comparison: $\frac{F_e}{F_G} \sim 10^{40}\ times \Rightarrow\ F_e \gg F_G$

3.2. The components of the electrostatic force

In the Cartesian system $(\vec{\imath}, \vec{\jmath}, \vec{k})$; the point M is identified by the position vector $\vec{r}$; where:

$$\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$$

Likewise, the distance r between two charges q_A and q_B is identified by:

$$\vec{r} = \overrightarrow{AB} = (x_B - x_A)\vec{\imath} + (y_B - y_A)\vec{\jmath} + (z_B - z_A)\vec{k}$$

and: $$r = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$$

The electrostatic force is therefore written:

$\overrightarrow{F_{A/B}} = k\frac{q_A q_B}{r^2}\overrightarrow{u_{AB}}$; where : $\overrightarrow{u_{AB}} = \frac{\overrightarrow{AB}}{AB} = \frac{\vec{r}}{r}$

$$\Rightarrow \boldsymbol{\overrightarrow{F_{A/B}} = k\frac{q_A q_B}{r^3}\vec{r}}$$

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Exercise-2

In the Cartesian system, two charges $q_A = 10^{-7}C$ and $q_B = -2 \times 10^{-7}C$; are located respectively in: $A(2,-1,3)$ and $B(-1,2,0)$. Determine the components of the electrostatic force $\overrightarrow{F_e}$.

Solution-2

We have: $\overrightarrow{F_e} = F_x\,\vec{\imath} + F_y\,\vec{\jmath} + F_z\,\vec{k}$

In addition: $\overrightarrow{F_{A/B}} = k\,\frac{q_A\,q_B}{r^3}\vec{r}$

with : $\begin{cases} \vec{r} = -3\,\vec{\imath} + 3\,\vec{\jmath} - 3\vec{k} \\ r = 3\sqrt{3} \end{cases}$; N.A : $\Rightarrow \begin{cases} F_x = \frac{-2\sqrt{3}\times 10^{-5}}{9} \\ F_y = \frac{2\sqrt{3}\times 10^{-5}}{9} \\ F_z = \frac{-2\sqrt{3}\times 10^{-5}}{9} \end{cases}$

3.3. Principle of superposition

If we have "n" electric charges at rest, and in a vacuum, the principle of superposition allows us to make the vector sum of the electrostatic forces exerted at point M, have a charge q_M.

$$\overrightarrow{F_M} = \sum_{i=1}^{n} \overrightarrow{F_i}$$

$$\overrightarrow{F_M} = \overrightarrow{F_1} + \overrightarrow{F_2} + \cdots + \overrightarrow{F_n}$$

$$\overrightarrow{F_M} = k\frac{q_1 q_M}{r_1^2}\overrightarrow{u_1} + k\frac{q_2 q_M}{r_2^2}\overrightarrow{u_2} + \cdots \ldots \ldots \ldots \ldots k\frac{q_n q_M}{r_n^2}\overrightarrow{u_n}$$

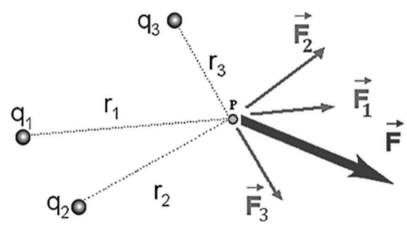

Figure 2. Sum vector of the electrostatic forces

4. Electrostatic field $\vec{E}$

In the presence of a charged particle the physical properties of the space surrounding this particle are changed, we then say that the electric charge "q" creates an electrostatic field $\vec{E}$.

4.1. Electrostatic field $\vec{E}$ created by a punctual electric charge

Let be an electric charge "q" acts on another electric charge "Q" placed at a point "M" in space by an electrostatic force $\overrightarrow{F_e}$, given by:

$$\overrightarrow{F_e} = k\,\frac{q\,Q}{r^2}\vec{u}$$

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We therefore write:

$$\overrightarrow{F_e} = Q\left[k\,\frac{q}{r^2}\vec{u}\right]$$

The term in parentheses presents the electrostatic field $\vec{E}$ created by the charge q at point M, and which takes the form:

$$\boldsymbol{\overrightarrow{E_M} = \frac{kq}{r^2}\vec{u}}$$

The unit of electrostatic field is expressed in (V/m)

The relation between the force and the electrostatic field is therefore:

$$\boldsymbol{\overrightarrow{F_e} = Q\,\vec{E}}$$

4.2. The electrostatic field lines

The field lines are oriented lines at which the field vector $\vec{E}$ is tangent at any point, where:

1- The lines are continued.
2- The lines start from the positive charge and end at the negative charge.
3- The arrows indicate the direction of $\vec{E}$.

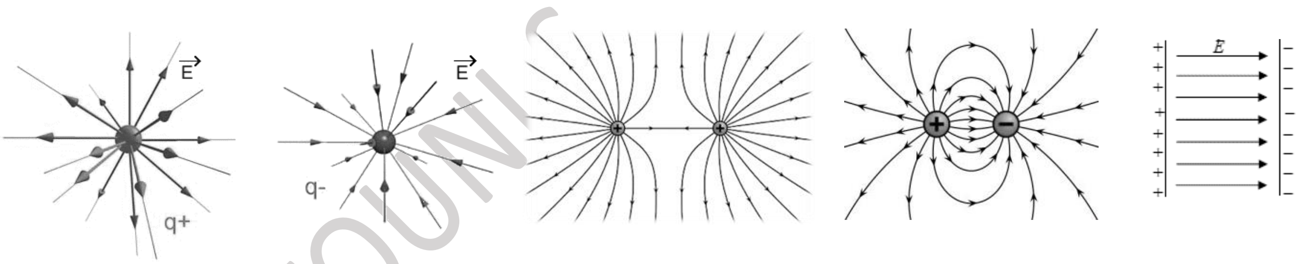
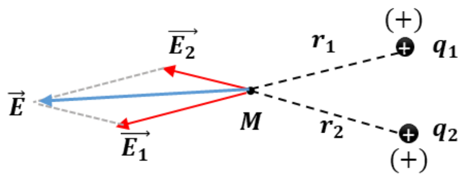


Figure 3. Electrostatic field lines

4.3. Electrostatic field $\vec{E}$ created by a set of charges

Likewise, if we have $''n''$ electric charges $''q_i''$, to determine the electrostatic field $\vec{E}$ produced by this set of charges at point $''M''$, we use the principle of superposition, which allows us to make the vector sum of the fields $\vec{E_i}$, we write:



$$\overrightarrow{E_M} = \sum_{i=1}^{n} \vec{E_i}$$

Figure 4. Sum vector of the electrostatic fields

$$\overrightarrow{E_M} = \vec{E_1} + \vec{E_2} + \cdots + \vec{E_n}$$

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$$\overrightarrow{E_M} = \frac{kq_1}{r_1^2}\overrightarrow{u_1} + \frac{kq_2}{r_2^2}\overrightarrow{u_2} + \cdots \ldots \frac{kq_n}{r_n^2}\overrightarrow{u_n}$$

4.4. Electrostatic field created by a continuous distribution of charges

Matter is formed by a large number of charged particles, these charges can be distributed uniformly along a straight line (line, wire), on a surface plane or in a volume.

Electrostatic equilibrium requires that the distribution of charges in matter is continuous, we therefore distinguish three types of charge distribution:

1- Linear distribution (l)
2- Surface distribution (S)
3- Volume distribution (V)

To simplify the study, we divide the distribution of charges into small parts charged with elementary charge "dq", then we calculate the elementary electrostatic field "dE" produced by "dq" and finally we make the integral according to the type of distribution. The elementary electrostatic field $\overrightarrow{dE}$ created at point "M" is given by:

$$\overrightarrow{dE} = \frac{k\,dq}{r^2}\,\vec{u}$$

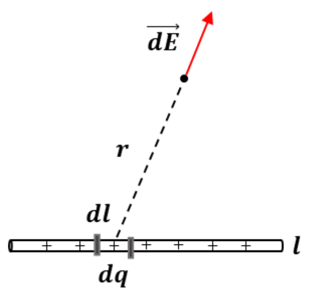
$$\overrightarrow{E_M} = \int \overrightarrow{dE}$$

$$\overrightarrow{E_M} = \int \frac{k\,dq}{r^2}\,\vec{u}$$

a) Linear distribution

If the charges are continuously distributed on a wire of length "l", we introduce the notion of the linear charge density "λ", where: $dq = \lambda\,dl$.

The elementary electrostatic field $\overrightarrow{dE}$ created at point "M" is given by:



$$\overrightarrow{dE} = \frac{k\,dq}{r^2}\,\vec{u}$$

$$\overrightarrow{E_M} = \int \overrightarrow{dE} = \int \frac{k\,\lambda\,dl}{r^2}\,\vec{u}$$

b) Surface distribution

If the charges are continuously distributed on a surface plane "S", we introduce the notion of the surface charge density "σ", where: $dq = \sigma\,dS$

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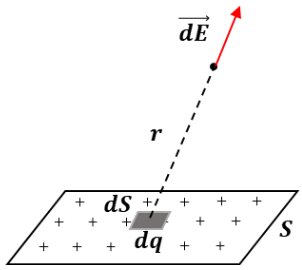
The elementary electrostatic field $\overrightarrow{dE}$ created at point "M" is given by:



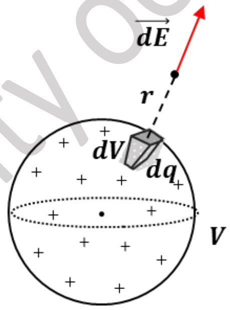
$$\overrightarrow{dE} = \frac{k\,\sigma\,dS}{r^2}\,\vec{u}$$

$$\overrightarrow{E_M} = \iint \frac{k\,\sigma\,dS}{r^2}\,\vec{u}$$

c) Volume distribution

If the charges are continuously distributed in a volume "V", we introduce the notion of the volume charge density "ρ", where: $dq = \rho\,dV$.

The elementary electrostatic field $\overrightarrow{dE}$ created at point "M" is given by:



$$\overrightarrow{dE} = \frac{k\,\rho\,dV}{r^2}\,\vec{u}$$

$$\overrightarrow{E_M} = \iiint \frac{k\rho\,dV}{r^2}\,\vec{u}$$

Table.1. Elements of length $\overrightarrow{dl}$, surface dS and volume dV in different coordinate systems

Cartesian coordinate system (x, y, z)	**Polar coordinate system (ρ, θ)**
$\overrightarrow{dl} = dx\,\vec{\imath} + dy\,\vec{\jmath} + dz\,\vec{k}$ $dS = dxdy$ $dV = dx\,dy\,dz$	$\overrightarrow{dl} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta}$, $dS = dr\,r\,d\theta$
Cylindrical coordinate system (ρ, θ, z)	**Spherical coordinate system (r, θ, φ)**
$\overrightarrow{dl} = d\rho\,\overrightarrow{u_\rho} + \rho d\theta\,\overrightarrow{u_\theta} + dz\,\vec{k}$ $dS_\rho = \rho d\theta dz$; (lateral Surface element) $dS_z = d\rho\rho d\theta$.(Surface element of Base) $dV = d\rho\,\rho\,d\theta\,dz$	$\overrightarrow{dl} = dr\,\overrightarrow{u_r} + r\,d\theta\,\overrightarrow{u_\theta} + r\,\sin\theta\,d\varphi\,\overrightarrow{u_\varphi}$ $dS = r^2 sin\theta d\theta d\varphi$ $dV = r^2\,dr\,\sin\theta\,d\theta\,d\varphi$

5. Electrostatic potential

In the mechanics of material point, we know that if the force $\overrightarrow{F_c}$ is conservative, the work of this force does not depend on the path followed and we say that this force F_c derives from a

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potential energy E_p. The electrostatic force $\vec{F_e}$ is a conservative force, it therefore derives from a potential energy E_p, and we write:

$$\vec{F_e} = -\overrightarrow{grad}\, E_p$$

5.1. Electrostatic potential energy

Let be a point charge "q" moves in an electrostatic field "$\vec{E}$" created by another fixed charge "Q", it therefore subjected to an electrostatic force $\vec{F_e}$, where:

$$\vec{F_e} = q\,\vec{E}$$

The elementary work carried out by the force $\vec{F_e}$ to make an elementary displacement $\vec{dl}$, is written as follows:

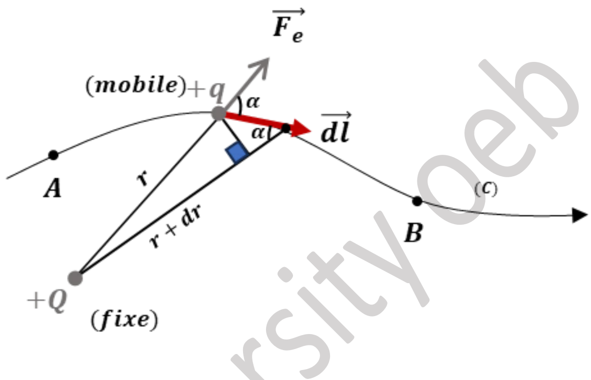

Figure 5. Interaction between fixed charge and moved charge

$$dW = \vec{F_e}.\vec{dl}$$

$$\Rightarrow dW = F_e.dl.\cos\alpha$$

According to the diagram: $\begin{cases} \cos\alpha = \frac{dr}{dl} \\ Fe = \frac{k\,q\,Q}{r^2} \end{cases} \Rightarrow \quad dW = \frac{k\,q\,Q}{r^2}\,dr$

$$\Rightarrow W(\vec{F_e}) = \int dW = \int \frac{k\,q\,Q}{r^2}\,dr$$

$$\Rightarrow W(\vec{F_e}) = -\frac{k\,q\,Q}{r} + c$$

$$if: r \to \infty, (W = 0) \Rightarrow (c = 0).$$

$$\Rightarrow \boldsymbol{W(\vec{F_e}) = -\frac{k\,q\,Q}{r}}$$

According to the principle of conservation of potential energy: $\Delta E_p = -\sum W(\vec{F_c})$

$$\Rightarrow \Delta E_p = -W(\vec{F_e}) = \frac{k\,q\,Q}{r}$$

Therefore, we write the expression of the potential energy of the electrostatic interaction between two charges q and Q as follows:

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$$\boldsymbol{E_p = \frac{k\, q\, Q}{r}}$$

The unit of electrostatic potential energy is expressed in Joule (J).

5.2. Electrostatic potential of a punctual charge

The electrostatic potential "V" is a scalar field directly related to the potential energy E_p of a charge "q" located in an electrostatic field "$\vec{E}$" created by a fixed charge "Q".

We have:

$$E_p = q\left[\frac{kQ}{r}\right]$$

$$\Rightarrow E_p = q\, V$$

$$\boldsymbol{V = \frac{kQ}{r}}$$

The unit of electrostatic potential is expressed in volts (V).

We consider a charge "q_A" located at point $A\,(x_A, y_A, z_A)$, the potential created by this charge at point $M\,(x, y, z)$ in the Cartesian system is given by:

$$V = \frac{kq_A}{\sqrt{(x - x_A)^2 + (y - y_A)^2 + (z - z_A)^2}}$$

5.3. Relation between field $\vec{E}$ and potential V

The relation between the electrostatic field and the electrostatic potential can be determined from the following relations:

$$\Rightarrow \begin{cases} W(\vec{F_e}) = -\Delta E_p \\ E_p = q\, V \end{cases} \qquad \Rightarrow \begin{cases} dW = -dE_p = -(q\, dV) \\ dE_p = q\, dV \end{cases}$$

$$dW = -q\, dV$$

$$\Rightarrow \vec{F_e}.\vec{dl} = -q\, dV$$

$$\Rightarrow dV = -\frac{\vec{F_e}}{q}.\vec{dl}$$

$$\Rightarrow dV = -\vec{E}.\vec{dl}$$

In the cartesian coordinate system:

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$$\begin{cases} \vec{E} = Ex\,\vec{i} + Ey\,\vec{j} + Ez\,\vec{k} \\ \vec{dl} = dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k} \\ dV = \frac{\partial V}{\partial x} dx\,\vec{i} + \frac{\partial V}{\partial y} dy\,\vec{j} + \frac{\partial V}{\partial z} dz\,\vec{k} \end{cases}$$

$$\Rightarrow \frac{\partial V}{\partial x} dx\,\vec{i} + \frac{\partial V}{\partial y} dy\,\vec{j} + \frac{\partial V}{\partial z} dz\,\vec{k} = -\left(E_x\,dx + E_y\,dy + E_z\,dz\right)$$

$$\frac{\partial V}{\partial x} = -E_x; \qquad \frac{\partial V}{\partial y} = -E_y; \qquad \frac{\partial V}{\partial z} = -E_z$$

The relation between $\vec{E}$ and V can be established in the following form:

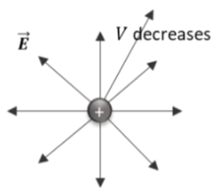
$$\vec{\boldsymbol{E}} = -\overrightarrow{\boldsymbol{grad}}\,\boldsymbol{V}$$

Table.2. $\vec{E} = -\overrightarrow{grad}\,V$ in different coordinate systems.

Cartesian coordinate system $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$	**Polar coordinate system** $(\boldsymbol{\rho}, \boldsymbol{\theta})$	**Cylindrical coordinate system** $(\boldsymbol{\rho}, \boldsymbol{\theta}, \boldsymbol{z})$	**Spherical coordinate system** $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$
$\begin{cases} E_x = -\frac{\partial V}{\partial x} \\ E_y = -\frac{\partial V}{\partial y} \\ E_z = -\frac{\partial V}{\partial z} \end{cases}$	$\begin{cases} E_r = -\frac{\partial V}{\partial r} \\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \end{cases}$	$\begin{cases} E_\rho = -\frac{\partial V}{\partial \rho} \\ E_\theta = -\frac{1}{\rho}\frac{\partial V}{\partial y} \\ E_z = -\frac{\partial V}{\partial z} \end{cases}$	$\begin{cases} E_r = -\frac{\partial V}{\partial r} \\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} \\ E_\varphi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi} \end{cases}$

Remarks:

1- The potential V decreases along the field line.



2- The sign (-) indicates that the field $\vec{E}$ is directed towards low potentials.

3- The field lines are perpendicular to the equipotential lines:

$$V = C^t \;\Rightarrow\; dV = 0 \;\Rightarrow\; \vec{E}.\vec{dl} = 0 \;\Longrightarrow\; \vec{E} \perp \vec{dl}$$

5.4. Electrostatic potential created by a set of charges

According to the principle of superposition, the potential V_M generated by "n" charges at point "M" is:

$V_M = V_1 + V_2 + V_3 + \cdots \ldots \ldots V_n$

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$$V_M = \sum_{i=1}^{n} V_i = \sum_{i=1}^{n} \frac{kq_i}{r_i}$$

5.5. Electrostatic potential created by a continuous distribution of charges

$$V_M = \int dV$$

$$V_M = \int \frac{kdq}{r}$$

5.6. Rotational of the electrostatic field $\vec{E}$

$$\overrightarrow{rot}\,\vec{E} = \vec{\nabla} \wedge \vec{E} = \begin{vmatrix} \vec{i} & -\vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

If the field $\vec{E}$ derives from a potential V: $\quad \vec{E} = -\overrightarrow{grad}\,V$

$$\Rightarrow \overrightarrow{rot}\,\vec{E} = -\overrightarrow{rot}\left(\overrightarrow{grad}\,V\right) = \vec{0}$$

Therefore; if $\overrightarrow{rot}\,\vec{E} = \vec{0}$, the field is irrotational

6. Electric dipole

6.1. Definition

The electric dipole is a system of two identical charges with different signs. We call $\vec{P}$ the dipole moment which is always directed from the negative charge $(-)$ to the positive charge $(+)$. (See **Table.3**). We write:

$$\vec{\boldsymbol{P}} = \boldsymbol{q}\,\vec{\boldsymbol{a}}$$

Table.3. Experimental dipole moment of some molecules.

Molecules	H_2O	HCl	CO_2	C_2H_5OH
$\boldsymbol{P}$ $(\boldsymbol{D})$	1.855	1.109	0	1.69

$1\ D\ (Debye) = 3.33564 \times 10^{-30}\ C.m\ (SI\ unit)$

Dipole moment of certain polar molecules

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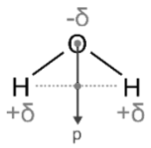

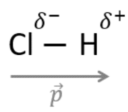

6.2. The field $\vec{E}$ and the potential V of an electric dipole

a) The electrostatic potential V created by two charges q_A and q_B at point M

Principle ofsuperposition: $V_M = V_A + V_B$

$$V_M = \frac{kq_A}{r_1} + \frac{kq_B}{r_2}$$

$$V_M = -\frac{kq}{r_1} + \frac{kq}{r_2} = kq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$V_M = kq\left(\frac{r_1 - r_2}{r_1 r_2}\right)$$

As long as $r >>> a$, we can consider:

$$\begin{cases} r_1 r_2 = r^2 \\ r_1 - r_2 = a\cos\theta \Rightarrow V_M = \frac{kqa\cos\theta}{r^2} \\ \vec{P} = q\,\vec{a} \end{cases}$$

$$\Rightarrow \boldsymbol{V_M = \frac{k\,P\cos\theta}{r^2}}$$

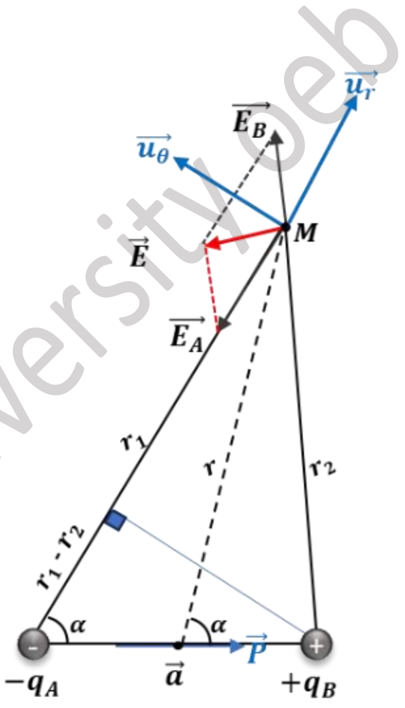

Figure 6. Electric dipole

b) The electrostatic field E_M created by two charges q_A and q_B at point M

We use the relation: $\vec{E} = -\overrightarrow{grad}\,V$ in the polar coordinate system:

$$\begin{cases} E_r = -\frac{\partial V}{\partial r} = \frac{2k\,P\cos\theta}{r^3} \\ E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{k\,P\sin\theta}{r^3} \end{cases};\quad E_M = \sqrt{{E_r}^2 + {E_\theta}^2}$$

$$\boldsymbol{E_M = \frac{k\,P}{r^3}\sqrt{1 + 3\,\cos^2\theta}}$$

Particular cases:

$$if\ :\ \theta = 0 \Rightarrow \begin{cases} E_r = \frac{2k\,P}{r^3} \\ E_\theta = 0 \end{cases} \quad ; \quad if : \theta = \frac{\pi}{2} \Rightarrow \begin{cases} E_r = 0 \\ E_\theta = \frac{k\,P}{r^3} \end{cases}$$

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6.3. Electric dipole in an external electric field $\overrightarrow{E_{ext}}$

When we place an electric dipole in an external field $\overrightarrow{E_{ext}}$, the dipole is subjected to electrostatic forces applied to its charges. These forces (equal and opposite) cause a couple moment $\vec{L}$ defined by:

$$\vec{L} = \vec{P} \wedge \overrightarrow{E_{ext}}$$

$$L = P\, E_{ext} \sin\alpha$$

Knowing that: $\begin{cases} \overrightarrow{F_A} = -q\, \overrightarrow{E_{ext}} \\ \overrightarrow{F_B} = +q\, \overrightarrow{E_{ext}} \end{cases} \Rightarrow \overrightarrow{F_A} + \overrightarrow{F_B} \neq \vec{0}$

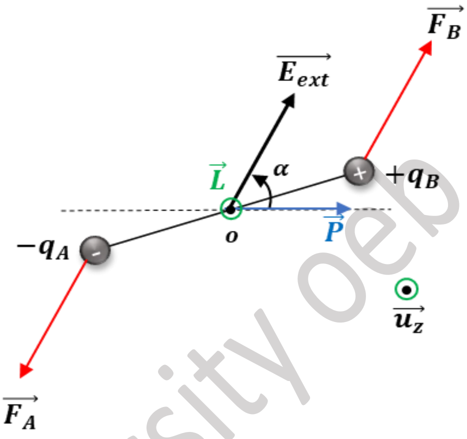

Figure 7. Electric dipole in an external electric field

6.4. Potential energy E_p of an electric dipole located in an external field.

we have: $E_p = q\,V \;\Rightarrow\; dE_p = q\,dV \;\Rightarrow\; dE_p = q\left(-\overrightarrow{E_{ext}} \cdot \overrightarrow{dl}\right) \;\Rightarrow E_p = -\,q\overrightarrow{E_{ext}} \cdot \vec{a}$

$$\boldsymbol{E_p = -\,\overrightarrow{E_{ext}} \cdot \vec{P}}$$

$$\begin{cases} if\ \alpha = 0 \quad \Rightarrow E_p = -E_{ext}.P \quad \Rightarrow E_{p,min}(stable) \\ if\ \alpha = \pi \quad \Rightarrow E_p = +E_{ext}.P \quad \Rightarrow E_{p,max}(unstable) \end{cases}$$

6.5. Field lines and equipotential surfaces

Equipotentials are closed lines that surround the charge and are perpendicular to the field lines.

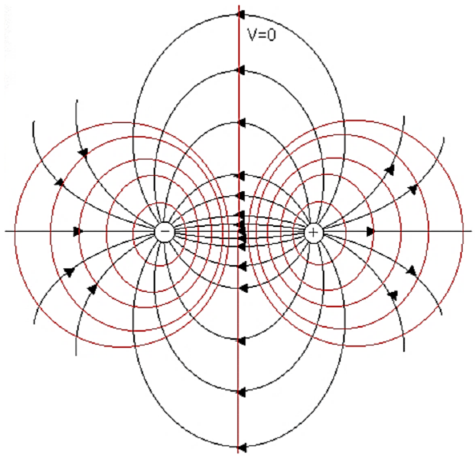

Figure 8. Field lines and equipotential surfaces

7. Circulation of the electric field

Consider a region of space where an electrostatic field $\vec{E}$ reigns, any charge q_0 moving in this field is subject to a force $\overrightarrow{F_e}$:

$$\overrightarrow{F_e} = q_0\vec{E}$$

The work done by $\overrightarrow{F_e}$, for the charge q_0 to move from A to B is given by:

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$$dW = \vec{F_e}.\vec{dl}$$

$$W_{AB} = \int_A^B dW = \int_A^B \vec{F_e}.\vec{dl}$$

$$W_{AB} = q_0 \int_A^B \vec{E}.\vec{dl}$$

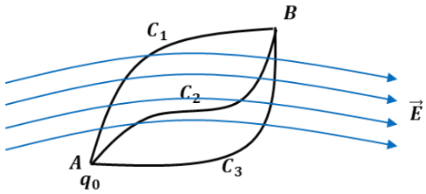
7.1. Definition

The integral $\int_A^B \vec{E}.\vec{dl}$ is called the circulation of the electric field along the curve $(A \rightarrow B)$, denoted "C", we write:

$$dC = \vec{E}.\vec{dl}$$

$$\boldsymbol{C = \int_A^B \vec{E}.\vec{dl}}$$

Remarks:



i. The circulation of the field is conservative, i.e.:

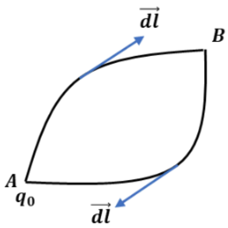
$$C_1 = C_2 = C_3$$

ii. The circulation of the electric field is equal to the difference in potential

$$dC = \vec{E}.\vec{dl} = -dV$$

$$\int_A^B dC = -\int_A^B dV$$

$$C = V_A - V_B$$



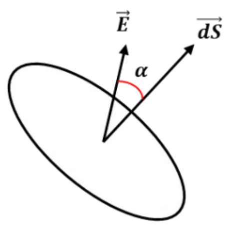
iii. The circulation of the electric field C along a closed curve is zero.

$$C = \oint \vec{E}.\vec{dl} = 0$$

We therefore say that the field E is irrotational ($\overrightarrow{rot}\,\vec{E} = \vec{0}$)

8. Flux of the electrostatic field $\emptyset$

8.1. Elementary surface vector $\vec{dS}$



$\vec{dS}$ is an elementary surface vector; it is always normal to the surface and directed towards the exterior:

$$\vec{dS} = dS\,\vec{n}$$

$\vec{n}$: normal unit vector

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8.2. Flux of the electrostatic field ∅

We call the flux of the electrostatic field ∅ across the surface the following quantity:

$$\boldsymbol{d\emptyset = \vec{E}.\overrightarrow{dS}}$$

$$\emptyset = \iint \vec{E}.\overrightarrow{dS}$$

$$\emptyset = \iint E.dS.\cos\alpha$$

The unit of ∅ is the weber $[Wb]$

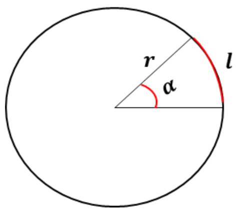
- $if: \alpha = \frac{\pi}{2} \Rightarrow \emptyset = 0$
- $if: \alpha > \frac{\pi}{2} \Rightarrow \emptyset < 0$
- $if: 0 < \alpha < \frac{\pi}{2} \Rightarrow \emptyset > 0$

8.3. Definition of solid angle *Ω*

a. Plane angle *α*

In the plane (2D), we can define the elementary plane angle $d\alpha$, by:

$$d\alpha = tg\,\alpha = \frac{\overrightarrow{AB}}{r} = \frac{dl}{r}$$

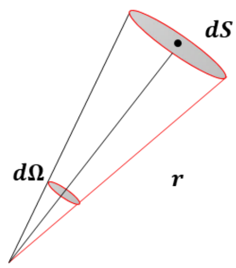


$$d\alpha = \frac{dl}{r}$$

b. Solid angle *Ω*

In space (3D), elementary solid angle $d\Omega$ is the space contained in an elementary conic surface dS of radius r. We write:

$$d\Omega = \frac{dS}{r^2}$$



$$\Omega = \frac{S}{r^2}$$

In spherical coordinate system: $dS = r^2 \sin\theta\, d\theta\, d\varphi$

$$dS = r^2 \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$d\Omega = \frac{dS}{r^2}$$

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$\Omega = 4\pi \quad (sr)$: Steradian

8.4. Relation between the flux ϕ and the solid angle Ω

We have:

$$d\emptyset = \vec{E}.\overrightarrow{dS} \qquad (\vec{E} \parallel \overrightarrow{dS})$$

$$d\emptyset = E\ dS = Ed\Omega\,\mathrm{r}^2$$

$$d\emptyset = kqd\Omega$$

Example: the flux created by $(+q)$ through space $(\Omega = 4\pi)$ is equal to:

$$d\emptyset = kqd\Omega$$

$$\emptyset = kq\Omega$$

$$\boldsymbol{d\emptyset = \frac{q}{\varepsilon_0}}$$

9. Gauss's theorem

Let "q" be a positive electric charge, it produces a radial electric field, directed outwards, where:

$$E = \frac{kq}{r^2}$$

To calculate the flux $\emptyset$ we must choose a spherical surface of radius r as a closed imaginary surface, called the Gaussian surface "$\boldsymbol{S_G}$".

$$d\emptyset = \vec{E}.\overrightarrow{dS}$$

$$d\emptyset = E.dS.\cos\alpha$$

$$\emptyset = E.S \quad : \quad S = 4\pi r^2; \qquad E = \frac{kq}{r^2}$$

$$\emptyset = \frac{q}{\varepsilon_0}$$

Note : $\forall r \quad : \quad \emptyset = \frac{q}{\varepsilon_0} \quad (the\ flux\ does\ not\ depend\ on\ r)$

Generalization:

If we consider $''n''$ electric charges whatever their signs, the elementary electric flux $''\emptyset''$ across the surface is given by:

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$$\emptyset = \frac{\sum q_i}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

9.1. Gauss's theorem statement

The flux of an electric field through a closed surface is equal to the algebraic sum of the charges found within the volume limited by this surface, divided by the vacuum permittivity:

$$\emptyset = \oiint \vec{\boldsymbol{E}}.\overrightarrow{\boldsymbol{dS}_G} = \frac{\sum \boldsymbol{q_{int}}}{\boldsymbol{\varepsilon_0}} = \frac{\boldsymbol{Q_{int}}}{\boldsymbol{\varepsilon_0}}$$

9.2. Conditions for applying Gauss's theorem

1) In practice, Gauss's theorem allows us to calculate the field "E" for a symmetric distribution of charges (i.e., a system with a high degree of symmetry).
2) Gauss's theorem allows us to calculate the field "E" in all points in space.
3) When we use the theorem, we must define an imaginary and closed surface on which the field $\vec{E}$ is constant and radial.
4) The chosen surface must depend on the symmetry of the problem (distribution of field $\vec{E}$).

10. Applications on Gauss's theorem

10.1. An infinite wire uniformly charged by a linear charge density ($\boldsymbol{\lambda}$)

By a symmetry, the field $\vec{\boldsymbol{E}}$ at any point ($\boldsymbol{r}$) in space is radial, perpendicular to the axis of the wire, and directed towards the exterior.

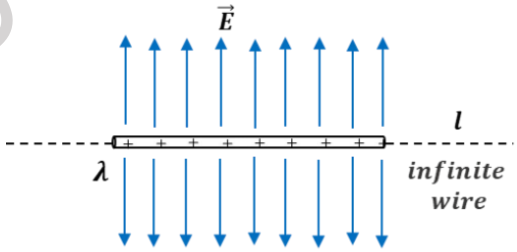

Figure 9. An infinite wire uniformly charged by a linear charge density

a) The total charge $\boldsymbol{Q_T}$

$Q_T = \int dq = \int_0^l \lambda \, dl = \lambda l \qquad \Rightarrow \boldsymbol{Q_T = \lambda l}$

b) The field $\boldsymbol{E}$ in all points of space

According to Gauss's theorem: $\boldsymbol{\phi} = \oiint \vec{\boldsymbol{E}} \, . \, \overrightarrow{\boldsymbol{dS}_G} = \frac{\boldsymbol{Q_{int}}}{\boldsymbol{\varepsilon_0}}$

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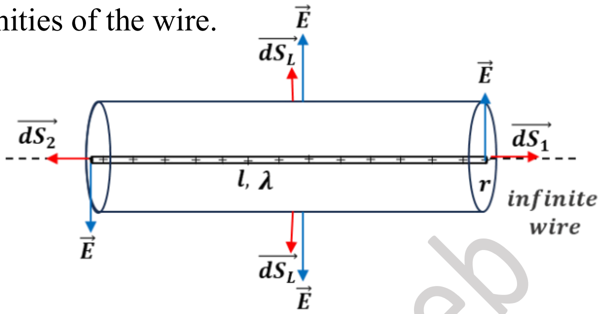
The Gaussian surface which is suitable here is a surface of a cylinder (r, h), where:

- The axis of the cylinder coincides with the wire.
- The two bases of the cylinder cover the extremities of the wire.



According to the diagram, there are three surfaces:

1- Base surface S_1 (disk) of $\overrightarrow{dS_1}$

2- Base surface S_2 (disk) of $\overrightarrow{dS_2}$

3- Lateral surface S_L (rectangular) of $\overrightarrow{dS_L}$

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

$$\text{Where:} \begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = 0; & (\vec{E} \perp \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \iint E.dS_2.\cos\theta = 0; & (\vec{E} \perp \overrightarrow{dS_2}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_L} = \iint E.dS_L.\cos\theta = E.S_L; & (\vec{E} \parallel \overrightarrow{dS_L}) \end{cases}$$

$$\emptyset_T = E.S_L = \frac{Q}{\varepsilon_0}$$

$$\text{Knowing that:} \begin{cases} S_L = 2\pi r h \\ Q_T = \lambda\, l \\ h = l \end{cases}$$

$$\boldsymbol{E(r) = \frac{\lambda}{2\pi\, \varepsilon_0\, r}}$$

10.2. An infinite plane uniformly charged by a surface charge density (σ)

By a symmetry, the field E at all points in space is: radial, perpendicular to the plane and directed towards the exterior.

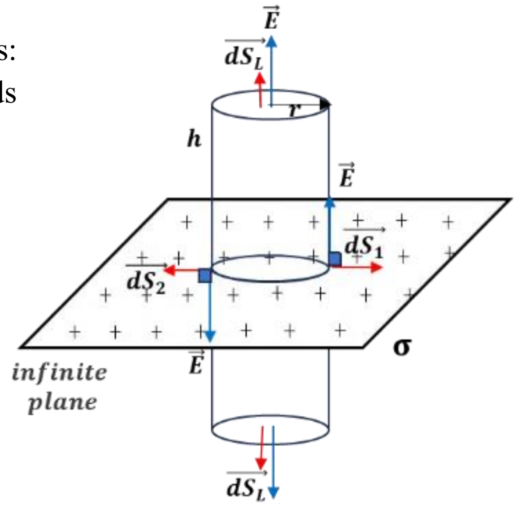


Figure 10. An infinite plane uniformly charged by a surface charge density

c) The total charge Q_T

$$Q_T = \int dq = \int \sigma\, dS = \sigma\, S \qquad \Rightarrow \boldsymbol{Q_T = \sigma\, S}$$

d) The field E at all points in the space

According to Gauss's theorem:

$$\boldsymbol{\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}}$$

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The most suitable Gauss surface $\boldsymbol{S_G}$ which gives a high degree of symmetry is a cylinder of (r, h).

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

Where: $$\begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = E.S; & (\vec{E} \parallel \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \iint E.dS_2.\cos\theta = E.S; & (\vec{E} \parallel \overrightarrow{dS_2}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_L} = \iint E.dS_L.\cos 90° = 0; & (\vec{E} \perp \overrightarrow{dS_L}) \end{cases}$$

Therefore:

$$\emptyset_T = 2E.S = \frac{Q_{int}}{\varepsilon_0}$$

Q_{int} : here presents the charge of the plane inside the cylinder (a disk of radius r).

$$\begin{cases} Q_{int} = \sigma\,\pi\,r^2 \\ S = \pi\,r^2 \end{cases}$$

$$\boldsymbol{E(r) = \frac{\sigma}{2\,\varepsilon_0}}$$

We see that the field created by an infinite charged plane is constant at every point in space.

10.3. A full sphere uniformly charged with a volume charge density ($\boldsymbol{\rho}$)

A full sphere with center O and radius R is uniformly charged by a positive and constant volume charge density ρ ($\rho > 0$).

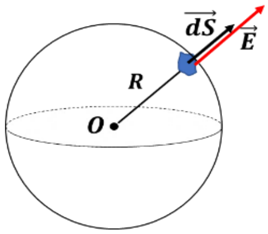

Figure 11. A full sphere uniformly charged with a volume charge density

a) The total charge $\boldsymbol{Q}$ of the sphere.

$$Q_T = \int dq = \iiint \rho dV = \rho V \rightarrow \boldsymbol{Q_T = \frac{4}{3}\pi R^3 \rho}$$

b) The expression of the electric field $\boldsymbol{E(r)}$ at any point $\boldsymbol{M}$ in space.

we use Gauss' theorem:

$$\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}$$

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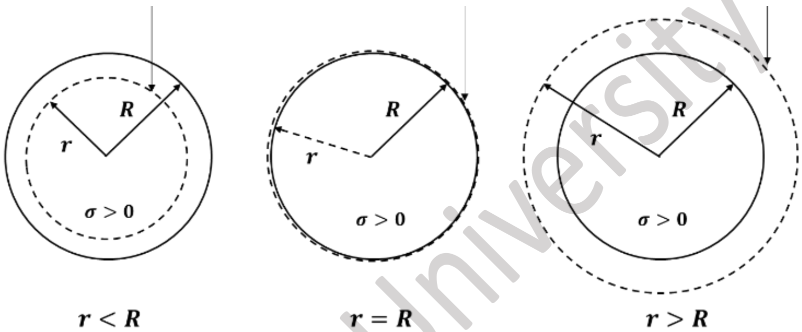
As the electrostatic field is radial, $\vec{E}$ and $\overrightarrow{dS}$ are collinear ($\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°$), we can then write:

$$\Rightarrow \oiint E.dS_G \cos\theta = \frac{Q_{int}}{\varepsilon_0}; (\vec{E} \parallel \overrightarrow{dS} \Rightarrow \theta = 0°)$$

$$\Rightarrow E.S_G = \frac{Q_{int}}{\varepsilon_0} \dots (*)$$

Due to spherical symmetry, the suitable closed Gauss surface S_G here is a surface of sphere with center O and radius r. ($S_G = 4\pi r^2$).

To determine the field at any point M in space, we distinguish 03-regions:



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Region − 1 $\boldsymbol{r < R}$: $\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi r^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow \boldsymbol{E_1 = \frac{\rho}{3\,\varepsilon_0} r}$

Region − 2 $\boldsymbol{r = R}$: $\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi R^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow \boldsymbol{E_2 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2}}\ r = R \Rightarrow \boldsymbol{E_2 = \frac{\rho R}{3\,\varepsilon_0}}$

Region − 3 $\mathbf{r > R}$: $\begin{cases} S_G = 4\pi r^2 \\ Q_{int} = \frac{4}{3}\pi R^3 \rho \end{cases} \Rightarrow l'eq(*) \Rightarrow E_3 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2} \Rightarrow \boldsymbol{E_3 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2}}$

Remarks:

We can obtain the expression of the field $\boldsymbol{E_2}$ on the surface of the sphere ($r = R$), by replacing $\boldsymbol{r}$ with R in the expression of $\boldsymbol{E_3}$ found outside the sphere or in $\boldsymbol{E_1}$ found inside the sphere.

c) The electric potential $V(r)$

To determine the potential, we use the relation: $\vec{E} = -\overrightarrow{grad}\, V$

E is radial $\Rightarrow E = -\frac{dV}{dr} \Rightarrow V = -\int E\, dr$

Region-1 $\boldsymbol{r < R}$ **:** $V_3 = -\int E_3\, dr = V = -\int \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r^2}\, dr = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r} + C_3$

To determine C_3, we use the limit conditions of the potential: $\lim\limits_{r \to \infty} V = 0$

$$\Rightarrow \lim_{r \to \infty} V_3 = 0 + C_3 = 0 \Rightarrow \boldsymbol{C_3 = 0} \Rightarrow \boldsymbol{V_3 = \frac{\rho}{3\,\varepsilon_0}\frac{R^3}{r}}$$

Region-2 $\boldsymbol{r = R}$ **:** $V_2 = -\int E_2\, dr = -\int \frac{\rho R}{3\,\varepsilon_0}\, dr = -\frac{\rho R}{3\,\varepsilon_0} r + C_2$

To determine C_2 , we use the condition of continuity of the potential at the interface $(r = R)$:

$$V_3(r = R) = V_2(r = R)$$

$$\begin{cases} V_2(R) = -\dfrac{\rho R^2}{3\,\varepsilon_0} + C_2 \\ V_3(R) = \dfrac{\rho R^2}{3\,\varepsilon_0} \end{cases} \Rightarrow C_2 = 2\frac{\rho R^2}{3\,\varepsilon_0} \Rightarrow \boldsymbol{V_2 = \frac{\rho R^2}{3\,\varepsilon_0}}$$

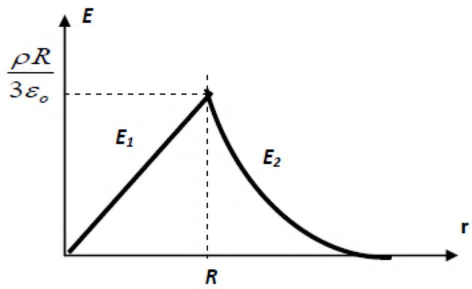
Region-2 $\mathbf{r > R}$ **:** $V_1 = -\int E_1\, dr = -\int \frac{\rho}{3\,\varepsilon_0} r\, dr = -\frac{\rho}{3\,\varepsilon_0}\frac{r^2}{2} + C_2$

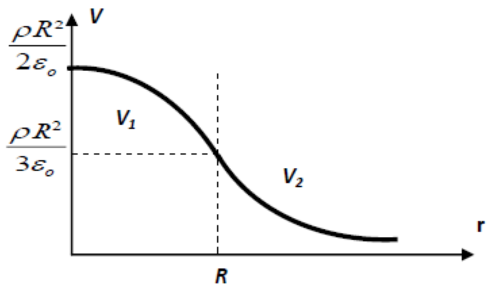
To determine C_1, we use the condition of continuity of the potential at the interface $(r = R)$:

$$V_1(R) = V_2(R)$$

$$\begin{cases} V_2(R) = \dfrac{\rho R^2}{3\,\varepsilon_0} \\ V_1(R) = -\dfrac{\rho}{3\,\varepsilon_0}\dfrac{R^2}{2} + C_2 \end{cases} \Rightarrow C_1 = \frac{\rho R^2}{2\,\varepsilon_0} \Rightarrow \boldsymbol{V_1 = \frac{\rho R^2}{3\,\varepsilon_0}\left[\frac{3}{2} - \frac{r^2}{2R^2}\right]}$$

d) The variation curve of $\boldsymbol{E(r)}$ and $\boldsymbol{V(r)}$.





11. Exercises

An electrostatic dipole is made up of two identical charges with different signs: $(q_A = -q)$ and $(q_B = +q)$ separated by a small distance $\boldsymbol{a}$, where $(r >> a)$. The dipole moment is defined by: $\vec{P} = q\,\overrightarrow{AB}$.

1- Find the potential V_M of this dipole at the point M distant r from its center.
2- Deduce the components E_r and E_θ of the electrostatic field E and its module $\|\vec{E}\|$ in the polar coordinate system.
3- We now assume that the dipole is subject to a uniform external field $\overrightarrow{E_{ext}}$. Give the expression for the electrostatic moment $\vec{L}$ and the potential energy of the dipole E_p.

(*The solution can be found in the* course)

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Chapter 2

Conductors in equilibrium

Summary

1. Definitions

- An electrical conductor is a body in which the electric charges can freely move when the voltage is applied.
- We say that a conductor is in electrostatic equilibrium if it charges are at rest.

2. Properties of a conductor in equilibrium

1) The electrostatic field inside the conductor in equilibrium is null ($\overrightarrow{\boldsymbol{E_{int}}} = \vec{\mathbf{0}}$)

As long as the electric charges inside the conductor in equilibrium are at rest, they are therefore not subject to any force:

$$\vec{F_e} = \vec{0}$$

$$\vec{F_e} = q\vec{E} = \vec{0} \quad \Rightarrow \quad \vec{E} = \vec{0}$$

2) The conductor in equilibrium constitutes an equipotential volume ($\boldsymbol{i.e.}\ \boldsymbol{V_{int}} = \boldsymbol{C^t}$).

We have: $dV = -\vec{E}.\overrightarrow{dl} = 0 \Rightarrow V = C^t$

This implies that the potential is constant at each point of the conductor in equilibrium (inside and at the surface).

3) The charge inside the conductor in equilibrium is null ($\boldsymbol{Q_{int}} = \mathbf{0}$).

Because: $Q^- + Q^+ = 0$

If the conductor is charged the added charges are located on the surface.

4) If the conductor is charged, the field vector $\vec{E}$ is perpendicular to the external surface of the conductor in equilibrium.

As long as $V = C^t$*,the external surface of the conductor is an equipotential surface, which proves that the field is perpendicular to the surface of the conductor.*

$$dV = -\vec{E}.\overrightarrow{dl} = 0 \quad \Longrightarrow \vec{E} \perp \overrightarrow{dl}$$

We can summarize the properties of the conductor in equilibrium in the following diagram:

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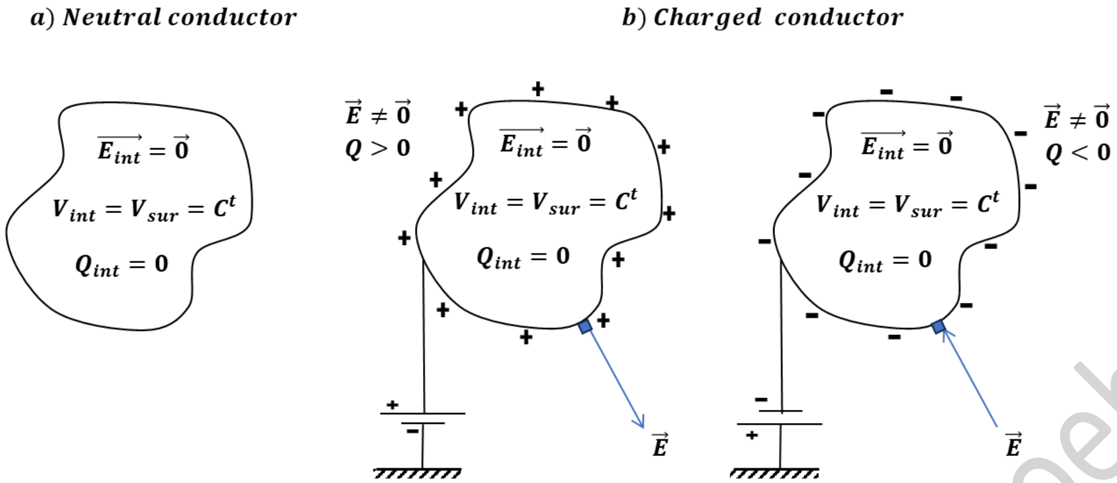

Figure 1. Summary of properties of a conductor in equilibrium (neutral and charged).

3. Coulomb's theorem: The electric field near a conductor

For a charged conductor at electrostatic equilibrium in a vacuum, the electric field just outside the surface:

- Is perpendicular to the surface,
- Has an intensity given by:

$$E = \frac{\sigma}{\varepsilon_0}$$

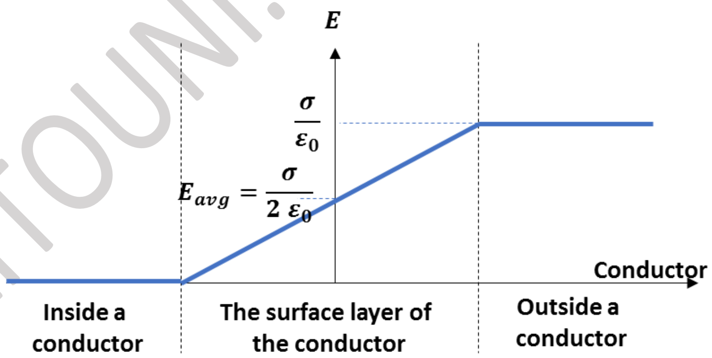

Figure 2. Change in electric field strength on either side of the conductor's surface.

Demonstration of Coulomb's theorem

We apply Gauss's Theorem:

$$\emptyset = \oiint E.dS_G = \frac{Q_{int}}{\varepsilon_0}$$

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S_G: The Gaussian surface which is suitable here is a surface of a cylinder (r, h); where:
$S_G: (S_1, S_2, S_L)$

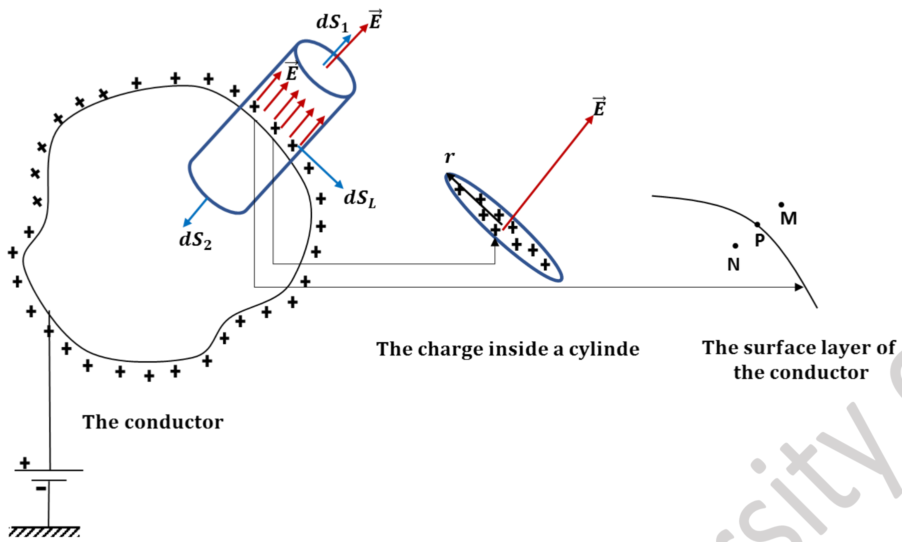

Figure 3. Demonstration of Coulomb's theorem

The total flux through all surfaces constituting the Gaussian cylinder is given by:

$$\emptyset_T = \emptyset_1 + \emptyset_2 + \emptyset_L$$

Where:

$$\begin{cases} \emptyset_1 = \iint \vec{E}.\overrightarrow{dS_1} = \iint E.dS_1.\cos\theta = \boldsymbol{ES}; & (\vec{E} \parallel \overrightarrow{dS_1}) \\ \emptyset_2 = \iint \vec{E}.\overrightarrow{dS_2} = \mathbf{0}; & (\overrightarrow{E_{int}} = \vec{0}) \\ \emptyset_L = \iint \vec{E}.\overrightarrow{dS_{L,int}} + \iint \vec{E}.\overrightarrow{dS_{L,ext}} = \mathbf{0}; & (\vec{E} \perp \overrightarrow{dS_{L,ext}}) \ and \ (\overrightarrow{E_{int}} = \vec{0}) \end{cases}$$

Therefore:

$$\emptyset_T = E.S = \frac{Q_{int}}{\varepsilon_0}$$

Knowing that: $\begin{cases} S = \pi r^2 \\ Q_{int} = \pi r^2 \sigma \end{cases}$

The electric field near a conductor is given by:

$$E = \frac{\sigma}{\varepsilon_0}$$
$$\vec{E} = \frac{\sigma}{\varepsilon_0}\ \vec{n}$$

$\vec{\boldsymbol{n}}$: Normal unit vector.

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- This formula describes the electric field just outside the surface of a conductor (at point M).
- Inside the conductor (at point N), the electric field is zero.
- At the surface itself (very close to it, point P), the field is often represented by an average value E_{avg}, given by:

$$E_{avg} = \frac{E_N + E_M}{2}$$

$$\boldsymbol{E = \frac{\sigma}{2\,\varepsilon_0}}$$

4. Electrostatic pressure

The charges located on the surface of the conductor are subjected to repulsive forces with other charges, which leads to appear a pressure on the surface of the conductor and thus:

$$p = \frac{F_e}{S} = \frac{QE}{S} = \sigma\, E$$

$$\boldsymbol{p = \frac{\sigma^2}{2\,\varepsilon_0}}$$

The unit of electrostatic pressure is the **Pascal** ($\boldsymbol{Pa}$).

5. The power of pointed surfaces

Experimentally, the electric charges have a tendency to accumulate on pointed surfaces with small radius of curvature. The surface charge density is then important in the pointed zone, the same reasoning for the electric field in the vicinity of these pointed surfaces.

When the σ becomes very large, charges can escape from the conductor, this is called field emission.

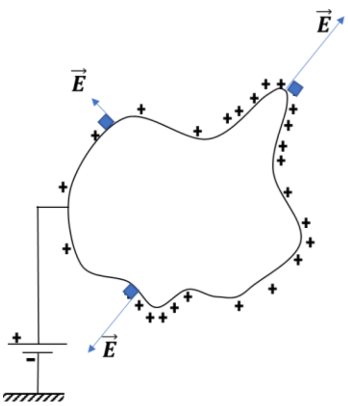

Figure 4. Pointed surfaces

Demonstration

Consider two spherical conductors (S_1, S_2), each with its own characteristics, connected by a wire.

The whole constituting a single conductor with a new state of equilibrium, where: $V = C^t$

$$V_1 = V_2$$

$$\frac{kQ_1}{R_1} = \frac{kQ_2}{R_2}$$

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$$\Rightarrow \frac{k\sigma_1 S_1}{R_1} = \frac{k\sigma_2 S_2}{R_2}$$

$$\Rightarrow \sigma_1 R_1 = \sigma_2 R_2$$

$$\Rightarrow \boldsymbol{\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}}$$

We have: $\boldsymbol{R_2 > R_1} \quad \Rightarrow \quad \boldsymbol{\sigma_2 < \sigma_1}$

This proves that charges tend to accumulate on pointed surfaces.

In life, we find the applications of pointed surfaces in the electrical discharge on the wings of planes, lightning arresters (antifoudre), lightning rods (paratonnerres), etc.

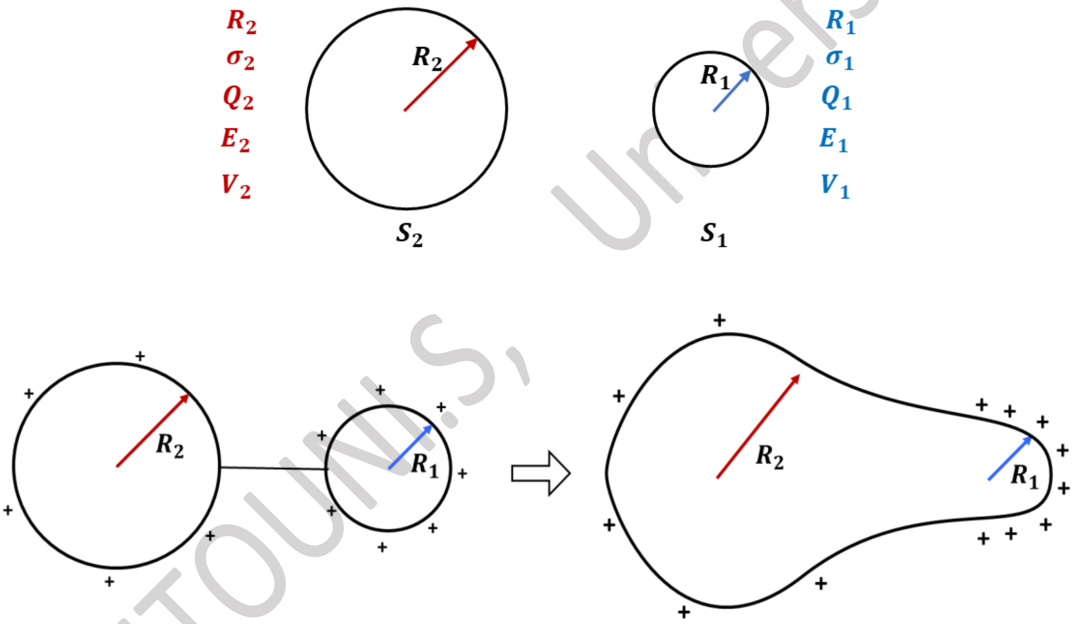

Figure 5. System of two connected spheres.

6. Electric Capacitance of a conductor

The electric capacitance "C" of an insulated conductor is the ratio between its charge "Q" and its potential "V":

$$C = \frac{Q}{V}$$

For the case of a spherical conductor placed in a vacuum, the capacitance is given by:

$$C = \frac{Q}{V} \quad and \quad V = \frac{kQ}{R}$$

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Therefore: $\boldsymbol{C = 4\,\pi\,\varepsilon_0\,R}$

If the insulating medium surrounding the spherical conductor is other than vacuum, its capacitance is then given by:

$$\boldsymbol{C = 4\,\pi\,\varepsilon\,R}$$

$\boldsymbol{\varepsilon}$: Dielectric permittivity of the medium.

For the case of two conductors carrying two charges $+Q$ and $-Q$, the capacitance of the system is given by:

$$C = \frac{Q}{U};\ U = V_1 - V_2$$

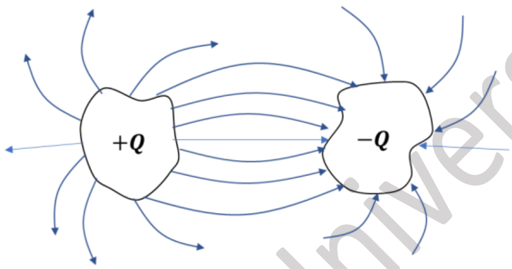

Figure 6. Capacitance of two conductors

Unit of capacitance C is the Farad (F).

$$[C] = \frac{[Q]}{[V]} = C/V = F$$

- $1\ mF = 10^{-3}\ F$
- $1\ \mu F = 10^{-6}\ F$
- $1\ nF = 10^{-9}\ F$
- $1\ pF = 10^{-12}\ F$

Example 1:

Calculate the capacitance C of the earth. We give: $R = 6400\ km;$ $\varepsilon_0 = 8.85 \times 10^{-12}\ (SI)$

Sol : $C = 710\ \mu\ F$

Example 2 :

Calculate the radius of a spherical conductor with capacitance $C = 1\ \mu F$, placed in a vacuum. **Sol** : $R = 900\ km$.

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7. Electrostatic Potential energy of a charged conductor

The electrostatic potential energy E_p stored in a conductor presents the work necessary to charge this conductor:

$$dE_p = dW = d(Vq) = Vdq = \frac{q}{C}dq$$

$$dE_p = \frac{1}{C}\int_0^Q qdq$$

$$\boldsymbol{E_p = \frac{1}{2}\frac{Q^2}{C};\ E_p = \frac{1}{2}CV^2;\ E_p = \frac{1}{2}QV}$$

8. Influence phenomena between conductors

What happens when we place a neutral and equilibrium conductor in an external electric field?

***Polarization*.**

Why: When we put an equilibrium conductor in an external electric field $\overrightarrow{E_{ext}}$, the free charges (at rest) move: the (+) charges move in the direction of $\overrightarrow{E_{ext}}$ while the (-) charges move in the opposite direction. It then occurs **a polarization of the conductor**.

8.1. Partial influence

We place the charge $(+q)$ in the vicinity of the neutral conductor (**figure** below). This means that all field lines starting from the punctual charge $(+q)$ do not reach the entire conductor, and this is what characterizes the partial influence.

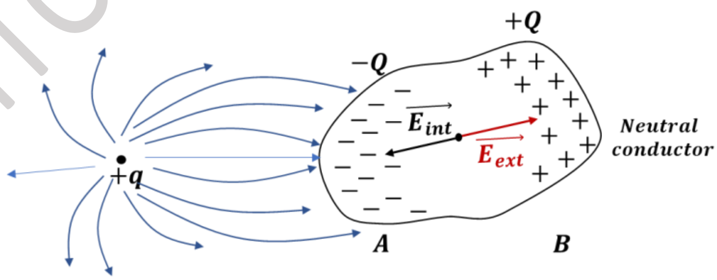

Figure 7. Partial influence

The charge (+q) produces an external field $\overrightarrow{E_{ext}}$, which forces the free electrons of the conductor to move towards face "A", this region becomes negatively charged. The electrons leaving the face "B" create a region of positive charge.

Then, the charges $+Q$ and $-Q$ in turn, produce an internal field $\overrightarrow{E_{int}}$ which opposes the direction of the external field $\overrightarrow{E_{ext}}$.

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When $\overrightarrow{E_{ext}} = \overrightarrow{E_{int}}$, the movement of electrons stops and the conductor becomes in electrostatic equilibrium, despite its polarization. Where:

$$\text{Inside a conductor:} \begin{cases} \overrightarrow{E_{ext}} + \overrightarrow{E_{int}} = \vec{0} \\ V = C^t \\ Q_T = 0 \end{cases}$$

Notes:

- If the conductor is connected to the earth (where the potential is zero: V=0), the positive charges (+) move towards earth and the potential of the conductor is canceled (zero).
- Once the conductor is disconnected from the ground, it keeps the charge it gained through induction.

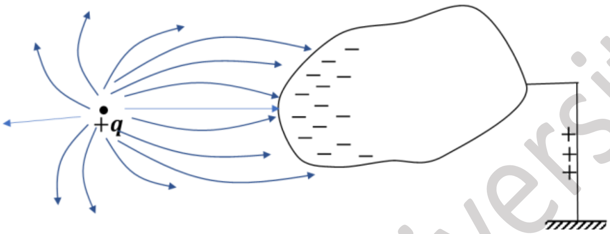

Figure 8. Polarized conductor connected to the earth.

8.2. Total influence

When a conductor (A) completely surrounds another conductor (B), all the lines of the field which are from (B) arrive at the conductor (A). This is what characterizes the total influence.

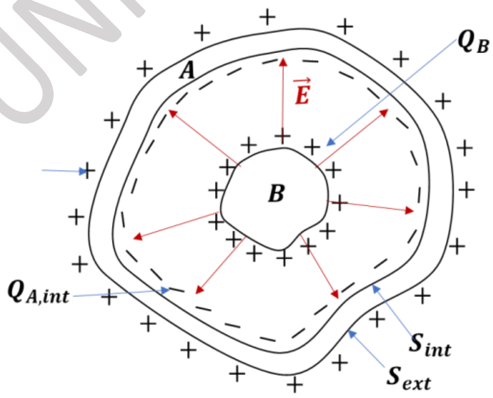

Figure 9. Total influence

a) If A is initially isolated and neutral

Principle of conservation of charge: $Q_{A,int} + Q_{A,ext} = 0$

According to the Theorem of corresponding element:

$$\begin{cases} Q_B = -Q_{A,int} \\ Q_{A,int} = -\,Q_{A,ext} \end{cases} \Rightarrow \boldsymbol{Q_B = Q_{A,ext}}$$

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b) If A is initially isolated and carries a charge Q_0

Principle of conservation of charge remains valid:

$$Q_0 = Q_{A,int} + Q_{A,ext}$$

According to the Theorem of corresponding element:

$$Q_B = -Q_{A,int}$$

$$\Rightarrow \boldsymbol{Q_{A,ext} = Q_B + Q_0}$$

9. Electric capacitors

The capacitor is an electronic component (device), made up of two conductors in total influence and separated by an insulator. Its main property is to store and condense electrical energy by accumulating electric charges on the surfaces of its conductors. The capacitor was originally known as the condenser.

9.1. Planar Capacitor (or Parallel Plate Capacitor)

A planar capacitor is made up of two flat, parallel metal plates separated by a distance ddd. Between the plates is an insulating material called a dielectric, which has a permittivity ε.

$$\boldsymbol{\varepsilon = \varepsilon_0\, \varepsilon_r}$$

The dielectric permittivity of the medium ε is the physical property which describes the response of the medium to the applied electric field.

$\boldsymbol{\varepsilon_0}$: The permittivity of a vacuum or free space, $\boldsymbol{\varepsilon_0 = 8.85 \times 10^{-12} F/m}$.

$\boldsymbol{\varepsilon_r}$: Relative permittivity of the insulator, also known as dielectric constant of an insulator

Table 1. Examples for relative permittivity of the same materials.

Insulator/material	Vacuum	Air	Glass	Polystyrene	Carbon disulfide
Relative permittivity	$\varepsilon_r = 1$	$\varepsilon_r = 1{,}0006$	$\varepsilon_r = 4.7$	$\varepsilon_r = 2.4 - 2.7$	$\varepsilon_r = 2.6$

1) The electrostatic field $\vec{E}$ between the plates

At the surface of a conductor in equilibrium, the electric field has an average value given by:

$$\boldsymbol{E = \frac{\sigma}{2\varepsilon};\ \varepsilon = \varepsilon_0\, \varepsilon_r}$$

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As long as the capacitor is made up of two conductors (plates), each plates produces a field $\vec{E_i}$:

$\vec{E} = \vec{E_1} + \vec{E_2}$; $\Rightarrow \vec{E} = 2\,E\,\vec{\imath} \Rightarrow \vec{E} = \frac{\sigma}{\varepsilon_0\,\varepsilon_r}\,\vec{\imath}$

$\Rightarrow \boldsymbol{\vec{E}} = \dfrac{\boldsymbol{Q}}{\boldsymbol{S\,\varepsilon_0\,\varepsilon_r}}\,\boldsymbol{\vec{\imath}}$; or $\boldsymbol{E} = \dfrac{\boldsymbol{Q}}{\boldsymbol{S\,\varepsilon_0\,\varepsilon_r}}$

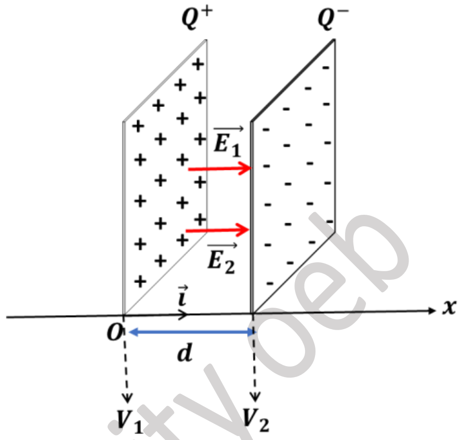

Figure 10. Planar capacitor

2) The potential difference U

We have: $U = V_1 - V_2$

Where $dV = -\vec{E}.\vec{dl} = -E\,dl \Rightarrow \int_{V_1}^{V_2} dV = -\int_0^d E\,dl$

$\Rightarrow \boldsymbol{U = Ed = \dfrac{Q\,d}{S\,\varepsilon_0\,\varepsilon_r}}$

3) The capacitance C

we have : $C = \dfrac{Q}{U} = \dfrac{\boldsymbol{S\,\varepsilon_0\,\varepsilon_r}}{\boldsymbol{d}}$

4) Energy E_p stored in the capacitor

we have: $E_p = \dfrac{1}{2}Q\,U \Rightarrow \boldsymbol{E_p = \dfrac{1}{2}\varepsilon_0\,\varepsilon_r\,E^2\,V}$

where: $V = d.S$ (volume)

5) The volume density of energy can be given by**:**

$$\boldsymbol{\frac{dE_p}{dV} = \frac{1}{2}\varepsilon_0\,\varepsilon_r\,E^2}$$

9.2. Spherical capacitor

The spherical capacitor is made up of two concentric spherical conductors, separated by an insulator.

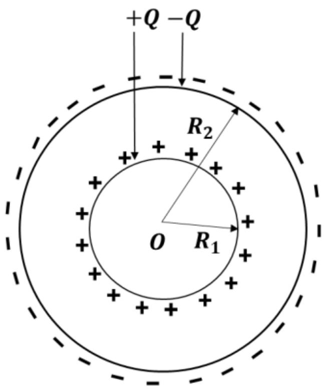

Figure 11. Spherical capacitor

1) The electrostatic field $\vec{E}$ between the spheres

T of Gauss: $\phi = \oiint \vec{E}\,.\,\vec{dS_G} = \frac{Q_{int}}{\varepsilon_0 \varepsilon_r}$

S_G : Gaussian sphere with radius r: $S_G = 4\pi\,r^2$

Region-1: $r < R_1 : Q_{int} = 0 \Rightarrow E_1 = 0$
Region -2: $r > R_2 : Q_{int} = 0 \Rightarrow E_2 = 0$
Region -3: $R_1 < r < R_2 : Q_{int} = +Q \Rightarrow E.4\,\pi\,r^2 = \frac{+Q}{\varepsilon_0 \varepsilon_r}$

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$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{Q}}{4\,\boldsymbol{\pi\varepsilon_0\,\varepsilon_r\,r^2}}$$

2) The potential difference U

We have: $U = V_1 - V_2$ and $\vec{E} = -\overrightarrow{grad}\,V \quad \Rightarrow dV = -E\,dr$

$$\Rightarrow \boldsymbol{U} = \frac{\boldsymbol{Q}}{4\pi\,\varepsilon_0\,\varepsilon_r}\left(\frac{R_1 - R_2}{R_1 R_2}\right)$$

3) Capacitance C

we have : $C = \dfrac{Q}{U}$ $\qquad \Rightarrow \boldsymbol{C} = 4\pi\,\varepsilon_0\,\varepsilon_r\left(\dfrac{R_1R_2}{R_1 - R_2}\right)$

4) Energy E_p

we have : $E_p = \dfrac{1}{2}Q\,U$ $\qquad \Rightarrow \boldsymbol{E_p} = \dfrac{Q^2}{4\pi\,\varepsilon_0\,\varepsilon_r}\left(\dfrac{R_1 - R_2}{R_1R_2}\right)$

9.3. Cylindrical capacitor

The cylindrical capacitor is made up of two coaxial and conductive cylinders, separated by an insulator.

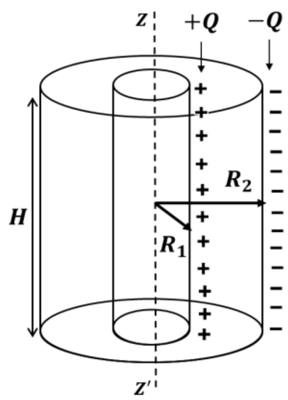

Figure 12. Cylindrical capacitor

1) The electrostatic field $\vec{E}$

T of Gauss: $\phi = \oiint \vec{E}\,.\,\overrightarrow{dS_G} = \frac{Q_{int}}{\varepsilon_0}$

$R_1 < r < R_2 : Q_{int} = +Q \Rightarrow E.2\,\pi\,r\,h = \frac{+Q}{\varepsilon_0}$

$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{Q}}{2\,\boldsymbol{\pi\varepsilon_0\;H\;r}}$$

2) The potential difference U

$U = V_1 - V_2$ et $\vec{E} = -\overrightarrow{grad}\,V \Rightarrow dV = -E\,dr \qquad \Rightarrow \boldsymbol{U} = \frac{Q}{2\pi\,\varepsilon_0 H}\,\boldsymbol{ln}\,\frac{R_2}{R_1}$

3) The capacitance C

we have : $C = \dfrac{Q}{U}$ $\qquad \Rightarrow \boldsymbol{C} = \dfrac{2\pi\,\varepsilon_0 H}{\boldsymbol{ln}\,\dfrac{R_2}{R_1}}$

4) Energy E_p

we have : $E_p = \dfrac{1}{2}Q\,U$ $\qquad \Rightarrow \boldsymbol{E_p} = \dfrac{Q^2}{2\pi\,\varepsilon_0\,H}\,\boldsymbol{ln}\,\dfrac{R_2}{R_1}$

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10. Association of capacitors

10.1. In series

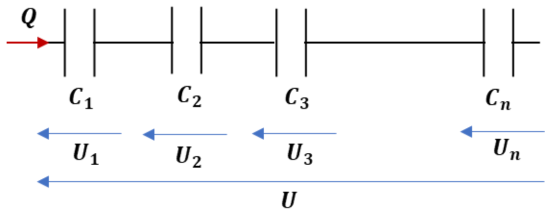


Figure 13. Association of capacitors in series

$$\begin{cases} U = U_1 + U_2 + U_3 \dots \dots \dots U_n \\ U = \dfrac{Q}{C} \end{cases}$$

$$\Rightarrow \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \dots \dots \dots \frac{Q}{C_n}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \dots \dots \dots \frac{1}{C_n}$$

$$\boldsymbol{\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}}$$

10.2. In parallel

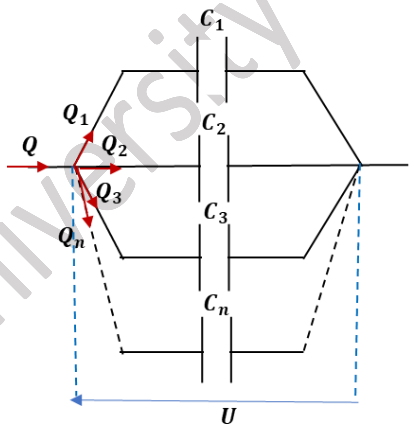


Figure 14. Association of capacitors in parallel

$$\begin{cases} U = U_1 = U_2 = U_n \\ Q = Q_1 + Q_2 + Q_3 \dots \dots \dots Q_n \\ \qquad Q = U.C \end{cases}$$

$$\Rightarrow U.C_{eq} = U.C_1 + U.C_2 + U.C_3 \dots \dots \dots U.C_n$$

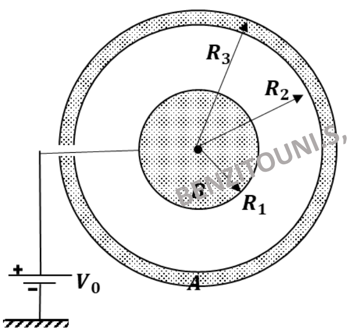
$$\Rightarrow C_{eq} = C_1 + C_2 + C_3 \dots \dots \dots C_n$$

$$\boldsymbol{C_{eq} = \sum_{i=1}^{n} C_i}$$

11. Exercises

Exercise-1

A hollow spherical conductor A, initially neutral, with interior radius $R_2 = 2R$ and exterior radius $R_3 = 4R$ surrounds a second spherical conductor B, with radius $R_1 = R$, brought to a potential V_0 via a generator. (See **figure** below). Conductor B carries a charge Q_0.



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1) What are the charges carried by the interior and exterior surfaces of the conductor A. justify.
2) By applying Gauss's theorem, determine the expression of the electric field E in the following four regions:

$$r < R, \qquad R < r < 2R, \qquad 2R < r < 4R, \qquad r > 4R.$$

3) Considering that V_A is the potential of conductor A and knowing that the electric potential is zero at infinity, determine the expression of the electric potential in the four regions.
4) Deduce the charge Q_0 as a function of R, V_0 and ε_0 .

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Chapter 3

Electrokinetics

Summary

1. Definitions

- Electrokinetics is the study of the movement of free charges in a conductive medium, under the effect of a potential difference applied between two points of this conductor.
- Electrokinetics is the study of the current passing through an electrical circuit in permanent regime.

2. Drift velocity V_D

When a potential difference (voltage) is applied between two points of the conductor, the charges located between these two points are subjected to an electric field $\vec{E}$, they are therefore accelerated under the effect of the electrostatic force $\vec{F_e}$ and oriented towards the opposite direction of the field $\vec{E}$.

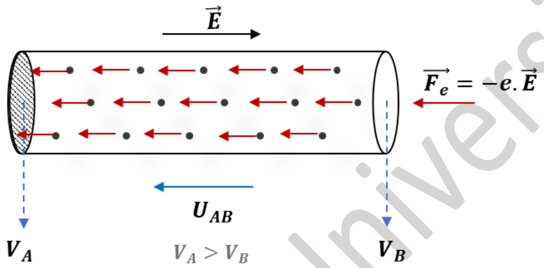

Figure 1. The movement of charges due to applied voltage.

When the charges pass through the conductor, they undergo a successive collision with the electrons strongly linked to the nucleus of the atoms of the conductor, so their velocities tend towards a limited value called the **Drift velocity V_D**.

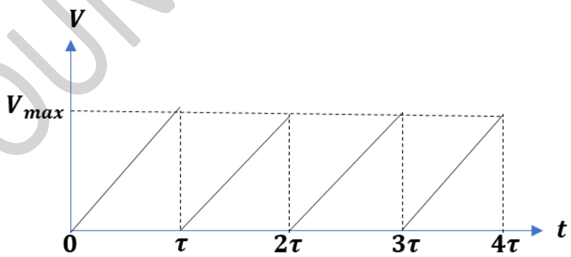

Figure 2. The drift velocity of charges

- τ: is the **mean free time** which presents the average time between two successive collisions (shocks) of the moving charge.
- The average distance separating two successive shocks of the moving charge during a time τ: is called "**Mean free path**".

Examples:

1) In an electric wire, the drift velocity of electrons is approximately in the order of: 1mm/s.
2) In an aqueous solution, the drift velocity of ions is lower than those of free electrons in metals.

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3) In a vacuum, the drift velocity of electrons is approximately $10^4\ km/s$, although it remains small compared to the speed of light: $c = 3 \times 10^5\ km/s$.

The Drift velocity of charges $\boldsymbol{V_D}$ is proportional to the intensity of applied electric field $\boldsymbol{E}$ according to the following relation:

$$V_D = kE$$

K: constant of proportionality, which is called the mobility of charges, denoted µ. We write:

$$\boldsymbol{V_D = \mu E}$$

Under the action of the field $\vec{E}$, the charges (electrons) acquire an acceleration γ. Therefore, according to the 2nd Law of Newton (FPD):

$$F_e = m\gamma$$

$$q.E = m\gamma$$

$$e.E = m_e\gamma = m_e\frac{dV}{dt}$$

$$\int_0^{V_{max}} dV = \frac{e\,E}{m_e}\int_0^{\tau} dt$$

$$\boldsymbol{V_{max} = \frac{e\,\tau}{m_e}E}$$

The Drift velocity V_D is expressed by:

$$\boldsymbol{V_D = \frac{e\,\tau}{2\,m_e}E}$$

Therefore, the electrons mobility is given by:

$$\mu = \boldsymbol{\frac{e\,\tau}{2\,m_e}}$$

The unit of mobility is expressed in: $m^2.V^{-1}.\ s^{-1}$

4. Electric current

Consider two conductors (A: charged) and (B: neutral) connected via a wire. To reach a new state of equilibrium, the charges dq move from “A” to “B” in a very short time dt, this is what we call electric current.

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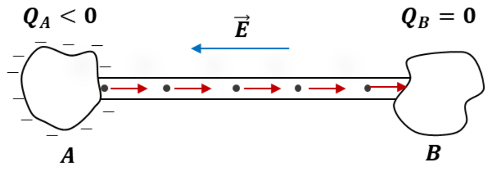

Figure 3. Electric current

4.1. Definition

Electric current is a collective and organized movement of charges (electrons) in a vacuum or through a conductive medium; the current appears when a potential difference U is applied between the extremities of the conductor.

4.2. Current intensity

a) Average current intensity: presents the quantity of charge Δq passing through a conductor during an interval of time Δt:

$$I = \frac{\Delta q}{\Delta t}$$

b) Instantaneous current intensity: presents the derivative of the charge with respect to time.

$$I = \frac{dq}{dt}$$

The unit of electric current is expressed in Ampere (A)

$$[I] = \frac{C}{s} = A$$

4.3. Direction of current

i. The current I always has the same direction of the applied electric field $\vec{E}$.
ii. The current I is always directed from the highest potential V_A to the lowest potential V_B.
iii. The conventional direction of the current opposes the direction of the negative electric charges.

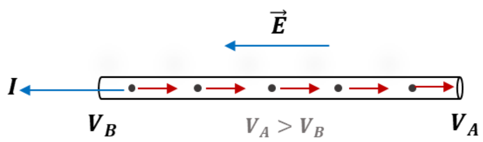

Figure 4. Direction of current.

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5. Electric current density

5.1. Current density *J*

We consider a conductor of section S and volume V, have n number of charges q per unit of volume, moving with a drift velocity V_D.

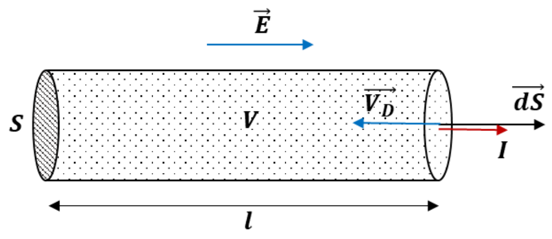

Figure 5. Electric current density

In a very short time, the displacement distance is given by:

$$\vec{dl} = \vec{V_D}.dt$$

On the other hand; the elementary charge dq contained in the elementary volume dV of the conductor is:

$$dq = n\,e\,dV$$

$$dV = \vec{S}.\vec{dl}$$

$$dq = n\,e\,\vec{S}.\vec{dl}$$

$$dq = n\,e\,\vec{S}.\vec{V_D}\,dt$$

$$\Rightarrow \frac{dq}{dt} = n\,e\,V_D\,S$$

$$\boldsymbol{I = n\,e\,V_D\,S}$$

We define the electric current density as the quantity J which presents the electric current I passing the surface S.

$$I = J.S$$

$$J = \frac{I}{S}$$

Or: $$J = n\,e\,V_D$$

The unit of current density is expressed in $(A.m^{-2})$

In local and vector form: $$\boldsymbol{\vec{J} = -n\,e\,\vec{V_D}}$$

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5.2. Conductivity

$$J = n\,e\,V_D$$

Knowing that: $$V_D = \mu\,E$$

$$J = n\,e\,\mu\,E$$

We finally write: $$\boldsymbol{J = \sigma\,E}$$

σ : presents the electrical conductivity, and we write:

$$\boldsymbol{\sigma = \frac{n\,e^2\,\tau}{2\,m_e}}$$

Or: $$\boldsymbol{\sigma = n\,e\,\mu}$$

The unit of electrical conductivity is expressed in $(\Omega.m)^{-1}$.

6. Ohm's law

6.1. Ohm's law

For a metallic conductor, under a constant temperature, the ratio between the potential difference U (or voltage) between its extremities, and the intensity I of the current passing through it, is constant and equal to the electrical resistance of the conductor.

$$R = \frac{U}{I} = C^t$$

The relation between (I, R, U) is known as: **Ohm's law**.

$$\boldsymbol{U = RI}$$

The unit of resistance is expressed in Ohm (Ω).

Notes:

- *To limit the intensity of the current passing through the powered devices, electrical resistors are placed in the circuit.*
- *A conductor is said to be ohmic if it obeys Ohm's law.*

6.2. Relation between the field $\boldsymbol{E}$, the current density $\boldsymbol{J}$ and resistance $\boldsymbol{R}$

We have: $$dV = -E.dl$$

$$\int_A^B dV = -E.\int_A^B dl$$

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$$V_A - V_B = El$$

Finally: $\begin{cases} U = El \\ U = RI \\ I = JS \end{cases}$

Thus, we obtain a new expression for the current density J :

$$\boldsymbol{J = \frac{l}{R\,S} E}$$

We have already defined the relation: $\boldsymbol{J = \sigma\, E}$

Which means that:

$$\boldsymbol{\sigma = \frac{l}{R\,S}}$$

6.3. Electrical resistivity

The conversely of conductivity is called the electrical resistivity of the conductor (or specific resistance).

$$\boldsymbol{\rho = \frac{1}{\sigma} = \frac{R\,S}{l} = \frac{2\,m_e}{n\,e\,\tau}}$$

The unit of resistivity is expressed in: $(\Omega.m)$

Thus, the expression of the resistance of a conductor can be written in the form:

$$\boldsymbol{R = \frac{l}{\sigma\,S} = \rho\frac{l}{S}}$$

Exercise 1

Consider a copper wire with molar mass $M_{Cu} = 63.54\ g/mol$ and volumetric mass $\rho = 8.8 \times 10^3\ Kg.m^{-3}$.

1- Calculate the number of electrons per unit volume m^3, knowing that each copper atom releases two electrons $(2\ e^-)$.
2- The copper wire of section $S = 10\ mm^2$, is crossed by a current $I = 30\ A$, calculate the current density J.
3- Deduce the drift velocity V_D of the electrons inside the copper crystal.

We give: $N_A = 6.023 \times 10^{23}\ atoms/mol$

Solution1

1) The number of electrons $\boldsymbol{n}$

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a) The number of atoms of Cu

$$N\,(Cu) = \frac{\rho\, N_A}{M} = 8.34 \times 10^{29}\ atomes/m^3$$

b) The number of electrons

$$Cu \rightarrow 2\, e^- + Cu^{2+}$$
$$n(e-) = 2\, N_{Cu} = 16.68 \times 10^{29}\ electrons\ /m^3$$

2) The current density J

We have: $\begin{cases} I = 30\ A \\ S = 10 \times 10^{-6}\ m^2 \end{cases} \Rightarrow J = \frac{I}{S} = 3 \times 10^{\,6} A.m^2$

3) The drift velocity V_D

We have: $J = n\, e\, V_D \quad \Longrightarrow \boldsymbol{V_D = 112.4\ \mu m/s}$

7. Joule effect

7.1. Definition

The passage of an electric current through a conductor produces its heating due to its resistance R, this phenomenon is called "***Joule Effect***" or ''**heating effect**''.

7.2. Electric power

Power is defined as the work done per unit of time.

$$P = \frac{dW}{dt}$$

$$dW = dE_p = Udq$$

$$P = U\frac{dq}{dt} = UI$$

$$\boldsymbol{P = UI} \qquad \text{Or} \qquad \boldsymbol{P = RI^2}$$

The unit of electric power is expressed in Watt (W).

7.3. Electric energy

Electrical energy consumed in the form of heat by the resistance R, during the time t is equal to:

$$dE = Udq \Rightarrow dE = RI^2dt$$

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$$\boldsymbol{E = RI^2 t}$$

The unit of energy is expressed in Joule (J).

8. Association of resistors

8.1. In series

$$\begin{cases} U = U_1 + U_2 + U_3 \ldots \ldots \ldots U_n \\ U = RI \end{cases}$$

$$\Rightarrow R_{eq} I = R_1 I + R_2 I + R_3 I \ldots \ldots \ldots R_n I$$

$$\Rightarrow R_{eq} = R_1 + R_2 + R_3 \ldots \ldots \ldots R_n$$

$$\boldsymbol{R_{eq} = \sum_{i=1}^{n} R_i}$$

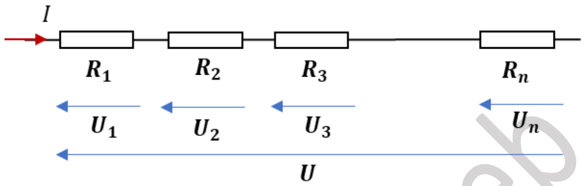

Figure 6. Association of resistors in series

8.2. In parallel

$$\begin{cases} U = U_1 = U_2 = U_n \\ I = I_1 + I_2 + I_3 \ldots \ldots \ldots I_n \\ I = U/R \end{cases}$$

$$\Rightarrow \frac{U}{R_{eq}} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} \ldots \ldots \ldots \frac{U}{R_n}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots \ldots \ldots \frac{1}{R_n}$$

$$\boldsymbol{\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}}$$

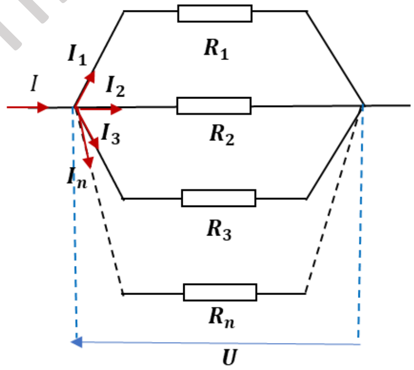

Figure.7. Association of resistors in parallel

9. Electrical generators

9.1. Definition

An electrical generator is a device placed in an electrical circuit, to ensure the circulation of current I and the transport of electrical energy. Knowing that the generator only converts some form of energy such as, mechanical, chemical, light, etc., in electrical energy. There are two types of generators:

- A voltage generator
- A current generator

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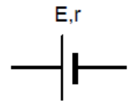
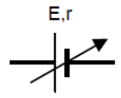
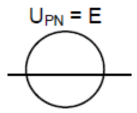
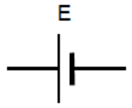
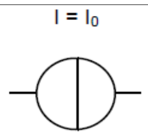
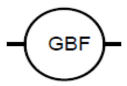
Table 1. This table summarizes the characteristics of different types of generators

Generator	Symbol	Utilization/example
Real voltage generator		Power supply for general use. e.g. Batteries, low-end stabilized electronic power supplies.
Adjustable voltage generator		Power supply for general use. e.g. Stabilized electronic power supply with adjustable output voltage.
Ideal voltage generator		Power supply for general use. e.g. a stabilized electronic power supply with a quality can be considered as an ideal voltage generator.
Current generator		Power supply used when constant current is required in the circuit. (e.g. charging the capacitor at constant current). This intensity is adjustable.
Low frequency generator		This generator is a voltage generator type. Its e.m.f. varies over time (sinusoidal, triangular, in slots, etc.). The amplitude and frequency of E are adjustable.

Note: In this course, we only consider the voltage generators.

9.2. Electromotive force (e.m.f.)

The generator must provide electrical work to the charges so that they can move in a closed electrical circuit.

We can define the electromotive force (e.m.f.), denoted E of a generator (G) (electrical source) as being the work dW done on charge dq to transport it in a very short time dt, in a closed circuit by:

$$E = \frac{dW}{dq}$$

And since the power of the generator G is: $P = \frac{dW}{dt}$

$$\Rightarrow P = \frac{dW}{dq} \cdot \frac{dq}{dt}$$

$$\Rightarrow P_G = E\ I$$

E : electromotive force of the generator, its unit is expressed in volts (V).

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9.3. Potential difference across a generator

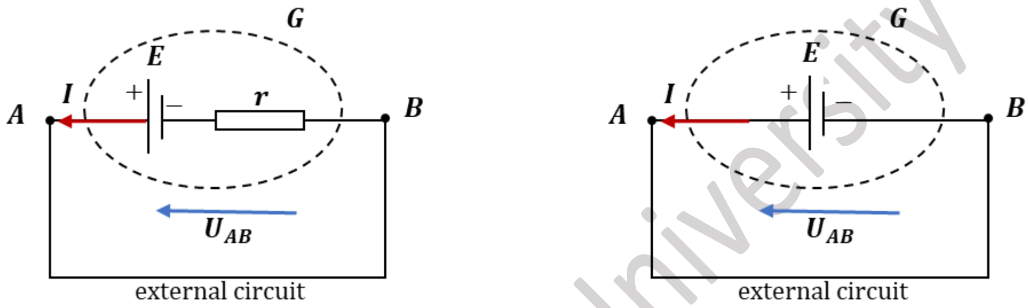
For an ideal voltage generator, the e.m.f. presents the potential difference across the generator:

$$P = UI$$

$$P_G = E\,I$$

$$E = U_{AB}$$

But the electromotive force E is different from the voltage U across the generator, due to the fact that the generator (G) has its own resistor, called the internal resistance, denoted $\boldsymbol{r}$.



a) Real voltage generator **b**) Ideal voltage generator

Figure 8. Direction of current I and voltage U in the case of generator.

The power consumed (P_{AB}) in the external circuit is given by:

$$P_{AB} = P_G - P_r$$

P_G : power supplied by the generator when it delivers a current I.

P_r : power dissipated in the generator by the internal resistance r.

So:

$$P_{AB} = P_G - P_r$$

$$U_{AB}I = EI - RI^2$$

$$U_{AB} = E - rI$$

9.4. Direction of current *I* and voltage *U*

- **i.** Outside the generator G, the current is directed from the $(+)$ pole to the $(-)$ pole.
- **ii.** Inside the generator G, the current is directed from the $(-)$ pole to the $(+)$ pole.
- **iii.** The voltage U and the current I are directed positively and in the same direction.

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10. Electrical loads

10.1. Definition

An electrical load is any electrical component or element that consumes electrical energy and converts it to another form of energy. There are two types of loads:

- **Active load**: converts an electrical energy to active energy (mechanical: motor; luminous: lamp).
- **Passive load**: converts an electrical energy to dissipated energy (heat: resistors).

10.2. The counter electromotive force (c.e.m.f)

A load exerts a resistance force on the electric charges which pass through it, the work of this force gives rise to a counter-electromotive force (c.e.m.f), denoted E'.

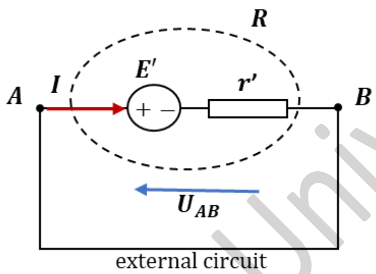

Figure 9. Direction of current I and voltage U in the case of load.

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The power transferred by the load can be written as follows:

$$P_{rec} = E'I$$

10.3. Potential difference across the load

In reality, the load has an internal resistance r; therefore, the total power transferred by the load is given by:

$$P_{AB} = P_{rec} + P_{r'}$$

$$\Rightarrow U_{AB}I = E'I + r'I^2$$

$$\Rightarrow U_{AB} = E' + r'I$$

10.4. Direction of current *I* and voltage *U*

i. Outside the load, current I is directed from the $(-)$ pole to the $(+)$ pole.
ii. The voltage U and the current I are always directed in two opposite directions.

11. Association of generators

11.1. In series

We have:

$$\begin{cases} U = U_1 + U_2 + U_3 \dots\dots\dots U_n \\ U = e - rI \end{cases}$$

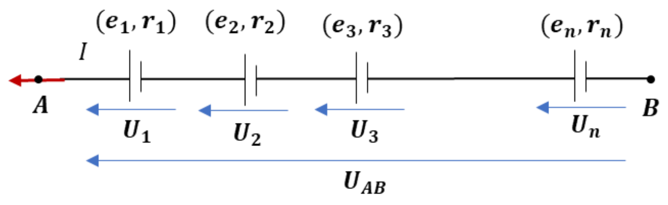


Figure.10. Association of generators in series

$$\Rightarrow R_{eq}I = R_1I + R_2I + R_3I \dots\dots\dots R_nI$$

$$\Rightarrow \left(e_{eq} - r_{eq}I\right) = (e_1 + e_2 + e_3 \dots\dots e_n) - (r_1 + r_2 + r_3 \dots\dots\dots r_n)I$$

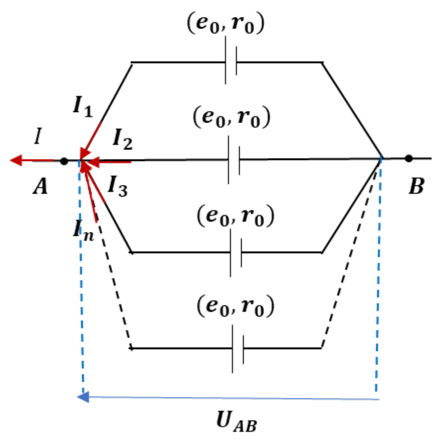
$$\boldsymbol{e_{eq} = \sum_{i=1}^{n} e_i}$$

$$\boldsymbol{r_{eq} = \sum_{i=1}^{n} r_i}$$

11.2. In parallel

We assume that the generators are identical $(\boldsymbol{e_0}, \boldsymbol{r_0})$

$$\begin{cases} U = U_1 = U_2 = U_n \\ I = I_1 + I_2 + I_3 \dots\dots\dots I_n \\ P = P_1 + P_2 + P_3 \dots\dots\dots P_n \\ P = eI - rI^2 \end{cases}$$



$$\boldsymbol{e_{eq} = e_0}$$

$$\boldsymbol{r_{eq} = \frac{r_0}{n}}$$

$$\boldsymbol{I = \sum_{i=1}^{n} I_i}$$

Figure.11. Association of generators in parallel

12. Kirchhoff's laws

12.1. Definitions

a. **The electrical circuit**: is a **closed path** that consists of electronic components like a resistor, a capacitor, an inductor, etc., connected by conductive wires through which electric current can flow.
b. **The electronic component**: is an electronic element which has an input terminal (pole: +) and an output terminal (pole: -). The components can be classified in:
   - **Passive components**: which consume an electrical energy, like the resistor.

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- **Active components**: which produce an electrical energy (current), like the generator.

c. **The branch**: is a set of elements associated in series between two nodes.
d. **The node (junction)**: is a common point between three branches at least.
e. **The loop**: is a set of electrical elements forming a closed path in circuit.
f. **The network**: is a set of electrical circuits.

Example:

Consider the following circuit:

The circuit contains:

- 2- nodes
- 3-branches
- 2-loops

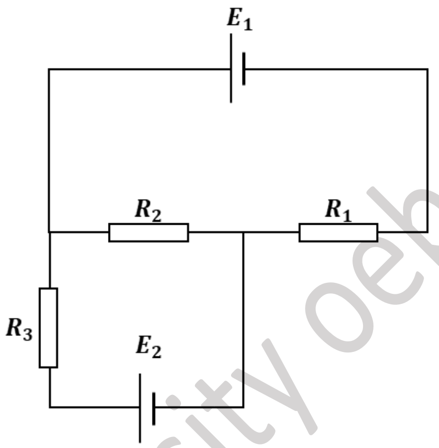

Figure 12. Electrical circuit

12.2. Kirchhoff's 1st law: current law (KCL)

This law, also called **the law of the junctions**. for any node (junction) in an electrical circuit, the sum of currents entering a node is equal to the sum of currents exiting a node:

$$\sum I_{ent} = \sum I_{ext}$$

This means, according to the principle of conservation of charges, that charges cannot accumulate in a node.

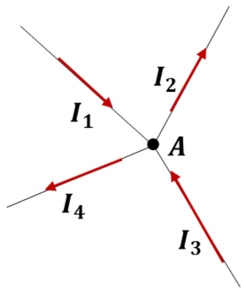

Figure.13. Kirchhoff's 1st law

Example:

we will have in a node A: $I_1 + I_3 = I_2 + I_4$

Or: $I_1 + I_3 - I_2 - I_4 = 0$

12.3. Kirchhoff's 2nd law: voltage law (KVL)

This law, also called **the law of the loops**. In a loop of a circuit, the algebraic sum of the voltages U is zero.

$$\sum U_i = 0$$

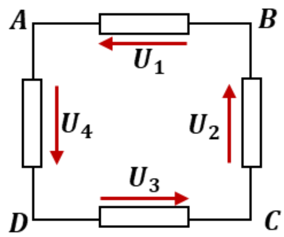

Figure.14. Kirchhoff's 2nd law

Example:

$U_1 + U_2 + U_3 + U_4 = 0$

$(V_A - V_B) + (V_B - V_C) + (V_C - V_D) + (V_D - V_A) = 0$

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12.4. How to apply Kirchhoff's laws (practical rules)

1) We arbitrarily choose on each branch the direction of the current I.
2) We determine the polarity of all the passive elements of the circuit: the (+) pole for the input current and the (-) pole for the output current.

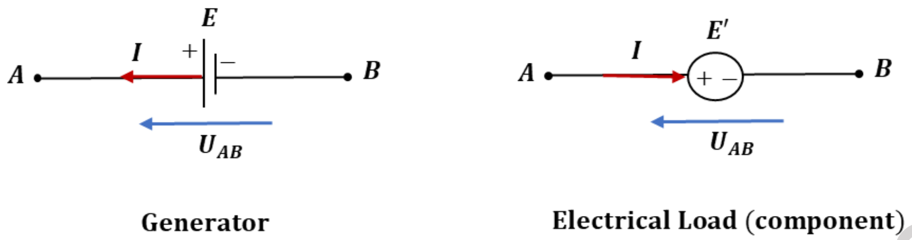


Figure 15. Direction of current I and voltage U in the case of generator and load.

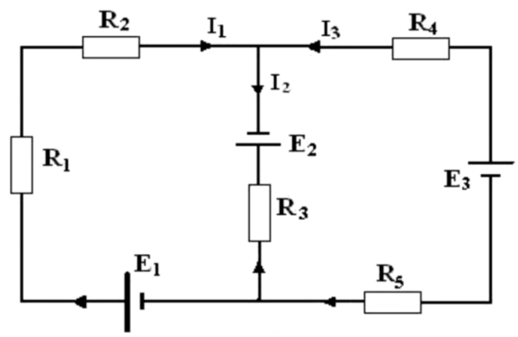
3) We determine the direction of the voltage U; the voltage is directed from the (-) pole to the (+) pole, i.e. in the opposite direction of the current.
4) We must choose a direction to go around the loop.
5) Using the law of the junctions (Kirchhoff's 1st law): $N_{equations} = n_{Nodes} - 1$
6) Using the law of the loops (Kirchhoff's 2nd law): $N_{equations} = n_{Loops}$
7) Total number of equations to solve: $\boldsymbol{N = n_{Nodes} + n_{Loops} - 1}$

Exercise-1

Calculate the current circulating in each branch.

We give:

$E_1 = 8V;\ E_2 = 6V, E_3 = 4V, R_1 = R_2 = 2\,\Omega, R_3 = R_4 = R_5 = 1\,\Omega.$



Sol : $I_1 = \frac{16}{7}\ A\ ; I_2 = \frac{34}{7}\ A\ ; I_3 = \frac{18}{7}\ A$

13. Thevenin's theorem

13.1. States

«Any linear circuit interposed between two terminals A and B, containing several generators and resistors can be simplified to a Thevenin-equivalent circuit with a single generator of electromotive force ($\boldsymbol{E_{Th}}$) and internal resistance ($\boldsymbol{R_{Th}}$)", connected in series with a load».

Specifically, the three components connected in series are:

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- E_{Th} : presents the electromotive force of the equivalent generator of Thevenin.
- R_{Th} : presents the equivalent resistance of Thevenin of the circuit between terminals (A and B).
- R_L : presents the resistance of load which exists between terminals (A and B).

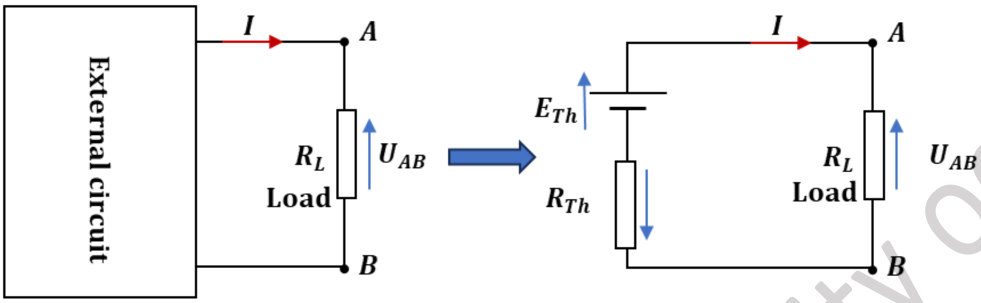


Figure 16. A complicate circuit with its Thevenin-equivalent circuit.

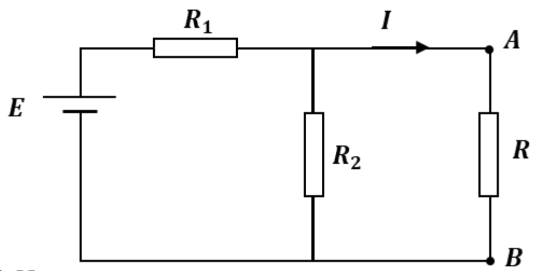
13.2. How to apply Thevenin's theorem and determine E_{Th} and R_{Th}

Calculating R_{Th} :

1- We remove the load R_L.
2- We turn off all voltage and current sources.
3- We designate a new shape of the new circuit which only contains resistors.
4- We calculate R_{eq} located between (A and B), $R_{Th} = R_{eq}$.

Calculating E_{Th} :

1- We open the circuit between A and B.
2- We remove the load R_L.
3- We calculate $E_{Th} = U_{AB}$.



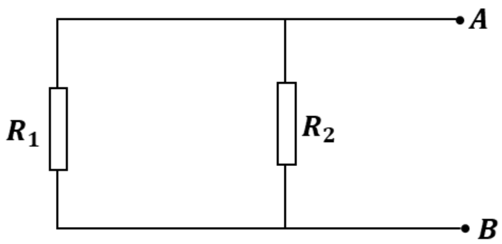
Exercise-2

Consider the circuit in the **opposite figure.**

We give: $R_1 = 1\,\Omega$; $R_2 = 3\,\Omega$; $R = 1\,\Omega$; $E = 10\,V$

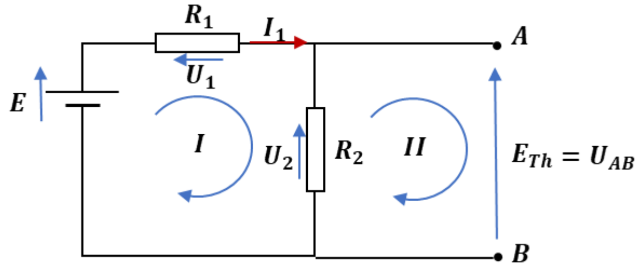
Solution

By applying Thevenin's Theorem



1°/ **Calculating R_{Th}**

$$\frac{1}{R_{Th}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \boldsymbol{R_{Th}} = \frac{\boldsymbol{R_1 R_2}}{\boldsymbol{R_1 + R_2}}$$



2°/ Calculating E_{Th}

We have : $E_{Th} = U_{AB}$

Loop I : $E - U_1 - U_2 = 0$

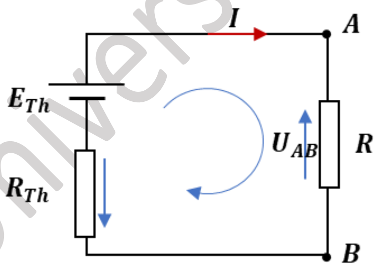
Loop II : $E_{Th} - U_2 = 0$

$$\Rightarrow \begin{cases} E - (R_1 - R_2)I_1 = 0 \\ E_{Th} = R_2 I_1 \end{cases} \Rightarrow I_1 = \frac{E}{R_1 + R_2}$$

$$\Rightarrow \boldsymbol{E_{Th} = \frac{R_2 E}{R_1 + R_2} = 7.5\ V}$$

2°/ Calculating I

To calculate the intensity I of the current we consider the equivalent generator of Thevenin which supplies the branch (A,B).



Loop: $E_{Th} - RI - R_{Th}I = 0$

$$\Rightarrow \boldsymbol{I = \frac{E_{Th}}{R + R_{Th}} = \frac{30}{11} A}$$

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Chapter 4

Electromagnetism

Summary

1. Magnetism

1.1. Magnetic field

In the vicinity of the magnet, the iron filings form a characteristic configuration that shows the effect of the magnet on the surrounding environment. From these formations, Michael Faraday had the idea to introduce the magnetic field ***B*** and the corresponding field lines.

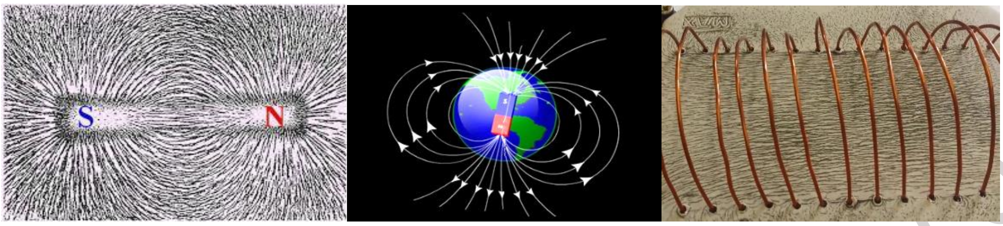

Figure 1. The different sources of the magnetic field B.

There are three primary sources of the magnetic field **B**:

1. The magnetic field produced by natural or artificial magnets
2. The Earth's magnetic field, which has an average strength of approximately $\mathbf{4.7 \times 10^{-5}\ T}$
3. The magnetic field generated by a conductor carrying an electric current

The magnetic field is measured in **tesla (T)**, the SI unit named after Nikola Tesla.

1.2. Magnets as a source of magnetic field

- A magnet is composed of magnetite or other magnetic substances, which naturally generates a magnetic field (magnetic induction) and capable of attracting metals such as: Fe, Cr, Co, Ni…. Etc.
- A magnet is characterized by a north pole (N) and a south pole (S), poles of different nature attract each other (N-S), while poles of similar nature repel each other (N-N) or (S-S).

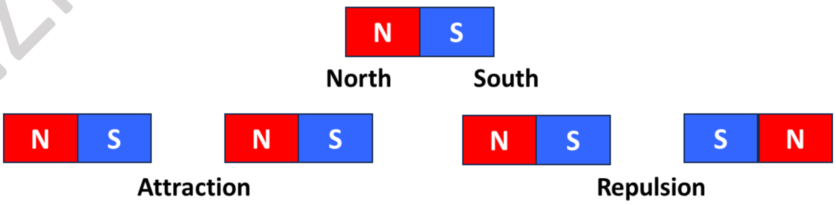

Figure 2. Attraction and repulsion between magnets.

1.3. Electric current as source of magnetic field

According to Oersted's experiment, the passage of a current in a wire leads to the deflection of the magnetic needle (compass), which means the existence of the magnetic field. Consequently, a wire carrying an electric current acquires magnetic properties.

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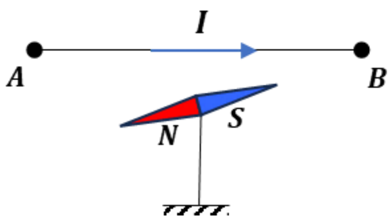

1.4. Magnetic field lines

The magnetic field lines run from the north pole to the south pole and on which the vector $\vec{\boldsymbol{B}}$ is tangent. The intensity of the field B is proportional to the number of lines crossing a surface normal to the field.

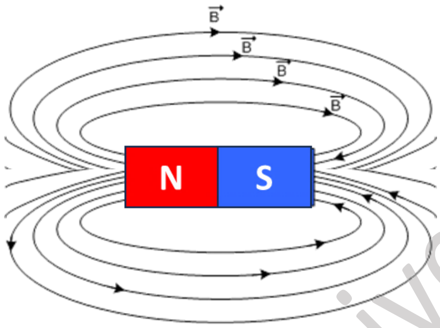

Figure 3. Magnetic field lines.

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2. Electromagnetism

2.1. Definition

Electromagnetism is the study of the relations between electric and magnetic phenomena created by moving electric charges, i.e. the passage of a current in a conductive wire.

2.2. The Lorentz force

According to Wilson's experiment, when an electric charge q moves with a velocity vector $\vec{V}$ in a magnetic field $\vec{B}$, it experiences a new force in addition to the electric force $\vec{F_e}$. This new force is the magnetic force $\vec{F_B}$, which is given by the formula:

$$\vec{F_B} = q\,\vec{V}\Lambda\vec{B}$$

$$\Rightarrow F_B = q\;V\;B\sin(\vec{V},\vec{B})$$

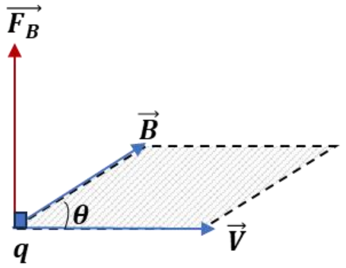
When a charge q moves in the presence of both an electric field $\vec{E}$ and magnetic field $\vec{B}$, the resultant of the forces $\vec{F_e}$ and $\vec{F_B}$ is given by Lorentz's law:

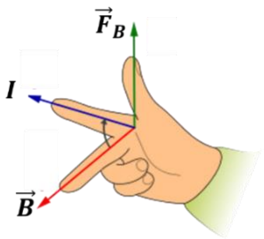
$$\vec{F_L} = q\vec{E} + q\,\vec{V}\Lambda\vec{B}$$

This force is called the **Lorentz force** or the **electromagnetic force**.

Remarks:

- $\overrightarrow{F_B}$: is perpendicular to the plane $(\vec{V}, \vec{B})$.
- The direction of $\overrightarrow{F_B}$ is determined by the **right-hand rule**.





2.3. The Laplace force

If a rectilinear conductor of length $\boldsymbol{l}$ carrying a current $\boldsymbol{I}$, is in a magnetic field $\vec{\boldsymbol{B}}$, the magnetic force applied $\overrightarrow{\boldsymbol{F_B}}$ to the unit of volume $\boldsymbol{dV}$ is given by:

$$\overrightarrow{dF} = \overrightarrow{F_B}\,.dV$$

$$\overrightarrow{dF} = q\,\vec{V}\Lambda\vec{B}\,dV$$

For n charges:

$$\overrightarrow{dF} = n\,q\,\vec{V}\Lambda\vec{B}\,S\,\overrightarrow{dl}$$

$$\Rightarrow \overrightarrow{dF} = I\,\overrightarrow{dl}\,\Lambda\,\vec{B}$$

$$\boldsymbol{F = I\,l\,B\sin\alpha}$$

α: Angle between $(\vec{B}, \overrightarrow{dl})$

This force is called **the Laplace force**.

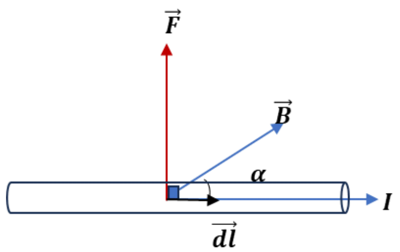


Figure 4. The Laplace force.

3. Magnetic field $\vec{B}$

3.1. Magnetic field created by a moving punctual charge q.

The magnetic field created at a point $\boldsymbol{M}$ by a particle of charge $\boldsymbol{q}$, moving with a velocity $\vec{\boldsymbol{V}}$ in a Galilean reference frame is given by:

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$$\overrightarrow{B_M} = \frac{\mu_0}{4\pi r^2}\, q\, \vec{V} \Lambda \vec{u}$$

Where:

- $\vec{u} = \frac{\vec{r}}{r}$: unit vector.
- M : a point of space.
- $\vec{B}$: magnetic field, its unit is: T.
- μ_0: Magnetic permeability in vacuum. $\mu_0 = 4\pi \times 10^{-7}\, T.m.A^{-1}$. This constant is also given by: $C^2 = \frac{1}{\mu_0\, \varepsilon_0}$, (C: speed of light).

3.2. Magnetic field created by a set of moving charges q_i.

Consider n particles of charges q_i having a velocity V_i. According to the principle of superposition, the magnetic field created at a point M is:

$$\vec{B} = \sum \vec{B_i} = \vec{B_1} + \vec{B_2} + \vec{B_3} + \cdots \ldots \ldots .. \vec{B_n}$$

$$\vec{B_i} = \frac{\mu_0}{4\pi r_i^2}\, q_i\, \vec{V_i} \Lambda \vec{u_i}$$

3.3. Magnetic field created by a conductor

3.3.1. The law of Biot and Savart

An electric current of intensity I passing through an element of length $\vec{dl}$ of a conductor produces an elementary magnetic field $\vec{dB}$ at a point M in space, which given by:

$$\overrightarrow{dB_M} = \frac{\mu_0 I}{4\pi r^2}\, \vec{dl} \Lambda \vec{u}$$

The total magnetic field $\vec{B}$ produced by the entire conductor:

$$\overrightarrow{B_M} = \frac{\mu_0 I}{4\pi} \int_c \frac{\vec{dl} \Lambda \vec{u}}{r^2}$$

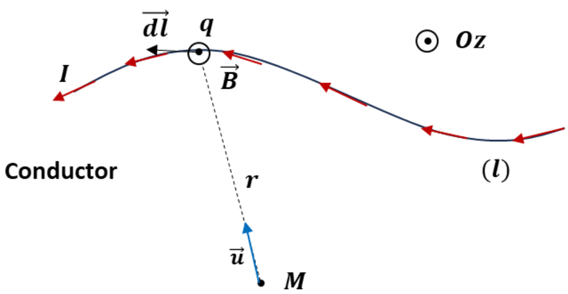


Figure 5. The law of Biot and Savart

3.3.2. Magnetic field created by an infinite wire (linear current)

We consider an infinite rectilinear wire carried by a current of intensity I.

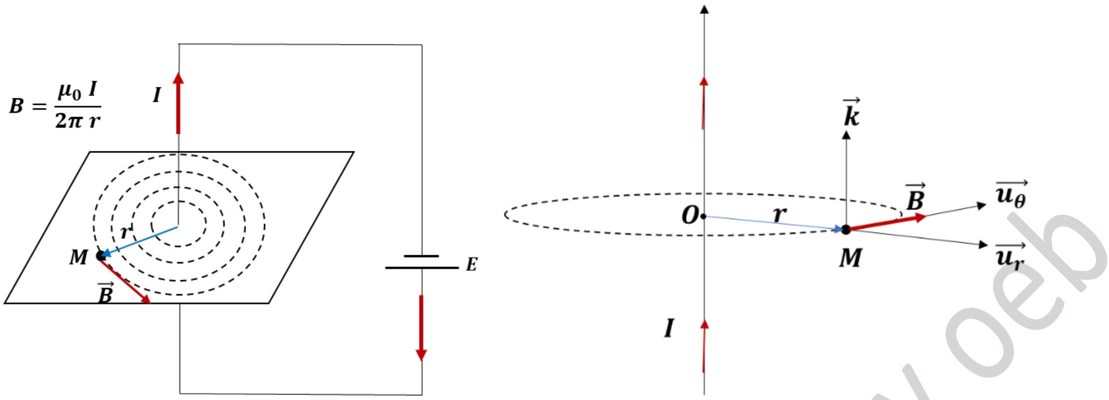


Figure 6. Magnetic field created by an infinite rectilinear wire.

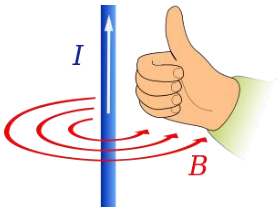
In cylindrical reference frame $(\vec{u_r}, \vec{u_\theta}, \vec{k})$, we write:

$$\vec{B} = B_r\, \vec{u_r} + B_\theta\, \vec{u_\theta} + B_Z\, \vec{k}$$

The direction of $\vec{B}$ is determined by **the corkscrew rule,** as shown in figure opposite:

The magnetic field created by the wire at point M is tangential to the circle of radius “r”, this means that:

$$\begin{cases} B_r = B_Z = 0 \\ B_\theta \neq 0 \end{cases}$$



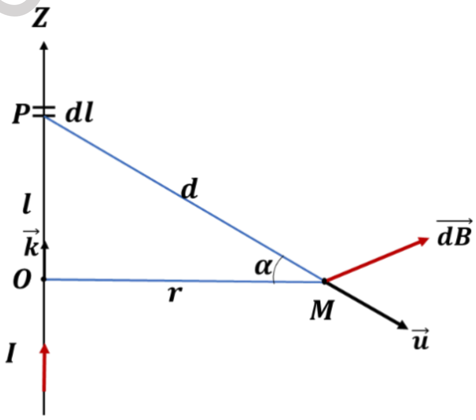


Figure 7. The magnetic field created by the wire at point M.

We therefore write:

$$\vec{B} = B_\theta\, \vec{u_\theta}$$

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According to the Biot-Savart law, we have:

$$\overrightarrow{dB_M} = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{dl} \wedge \vec{u}}{d^2}$$

According to the diagram: $\begin{cases} d = PM \\ \vec{u} = \frac{\overrightarrow{PM}}{PM} \end{cases}$

We then write:

$$\overrightarrow{dB_M} = \frac{\mu_0 I}{4\pi} \frac{\overrightarrow{dl} \wedge \overrightarrow{PM}}{PM^3}$$

where:
$$\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$$

$$\Rightarrow \begin{cases} \overrightarrow{PM} = -l\,\vec{k} + r\,\overrightarrow{u_r} \\ \overrightarrow{dl} = dl\,\vec{k} \end{cases}$$

The vector product:

$$\overrightarrow{dl} \wedge \overrightarrow{PM} = \begin{pmatrix} 0 \\ 0 \\ dl \end{pmatrix} \wedge \begin{pmatrix} r \\ 0 \\ -l \end{pmatrix} = \begin{vmatrix} \overrightarrow{u_r} & -\overrightarrow{u_\theta} & \vec{k} \\ 0 & 0 & dl \\ r & 0 & -l \end{vmatrix}$$

$$\Rightarrow \overrightarrow{dl} \wedge \overrightarrow{PM} = r\,dl\,\overrightarrow{u_\theta}$$

The elementary magnetic field has become:

$$\overrightarrow{dB_M} = \frac{\mu_0 I}{4\pi} \frac{r\,dl}{d^3} \overrightarrow{u_\theta}$$

As long as l is infinite, the angle α is between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, we therefore change the variable from dl to $d\alpha$. According to the diagram:

$$\begin{cases} \cos\alpha = \frac{r}{d} \\ \tan\alpha = \frac{l}{r} \end{cases} \Rightarrow \begin{cases} d = \frac{r}{\cos\alpha} \\ l = r\tan\alpha \end{cases} \Rightarrow \begin{cases} d = \frac{r}{\cos\alpha} \\ dl = \frac{r}{\cos^2\alpha}\,d\alpha \end{cases}$$

After replacing d and dl in eq of dB_M:

$$\int dB = \frac{\mu_0 I}{4\pi r} \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos\alpha\,d\alpha$$

$$\Rightarrow \boldsymbol{\vec{B}} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi r}} \overrightarrow{\boldsymbol{u_\theta}}$$

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3.3.3. Magnetic field created by a ring (circular current)

We consider a ring traversed by an electric current of constant intensity I. We want to find the magnetic field along the axis of symmetry Oz of the ring.

We choose an elementary length $\overrightarrow{dl}$ on the ring, then we calculate the elementary magnetic field $\overrightarrow{dB}$ produced at point P, located on the axis of symmetry Oz of the ring.

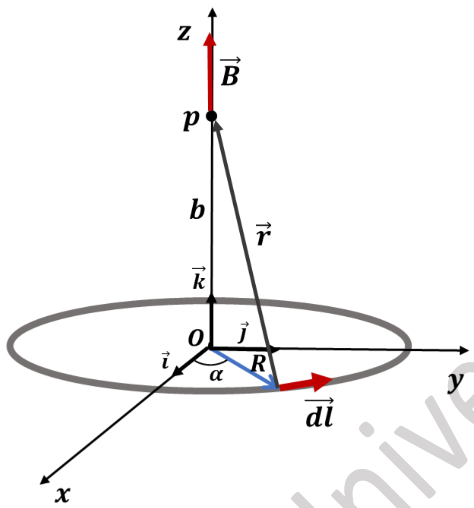

Figure 8. Magnetic field created by a ring.

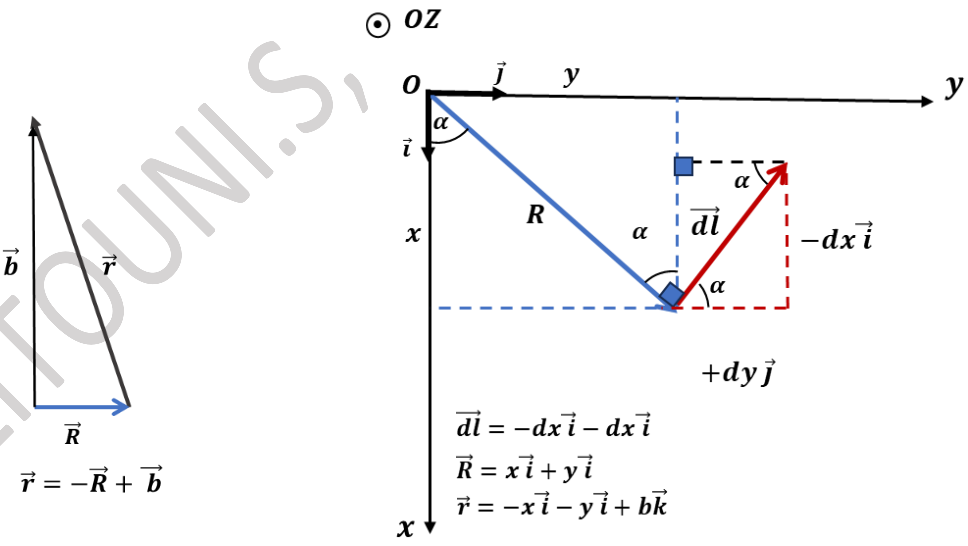

Figure 9. Determination of $\overrightarrow{dl}$ and $\vec{r}$ vectors.

From diagram:

$$\overrightarrow{dl} = -dx\,\vec{\imath} + dy\,\vec{j}$$

$$\overrightarrow{dl} = -dl\sin\alpha\ \vec{\imath} + dl\cos\alpha\ \vec{j}$$

Where: $$dl = R\,d\alpha$$

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$$\Rightarrow \overrightarrow{dl} = -R\, d\alpha \sin\alpha\ \vec{\imath} + R\, d\alpha \cos\alpha\ \vec{\jmath}$$

On the other hand,

$$\vec{r} = -\vec{R} + \vec{b}$$

$$\Rightarrow \vec{r} = -x\,\vec{\imath} - y\,\vec{\jmath} + \vec{b}$$

By applying the Biot and Savard law:

$$\overrightarrow{dB_P} = \frac{\mu_0 I}{4\pi}\frac{\overrightarrow{dl}\wedge\vec{r}}{r^3}$$

$$\Rightarrow \overrightarrow{dB_P} = \frac{\mu_0 I}{4\pi r^3}(Rb\cos\alpha\ d\alpha)\,\vec{\imath} + \frac{\mu_0 I}{4\pi r^3}(Rb\sin\alpha\ d\alpha)\,\vec{\jmath} + \frac{\mu_0 I}{4\pi r^3}(R^2\, d\alpha)\,\vec{k}$$

It appears to us that $\overrightarrow{dB}$ has three components: $\overrightarrow{dB_p} = dB_x\vec{\imath} + dB_y\,\vec{\jmath} + dB_z\,\vec{k}$

In order to obtain the total field, we must integrate all components:

$$B_x = \int_0^{2\pi} dB_x = \int_0^{2\pi}\frac{\mu_0 IRb}{4\pi r^3}\cos\alpha\ d\alpha = 0$$

$$B_y = \int_0^{2\pi} dB_y = \int_0^{2\pi}\frac{\mu_0 IRb}{4\pi r^3}\sin\alpha\ d\alpha = 0$$

$$B_z = \int_0^{2\pi} dB_z = \int_0^{2\pi}\frac{\mu_0 IR^2}{4\pi r^3}\, d\alpha = \frac{\mu_0 IR^2}{4\pi r^3}.2\pi$$

$$\boldsymbol{\overrightarrow{B_P} = \frac{\mu_0 IR^2}{2\,(R^2+b^2)^{\frac{3}{2}}}\,\vec{k}}$$

Remarks:

- At the center of the ring $(b = 0)$: $B = \frac{\mu_0 I}{2\,R}$
- If the ring has a very small radius R compared to b : $B = \frac{\mu_0\, IR^2}{2\,b^3}$
- In the case of a flat coil, consists of N turns: $B = \frac{\mu_0\, IR^2}{2\,b^3}.N$

4. Ampere's theorem

The circulation of the magnetic field along a closed curve that encloses several currents: $I_1, I_2 \ldots\ldots I_n$, is expressed by Ampère's Circuital Law:

$$\oint \vec{B}.\overrightarrow{dl} = \mu_0 \sum I_i$$

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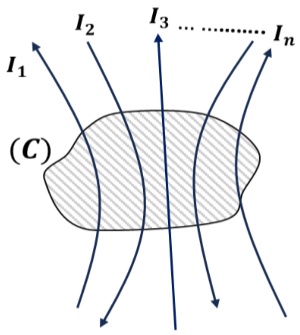


Figure 10. Ampere's theorem.

5. Magnetic dipole

5.1. Definition and modeling

A magnetic dipole is typically represented by a small current loop (often circular R), with very small dimensions compared to the distance r of the point "M" in the space, where we calculate its effects, i.e., the magnetic field $\vec{B}$. We distinguish two types:

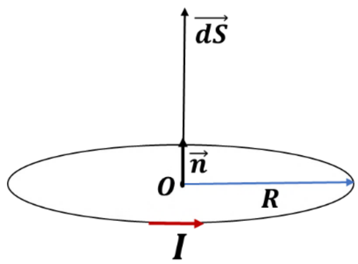
- An active dipole ($r < R$)
- A passive dipole: (r≫R)

5.2. Oriented surface

Considering a turn traversed by a current of intensity "I" (current loop), we define the oriented surface by an elementary surface vector $\overrightarrow{dS}$:

$$\overrightarrow{dS} = dS\,\vec{n}$$

$$\vec{S} = \int dS\,\vec{n}$$

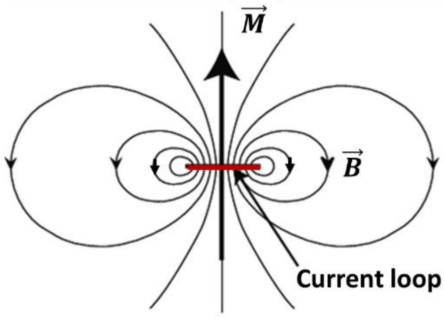


5.3. Magnetic moment $\vec{M}$

The magnetic moment $\vec{M}$ of a current loop of oriented surface S, traversed by a current I is defined by:

$$\vec{M} = I.\vec{S}$$

$$\vec{M} = I\,S\,\vec{n}$$



Example: The magnetic moment of circular turn

$$\vec{M} = IS\,\vec{n}$$

$$\Rightarrow \vec{M} = I\,\pi\,R^2\,\vec{k}$$

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- The direction of moment $\vec{M}$ is determined by the Corkscrew rule.
- M is expressed in $A.m^2$.

5.4. Magnetic field $\vec{B}$ created by a magnetic dipole

$$\begin{cases} B_r = \dfrac{\mu_0 M \cos\theta}{2\pi\, r^3} \\ B_\theta = \dfrac{\mu_0 M \sin\theta}{4\pi\, r^3} \end{cases}$$

$$\overrightarrow{B_M} = \frac{\mu_0 M}{4\pi\, r^3}[2\cos\theta\,\overrightarrow{u_r} + \sin\theta\,\overrightarrow{u_\theta}]$$

5.5. Effect of an external and uniform magnetic field $\overrightarrow{B_{ext}}$ on a magnetic dipole

a. The Laplace force $\overrightarrow{F_L}$

Any circuit traversed by a current and immersed in an external magnetic field subjects to Laplace force $\overrightarrow{F_L}$.

We have:

$$\overrightarrow{dF} = I\,\overrightarrow{dl} \wedge \vec{B}$$

The total force exerted on the loop is given by:

$$\vec{F} = \oint I\,\overrightarrow{dl} \wedge \vec{B}$$

$$\oint \overrightarrow{dl} = 0$$

$$\overrightarrow{F_L} = \vec{0}$$

b. Moment exerted on a magnetic dipole

The moment $\vec{\tau}$ exerted on a magnetic dipole placed in an external magnetic field $\overrightarrow{B_{ext}}$ is given by:

$$\vec{\tau} = \vec{M} \wedge \overrightarrow{B_{ext}}$$

c. Potential energy of a dipole placed in an external magnetic field

$$E_p = -\vec{M}.\overrightarrow{B_{ext}}$$

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6. Electromagnetic induction

6.1. Magnetic flux

The flux of the magnetic field ϕ_m through a surface of the element $\overrightarrow{dS}$ is given by:

$$\phi_m = \iint \vec{B}.\overrightarrow{dS} = \iint \vec{B}.\vec{n}\ dS = \int B.\cos\theta.dS$$

The flux is expressed in T.m^2 or in Weber (Wb).

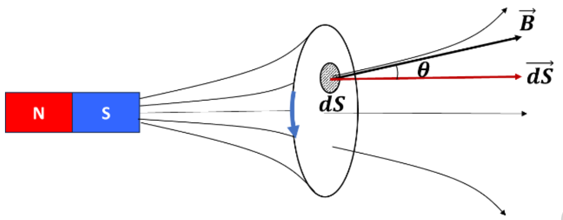
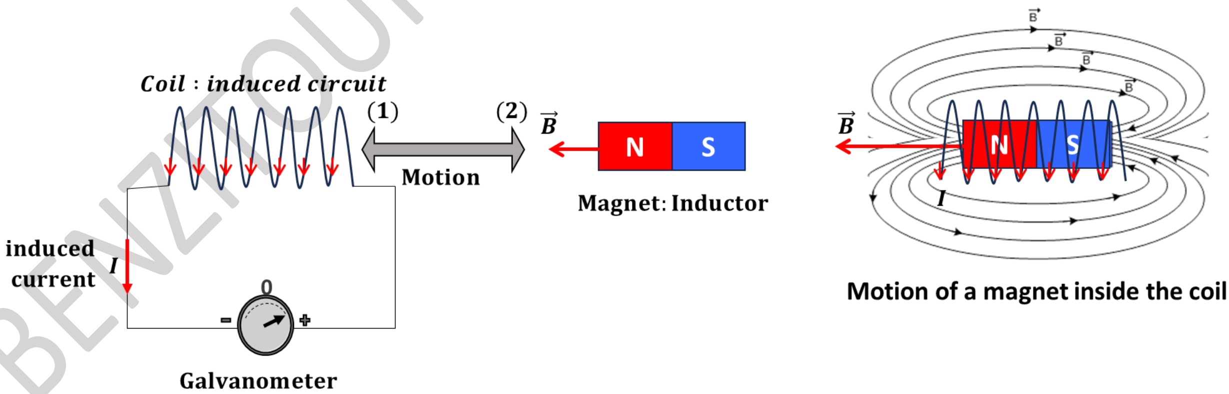


Figure 11. Magnetic flux through a surface.

6.2. Electromagnetic induction phenomena

- The figure below presents a hollow coil, composed of a large number of turns, connected to a very sensitive galvanometer.
- A magnet is placed at rest, aligned along the coil's axis — no current is detected.
- When the magnet is moved into the coil, the galvanometer shows a brief current.
- The current disappears as soon as the magnet stops moving.
- When the magnet is pulled out, the galvanometer detects another brief current, but in the opposite direction.



(1): Appearance of an induced current
(2): Appearance of an induced current in the opposite direction
Static magnet: no current

Figure 12. Electromagnetic induction

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- The recorded current is called the induced current, the magnet is the inductor, and the coil is the induced circuit.
- An induced current can be obtained by rotating the coil in front of the fixed magnet.

6.2.1. Faraday's law

Statement of the law:

''The electromotive force E induced is the negative change in the magnetic flux Ø_m through a closed-circuit per unit time t '':

$$E = -\frac{d\emptyset_m}{dt}$$

6.2.2. Lenz's law

Statement of the law:

"The direction of the induced current in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field."

Example:

The magnetic flux in a circuit is given by the following expression:

$$\emptyset = 50t + 10$$

Determine the e.m.f. E created by this magnetic flux.

Solution: We have: $E = -\frac{d\emptyset}{dt} \Rightarrow E = -50\ volts$

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Tutorials

Summary

Tutorial n°1: Electrostatics

Part-I

Exercise 0

1- In a Cartesian reference frame, a point M (x, y, z) is identified by the following position vector:

$\overrightarrow{OM} = \vec{r} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$, where: $r = \sqrt{x^2 + y^2 + z^2}$

Calculate: $\overrightarrow{grad}\, r, \overrightarrow{grad}\left(\frac{1}{r}\right), \overrightarrow{grad}\,(\ln r),\ div\,\vec{r}$ and $\overrightarrow{rot}\,\vec{r}$

2- In a spherical reference frame, let $f\,(r,\theta,\varphi)$ be a scalar function, its gradient is given by:

$$\overrightarrow{grad}\, f = \frac{\partial f}{\partial r}\,\overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\overrightarrow{u_\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\,\overrightarrow{u_\varphi}$$

Calculate $\overrightarrow{grad}\, f$ for these cases: $f_1 = r,\ f_2 = \frac{1}{r}$ and $f_3 = k\frac{\cos\theta}{r^2}$

Exercise 1

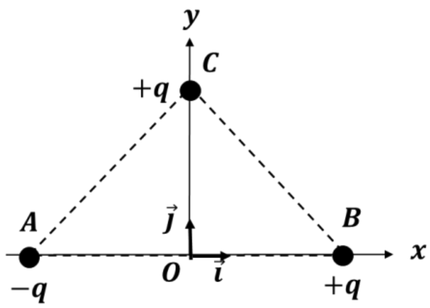
I. Calculate the electrostatic field (E) and the electrostatic potential (V) produced by an electric charge $q = 6 \times 10^{-10} C$ in point M at a distance d = 3 cm.

II. In a Cartesian reference frame, two electrical charges $q_A = 3 \times 10^{-7} C$ and $q_B = -10^{-7} C$ are placed respectively in points $A\,(1, 0, 2)$ and $B\,(3, -2, 4)$: (m). Determine the components of the electrostatic force and calculate its magnitude.

We give: $k = 9 \times 10^9\, N.m^2.C^{-2}$

Exercise 2

We consider three identical punctual charges $q_A = -q, q_B = +q$ and $q_C = +q$ placed at the tops of a triangle, in $A\,(-R, 0)$, $B\,(R, 0)$ and $C\,(0, R)$, respectively.



1- Give the expression of the electric field $\overrightarrow{E_O}$ at point O, and calculate its value.

2- Give the expression of electric potential V_O at point O, and calculate its value.

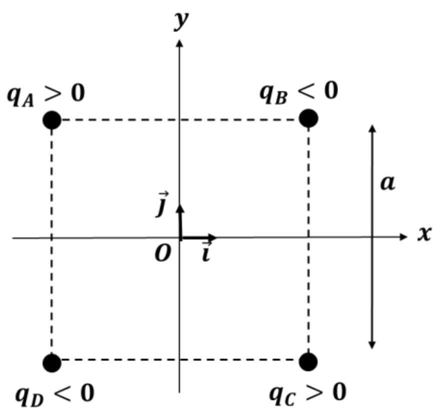
3- Calculate the electrostatic force acting on the charge $q_O = q/2$ placed in O.

4- Calculate the potential energy of the charge q_O.

We give: $k = 9 \times 10^9\, N.m^2.C^{-2}$; $q = 2 \times 10^{-9} C$; $R = 3\, cm$.

Exercise 3 (In the course)

We place four electric charges q_A, q_B, q_C and q_D at the tops ABCD of a square, with side a and center O, origin of an orthonormal base $(O, \vec{i}, \vec{j})$.



1- Give the expression of the electric field $\overrightarrow{E_O}$ at point O.

2- Give the expression of electric potential V_O at point O.

We give: $q_A = q,\ q_B = -2q,\ q_C = 2q$ et $q_D = -q$.

Tutorial n°2: Electrostatics

Part-II

Exercise 1

Find the total charge Q for the following distributions:

1- Linear charge distributed uniformly on a ring of radius a
2- Surface charge distributed uniformly on a sphere of radius R.
3- Volume charge distributed uniformly on a cylinder of radius R and height h.
4- Volume charge distributed uniformly in a sphere of radius R.

Exercise 2

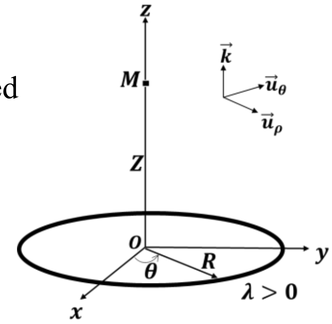
I. We consider a wire of length **2L**, of negligible diameter, uniformly charged with a positive and constant linear charge density $\boldsymbol{\lambda}$.

1- Determine the total charge Q of the wire.
2- Determine the electric field E created by this wire at a point M located on its axis of symmetry Oy: ($\overrightarrow{OM} = b\,\vec{j}$). Deduce the electric field created by an infinite wire.

II. We consider a disk of radius $\boldsymbol{R}$, uniformly charged with a positive and constant surface charge density $\boldsymbol{\sigma}$.

1- Determine the total charge Q of the disk.
2- Determine the electric field E created by this disk at a point N located on its axis of symmetry Ox: ($\overrightarrow{ON} = a\,\vec{i}$). Deduce the electric field created by an infinite plane.

Exercise 3



A ring (circular wire) with center O, radius R and axis Oz, is uniformly charged with a positive and constant linear charge density $\boldsymbol{\lambda}$. Give the expression of:

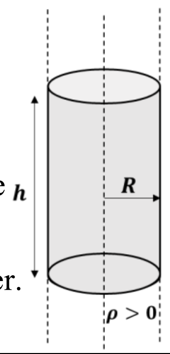
1- The total charge Q of the ring.
2- The electrostatic field created by the ring at a point M located on its axis of symmetry Oz. Deduce the electrostatic field at point O.
3- The electrostatic potential at point M.
4- The electrostatic force exerted by the ring on a punctual charge $\boldsymbol{q}$, placed at M.

Exercise 4

A hollow sphere with center O and radius R is uniformly charged by a positive and constant surface charge density σ ($\sigma > 0$).

1- Calculate the total charge Q of the sphere.
2- Determine the expression of the electric field $E(r)$ at any point M in space.
3- Deduce the expression of the electric potential $V(r)$.
4- Graph (draw) the variation curve of $E(r)$ and $V(r)$. Is there continuity?

Exercise 5



Let C be a solid cylinder of the axis of revolution (Oz), of radius R and of infinite length, uniformly charged by a positive and constant volume charge density ρ.

1- Determine the electrostatic field inside, on the surface and outside of the cylinder. Check that the electric field is continuous at the surface of the cylinder.

Tutorial n°3: Conductors in equilibrum

Exercise 1

1- What are the principal properties of a **conductor in electrostatic equilibrium**?

2- Consider a conductive sphere (S_1) of radius $R_1 = 2R$ carrying a charge $\boldsymbol{Q_1} = +\boldsymbol{Q}$. Calculate its potential $\boldsymbol{V}$, its capacitance $\boldsymbol{C}$ and its surface charge density $\boldsymbol{\sigma}$.

3- A second conductive sphere (S_2) initially neutral, of radius $R_2 = R$, is placed at a distance of (S_1), sufficiently far away so that the influence phenomena are negligible. The two spheres are connected by a conductive wire of negligible capacity.

- **a)** Determine the new distribution of charges Q_1' and Q_2' after equilibrium, as a function of $\boldsymbol{Q}$.
- **b)** Express, as a function of R_1 and R_2, the ratio of charge densities σ_1/σ_2 .
- **c)** Determine the ratio of the electrostatic fields E_1/E_2 near the surfaces (S_1) and (S_2). Conclude.

Exercise 2

A **planar capacitor** is made up of two metal plates of surface S, separated by an insulator of thickness d with relative permittivity ε_r . We will admit that the two plates are in total influence (the edge effects are negligible). The capacitor is charged by applying a potential difference of $U = V_1 - V_2$.

1- Determine the surface charge density $\boldsymbol{\sigma}$ as a function of Q and S.

2- Determine the electrostatic field $\vec{\boldsymbol{E}}$ between the plates of capacitor as a function of Q.

3- Determine the potential difference $\boldsymbol{U}$ between the plates of the capacitor.

4- Deduce the capacitance $\boldsymbol{C}$ of the capacitor.

5- Find the electrostatic potential energy $\boldsymbol{E_p}$ stored in this capacitor, as a function of $\boldsymbol{\varepsilon_0}, \boldsymbol{\varepsilon_r}, \boldsymbol{E}$ and $\boldsymbol{V}$.

6- A new metal plate of thickness $\boldsymbol{a}$ is introduced parallel between the plates of capacitor. Determine the equivalent capacitance $\boldsymbol{C_{eq}}$.

Exercise 3

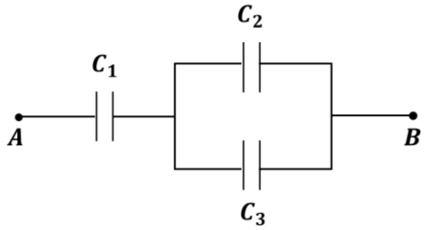
I. Consider **the grouping of capacitors** as shown in figure opposite. We apply between A and B a potential difference U_{AB} of 10 V.

1- Calculate the equivalent capacitance of the capacitors.

2- Calculate the quantity of charge Q_{AB} that has circulated.

3- Calculate the tension across each capacitor.

4- Deduce the charge carried for each capacitor.



II. We disconnect the source and then create a short circuit between A and B. Calculate the energy liberated during this operation.

We give: $C_1 = 2\,\mu F$; $C_2 = 7\,\mu F$; $C_3 = 3\,\mu F$

Tutorial n°4: Electrokinetics

Exercise 1

We consider a copper wire of section $\mathbf{S = 10\ mm^2}$ and length $\mathbf{l = 1m}$ traversed by a current of intensity $\mathbf{I = 1\ A}$ under a voltage $\mathbf{U = 20\ V}$, whose charge carriers are the electrons.

1- Calculate the number of electrons $\boldsymbol{n}$ per unit volume (assuming that each copper atom releases two electrons).
2- Calculate the current density $\boldsymbol{J}$ in the copper wire.
3- Deduce the drift velocity $\boldsymbol{V_D}$ of the electrons in the copper wire.
4- Calculate the conductivity $\boldsymbol{\sigma}$ of the copper wire.

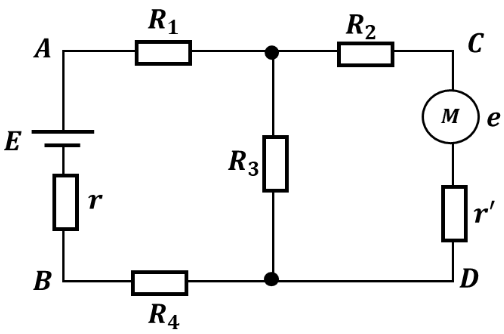
We give:

$$\boldsymbol{M_{Cu}} = 63.5 g/mol\ ;\ \boldsymbol{N_A} = 6.02 \times 10^{23}\ atoms.mol^{-1}\ ;\ \boldsymbol{\rho} = 8800\ kg/m^3$$

Exercise 2

Consider the electrical circuit shown in the **figure opposite**. The current circulating in each branch is given by:

$I_1 = 1A;\ \ I_2 = 0.5\ A;\ \ and\ \ I_3 = 0.5\ A$.



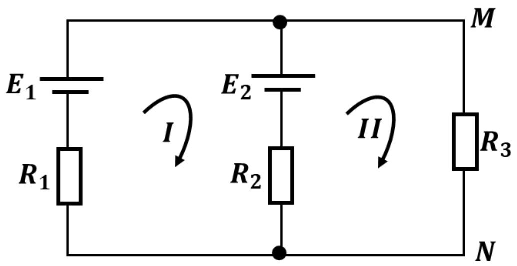
1- Calculate the potential differences U_{AB} and U_{CD}.
2- Calculate the efficiency of the generator and the motor.
3- Calculate the power dissipated by the Joule effect in the circuit.

We give:

$$E = 10\,V,\ e = 2V,\ r = r' = 1\Omega,\ R_1 = 4\Omega,\ R_2 = 1\Omega,\ R_3 = 6\Omega\ and\ \ R_4 = 2\Omega$$

Exercise 3

Consider the electrical circuit illustrated in the **figure opposite**



1- Calculate current intensities I_1, I_2 and I_3 circulating respectively in the resistors R_1, R_2 and R_3 using **Kirchhoff's laws.**
2- Calculate the current I_3 circulating in the MN branch using **Thevenin's theorem**.

We give:

$$E_1 = 10\,V, E_2 = 8\,V, R_1 = R_2 = R_3 = 2\,\Omega.$$

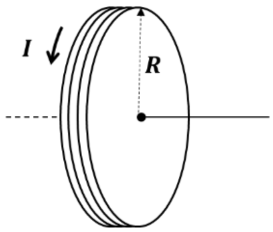
N.B for exercise 2. You can recalculate the current in each branch, using **Kirchhoff's laws**.

Tutorial n°5: Electromagnetism

Exercise 1

A flat circular coil of negligible thickness has $\boldsymbol{N}$ circular turns of radius $\boldsymbol{R}$ traversed by a current of intensity $\boldsymbol{I}$. (**Figure opposite**). Using the law of **Biot and Savart**:

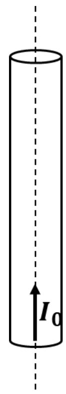
1- Determine the magnetic field $\boldsymbol{B}$ created at the center of the coil.



Exercise 2

We consider a cylindrical conductor of radius $\boldsymbol{R}$ and infinite length $\boldsymbol{l}$ traversed by a current of intensity $\boldsymbol{I_0}$. The current density $\boldsymbol{J}$ is constant across the entire section of the cylinder and parallel to the axis Oz. Using **Ampère's theorem**:

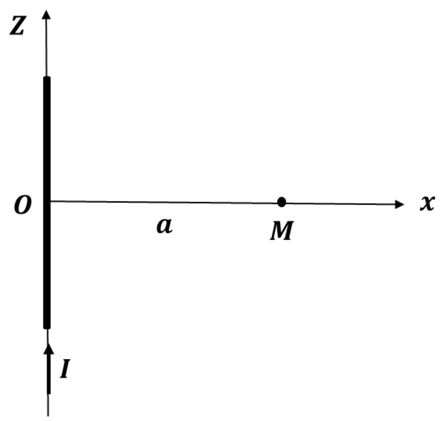
1- Determine the magnetic field $\boldsymbol{B}$ outside the cylinder ($r \geq R$).
2- Determine the magnetic field $\boldsymbol{B}$ inside the cylinder ($r < R$).
3- Draw the curve $\boldsymbol{B(r)}$.



Exercise 3 (Do in the course)

We consider a wire of infinite length and negligible diameter, traversed by an electric current of intensity $\boldsymbol{I}$ (**Figure below**). Using the law of **Biot and Savart**:

1- Determine the magnetic field $\vec{\boldsymbol{B}}$ created by this wire at a point $\boldsymbol{M}$ located on the $\boldsymbol{Ox}$ axis.



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