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Observer-based adaptive control of robot manipulators: Fuzzy systems approach

Noureddine Goléa [a,*], Amar Goléa [b], Kamel Barra [a], Tarek Bouktir [a]

[a] Electrical Engineering Institute, Oum El-Bouaghi University, 04000 Oum El-Bouaghi, Algeria
[b] Electrical Engineering Department, Biskra University, 07000 Biskra, Algeria

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Abstract

This paper presents a fuzzy adaptive control suitable for motion control of multi-link robot manipulators with structured and unstructured uncertainties. When joint velocities are available, full state fuzzy adaptive feedback control is designed to ensure the stability of the closed loop dynamic. If the joint velocities are not measurable, an observer is introduced and an adaptive output feedback control is designed based on the estimated velocities. In the proposed control scheme, we need not derive the linear formulation of robot dynamic equation and tune the parameters. To reduce the number of fuzzy rules of the fuzzy controller, we consider the properties of robot dynamics and the decomposition of the uncertainties terms. The proposed controller is robust against uncertainties and external disturbance. Further, it is shown that required stability conditions, in both cases, can be formulated as LMI problems and solved using dedicated software. The validity of the control scheme is demonstrated by computer simulations on a two-link robot manipulator.

Keywords: Robot manipulators; Fuzzy systems; Adaptive control; Observer; Stability

1. Introduction

Robots are one of the most important pieces of machinery for industrial automation nowadays. Design of robust adaptive controllers suitable for real-time control of robot manipulators is one of the most challenging tasks for many control engineers, especially when manipulators are required to maneuver very quickly under external disturbances. Motion control of robot manipulators has been studied using various approaches (see e.g., [1–8] and references therein). The traditional proportional and derivative (PD) controller is very simple and does not require any knowledge of the robot dynamics. However, it requires very large actuation to achieve precise control, which is not practical but highly demanded in many cases. Robot manipulators are multivariable nonlinear coupling systems and are frequently subjected to structured and/or unstructured uncertainties even in a well-structured setting for industrial use. Structured uncertainties are mainly caused by imprecision in the manipulator link properties, unknown loads, and so on. Unstructured uncertainties are caused by unmodeled dynamics, e.g., nonlinear friction, disturbances, and high-frequency models of the dynamics. As a result, it is difficult to obtain an accurate mathematical model so that computed torque controllers [1,6] or other model-based controllers [3,8] can be accurately applied. Although adaptive controllers [1,5,7] can achieve fine control and compensate for partially unknown manipulator dynamics (i.e., structured uncertainties), they often suffer from incapacity to deal with unstructured uncertainties. Hence, there is a need for model-free adaptive control strategies.

The application of fuzzy systems to robots dynamic control is not new [9–11]. Though the proposed methods have been practically successful, it has proved extremely difficult to develop a general analysis and design theory for conventional fuzzy control systems. During the last few years, a number of papers have been presented to deal with the problem of robot adaptive control [12–16]. The basic idea of these methods is to design the feedback controller based on the computed torque principle, and to use an adaptive fuzzy system to approximate the robot nonlinearities involved in the control input design. However, most of the above designs present two drawbacks.

* Corresponding author.
E-mail addresses: nour_golea@yahoo.fr (N. Goléa), goleaamar1@yahoo.fr (A. Goléa).

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First, the robot dynamic is presented as single nonlinearity approximated by a single fuzzy system with the robot real and desired positions and velocities as inputs, which results in large number of rules with lot of parameters to be tuned. Second, the feedback control design is setup under the constraint that the states of the robot are available, which is not true in many practical cases where the joint velocities are not measurable.

In this paper, we develop an adaptive fuzzy control for rigid robot manipulators, with reduced complexity. This work can be classed in the context of observer-based fuzzy adaptive control of MIMO nonlinear systems where few approaches were proposed (see e.g., [17–19]). Ref. [17] proposes a hybrid direct–indirect fuzzy adaptive approach, while [17,18] investigate the indirect one. In [17], a linear observer is used to estimate the tracking errors, while in [18,19] a nonlinear observer, based on the system estimated fuzzy model, is used to estimate the system state vector. The three approaches share the introduction of robustifying terms to ensure the convergence. Further, [17,19] share the assumption that strict positive real (SPR) condition is fulfilled for the estimation error dynamic, which is not true due to the triplet (A, B, C) structure. This assumption is meet in [18] by appropriately filtering the estimation error dynamic. The present work differs from [17–19] in several aspects. The first one is that this work is dedicated to robot manipulators dynamic control and exploits the robots dynamic structure in the control design, while [17,18] are of general purpose. The second aspect is that, compared to [17–19] where all input gain matrix elements are estimated, only the diagonal elements are considered here which reduces the used fuzzy approximators number and complexity. The third aspect is that no robustifying term is used, and the tracking and stabilization is ensured by the fuzzy adaptive controller. The last aspect is that, as in [17], a linear observer is used to estimate the tracking errors, which in turn are used in the parameters update laws. This technique avoids the SPR condition problem encountered in [17,19] and the filtering approach used in [18].

The paper is organized as follows. In Section 2, the rigid robot control problem is formulated, and a brief description of fuzzy systems is presented. In Section 3, based on full state information assumption, a fuzzy adaptive state feedback is designed and it's stability is analyzed. In Section 4, a linear observer is used to estimate the joint velocities, and an observer-based output-feedback control law is developed. In Section 5, simulation tests for a two-link robot under uncertainties and disturbances are presented to confirm the effectiveness and applicability of the proposed method. Finally, conclusions are included in Section 6.

2. Preliminaries

Standard notation is used in this paper. Let R be the real number set, R^n be the n-dimensional vector space, $R^{n\times n}$ be the $n \times n$ real matrix space. The norm of vector $x \in R^n$ and that of matrix $A \in R^{n\times n}$ are defined, respectively, as $|x| = \sqrt{x^T x}$ and $|A|^2 = \mathrm{tr}\,[A^T A]$. If y is a scalar, then $|y|$ denotes the absolute value.

2.1. Robot control problem

The dynamic equations of the robot manipulator are a set of highly nonlinear coupled differential equations. Using the Lagrange–Euler formulation, the dynamic equation of an n-joint robot arm can be expressed as

$$M(q)\ddot{q} + c(q,\dot{q}) + g(q) + \tau_c(q,\dot{q}) + \tau_d(q,\dot{q}) = u \tag{1}$$

where $M(q) \in R^{n\times n}$ is the bounded positive definite inertia matrix; $c(q,\dot{q}) \in R^n$ the vector representing centrifugal and Coriolis effects; $g(q) \in R^n$ the vector representing gravitational torques; and $\tau_c(q,\dot{q}) \in R^n$, $\tau_d(q,\dot{q}) \in R^n$ are the vectors representing the dynamic effects as nonlinear frictions, small joint and link elasticities, backlash and bounded torque disturbances. Here the uncertainties effect is decomposed as continuous part $\tau_c(q,\dot{q})$ and discontinuous part $\tau_d(q,\dot{q})$.

$u \in R^n$ is the vector of joint torques supplied by the actuators; $q \in R^n$ the vector of joint positions; $\dot{q} \in R^n$ the vector of joint velocities; and $\ddot{q} \in R^n$ is the vector of joint accelerations.

Taking as state vector $x^T = [\,x_1^T \quad \cdots \quad x_n^T\,]$ with $x_i^T = [\,q_i \quad \dot{q}_i\,]$, the robot model (1) can be rewritten as

$$\dot{x} = Ax + B[F(q,\dot{q}) + G(q)u + d(q,\dot{q})] \tag{2}$$

where

$$F(q,\dot{q}) = \begin{bmatrix} f_1(q,\dot{q}) \\ \vdots \\ f_n(q,\dot{q}) \end{bmatrix} := -M^{-1}(q)[c(q,\dot{q}) + g(q) + \tau_c(q,\dot{q})]$$

$$G(q) = \begin{bmatrix} g_{11}(q) & \cdots & g_{1n}(q) \\ \vdots & \ddots & \vdots \\ g_{n1}(q) & \cdots & g_{nn}(q) \end{bmatrix} := M^{-1}(q)$$

$$d(q,\dot{q}) = \begin{bmatrix} d_1(q,\dot{q}) \\ \vdots \\ d_n(q,\dot{q}) \end{bmatrix} := -M^{-1}(q)\tau_d(q,\dot{q})$$

and $A = \mathrm{diag}[A_1, \ldots, A_n]$, $B = \mathrm{diag}[b_1, \ldots, b_n]$ with

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, b_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, i = 1, \ldots, n$$

The tracking control problem can be stated as: for given bounded reference trajectories q_r, $\dot{q}_r$ and $\ddot{q}_r \in R^n$, (with $|\ddot{q}_r| \leq q_0$ form some known constant q_0) design the control input torques u such as the robot's states follow the reference trajectories, with all involved signals in closed loop remain bounded.

2.2. Fuzzy systems

A fuzzy system (FS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for the FS comprises a collection of fuzzy If–then rules of the following form:

$$R_j : \text{If } z_1 \text{ is } Z_1^j \text{ and } z_2 \text{ is } Z_2^j \text{ and}, \ldots, \text{and } z_p \text{ is } Z_m^j \quad \text{then } y \text{ is } Y^j, \quad j = 1, \ldots, m \tag{3}$$

where $z = (z_1, \ldots, z_p) \in R^p$, $y \in R$ are the fuzzy system input and output, respectively. The fuzzy sets Z_l^j and Y^j, associated with the membership functions $\mu_{Z_l^j}(z_j)$ and $\mu_{Y^j}(y)$, respectively, operate fuzzy partitions of the fuzzy system input and output spaces Z and Y, respectively. m is the rules number.

The output of the FS with weighted-center defuzzifier, product inference, and singleton fuzzifier is of the following form:

$$y = \sum_{j=1}^{m} \phi_j(z) w_j \tag{4}$$

where $\phi_j(z) = \prod_{l=1}^{p} \mu_{Z_l^j}(z_l)$ and w_j is the point in Y at which $\mu_{Y^j}(y)$ achieves its maximum value. In more compact form, (4) can be arranged as

$$y = \theta^T \phi(z) \tag{5}$$

where $\theta^T = [\, w_1 \quad \ldots \quad w_m \,]$ and $\phi^T(z) = [\, \phi_1(z) \quad \ldots \quad \phi_m(z) \,]$.

In this work, the membership functions are chosen as gaussian function; that is

$$\mu_{Z_l^j}(z_l) = \exp\left(-\frac{(z_l - c_{jl})^2}{2\sigma_{jl}^2} \right) \tag{6}$$

where c_{jl} and σ_{jl} are the membership function center and variance, respectively. The above membership function could be replaced by any other function, but as was shown in [20], this function has the best approximation property.

To maintain consistent performance of the fuzzy systems in situations where there is a large uncertainty or unknown variation in plant parameters and structures, the fuzzy systems should be adaptive. To reduce the processing requirements, the membership functions parameters are fixed (i.e., regular membership functions distribution is used), and the vector θ is chosen as the free adjustable parameter.

3. Fuzzy adaptive state feedback

In this section, it is assumed that robot joint velocities are available to measurement. Then, adaptive fuzzy state feedback can be designed using full state vector information.

Theoretical results [20,21] have shown that fuzzy systems (5) are universal approximators, i.e., they can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear terms in (2) can be approximated as

$$\begin{aligned} f_i(q, \dot{q}) &= \theta_{f_i}^{*T} \phi_i(q, \dot{q}) + \epsilon_i(q, \dot{q}), \\ g_{ii}(q) &= \theta_{g_i}^{*T} \psi_i(q) + \varepsilon_i(q), \quad i = 1, \ldots, n \end{aligned} \tag{7}$$

where $\theta_{f_i}^{*T} \phi_i(q, \dot{q})$ and $\theta_{g_i}^{*T} \psi_i(q)$ are fuzzy systems of the from (5), and $\epsilon_i(q, \dot{q})$, $\varepsilon_i(q)$ are the inherent approximation errors due to the finite number of rules in the fuzzy systems. The optimal weights $\theta_{f_i}^*$ and $\theta_{g_i}^*$ defined above are quantities required only for analytical purpose. Typically $\theta_{f_i}^*$ and $\theta_{g_i}^*$ are chosen to minimize $\epsilon_i(q, \dot{q})$ and $\varepsilon_i(q)$ over the compact regions Ω_f and Ω_g respectively, that is

$$\theta_{f_i}^* = \arg\min \{ \sup_{q, \dot{q} \in \Omega_f} | f_i(q, \dot{q}) - \theta_{fi}^T \phi_i(q, \dot{q}) | \}$$

$$\theta_{g_i}^* = \arg\min \{ \sup_{q \in \Omega_g} | g_{ii}(q) - \theta_{gi}^T \psi_i(q) | \}$$

Assumption 1. The fuzzy systems approximation errors are bounded by $|\epsilon_i(q, \dot{q})| \le \epsilon_{0i}$ and $|\varepsilon_i(q)| \le \varepsilon_{0i}$, $i = 1, \ldots, n$, for some constants ϵ_{0i} and ε_{0i}.

Assumption 1 results from the universal approximation property of fuzzy systems, that can approximate any well-defined function over a compact space with finite approximation error.

Using (7) in (2), the robot dynamic can be written as

$$\dot{x} = Ax + B[\Theta_f^* \Phi(q, \dot{q}) + \Theta_g^* \Psi(q) u + H(q) u + \omega(q, \dot{q})] \tag{8}$$

where
$\Phi(q, \dot{q}) = \text{block-diag}[\phi_1(q, \dot{q}), \ldots, \phi_n(q, \dot{q})]$,-
$\Psi(q, \dot{q}) = \text{block-diag}[\psi_1(q, \dot{q}), \ldots, \psi_n(q, \dot{q})]$,
$\Theta_f^* = \text{block-diag}[\theta_{f_1}^{*T}, \ldots, \theta_{f_n}^{*T}]$,
$\Theta_g^* = \text{block-diag}[\theta_{g_1}^{*T}, \ldots, \theta_{g_n}^{*T}]$, $\omega(q, \dot{q}) = \epsilon + d(q, \dot{q})$, with $\epsilon^T = [\, \epsilon_1 \quad \cdots \quad \epsilon_n \,]$, and

$$H(q) = \begin{bmatrix} \varepsilon_1 & g_{12}(q) & \cdots & g_{1n}(q) \\ g_{21}(q) & \ddots & & \vdots \\ \vdots & & \ddots & g_{(n-1)n}(q) \\ g_{n1}(q) & \cdots & g_{n(n-1)}(q) & \varepsilon_n \end{bmatrix}$$

Based on (7) and (8), the control inputs are defined as

$$u = [\Theta_g \Psi(q)]^{-1} [-\Theta_f \Phi(q, \dot{q}) + \ddot{q}_r + K\mathrm{e}] \tag{9}$$

where $\mathrm{e}^T = [\, (q_r - q)^T \quad (\dot{q}_r - \dot{q})^T \,]$ is the tracking error vector, Θ_g, Θ_f are the estimated fuzzy systems parameters, and $K = \text{diag}[K_1, \ldots, K_n]$ with $K_i \in R^2$ is PD regulator gain vector, chosen such as the matrix $A_c = A - BK$ is Hurwitz.

Then, introducing the control input (9) in (8) yields

$$\dot{\mathrm{e}} = A_c \mathrm{e} - B[\bar{\Theta}_f \Phi(q, \dot{q}) + \bar{\Theta}_g \Psi(q) u + H(q) u + \omega(q, \dot{q})] \tag{10}$$

where $\bar{\Theta}_f = \Theta_f^* - \Theta_f$ and $\bar{\Theta}_g = \Theta_g^* - \Theta_g$ are the parameters estimation errors.

From (10), it can be seen that the tracking error vector is driven by the coupling terms and the finite approximation accuracy effects reflected by $H(q)$ and the uncertainty term $\omega(q, \dot{q})$.

To design the fuzzy systems parameters update laws and to ensure boundedness of the involved signals in the closed loop robot control, the following assumptions are used:

Assumption 2. The diagonal elements of $G(q)$ are bounded such as $g_m \le \text{diag}[g_{11}(q), \ldots, g_{nn}(q)] \le g_M$, the matrix $H(q)$ is bounded by $|H(q)| \leq h_0$, and the disturbance term $\omega(q, \dot{q})$ is bounded by $|\omega(q, \dot{q})| \leq \omega_0$.

Assumption 3. The fuzzy systems parameters are bounded by the constraint sets Ω_f and Ω_g such that: $\Omega_f = \{\Theta_f^* | |\Theta_f^*| \leq f_M\}$ and $\Omega_g = \{\Theta_g^* | g_m \leq |\Theta_g^*| \leq g_M\}$, respectively, where f_M, g_m, and g_M are some known constants.

The first part of Assumption 2 follows from the fact that $M(q)$ is bounded positive definite matrix, the second part follows from the boundedness of $M(q)$ and $\varepsilon_i(q)$. Finally, the third part follows from boundedness of $M(q)$, τ_d and $\epsilon_i(q, \dot{q})$. The bounds used in Assumption 3 result from the Assumptions 1 and 2 and are used to ensure the boundedness of the fuzzy systems outputs.

In order to constraint the parameters Θ_f and Θ_g within the sets Ω_f and Ω_g, respectively, we use the following parameter projection algorithm:

$$\dot{\Theta}_f = \begin{cases} -\gamma_1 W_f \\ \quad \text{if } |\Theta_f| < f_M \text{ or } (|\Theta_f| = f_M \text{ and tr}\,[W_f \Theta_f^T] \geq 0) \\ -\gamma_1 W_f + \gamma_1 \text{tr}[W_f \Theta_f^T] \left(\dfrac{1 + |\Theta_f|}{f_M} \right)^2 \Theta_f \\ \quad \text{if } |\Theta_f| = f_M \text{ and tr}[W_f \Theta_f^T] < 0 \end{cases} \tag{11}$$

and

$$\dot{\Theta}_g = \begin{cases} -\gamma_2 W_g \\ \quad \text{if } |\Theta_g| < g_M \text{or } (|\Theta_g| = g_M \text{ and tr}\,[W_g \Theta_g^T] \geq 0) \\ -\gamma_2 W_g + \gamma_2 \text{tr}\,[W_g \Theta_g^T] \left(\dfrac{1 + |\Theta_g|}{g_M} \right)^2 \Theta_g \\ \quad \text{if } |\Theta_g| = g_M \text{ and tr}[W_g \Theta_g^T] \leq 0 \end{cases} \tag{12}$$

where $\gamma_1, \gamma_2 > 0$ are design parameters, $W_f = B^T P e \Phi^T(q, \dot{q})$, $W_g = B^T P e u^T \Psi^T(q)$, and $P = P^T > 0$ is the solution, for a given $Q = Q^T > 0$, of the Lyapunov equation:

$$A_c^T P + P A_c = -Q \tag{13}$$

Moreover, in order to guarantee $|\Theta_g| \geq g_m$ such that an inverse of $\Theta_g \Psi(q)$ always exists, we use the following law to adjust the parameter Θ_g.

1. Whenever any element $[\Theta_g]_{ij} = g_m$ use

$$[\dot{\Theta}_g]_{ij} = \begin{cases} -\gamma_2 [W_g]_{ij} & \text{if } [W_g]_{ij} < 0 \\ 0 & \text{if } [W_g]_{ij} \geq 0 \end{cases} \tag{14}$$

2. Otherwise, use (12).

where $[A]_{ij}$ stands for the ij th element of the matrix A.

The stability properties of the proposed fuzzy adaptive state feedback are summarized by the following theorem.

Theorem 1. *The robot adaptive control composed by the robot dynamic* (2), *the control input* (9), *the update laws* (11)–(12) *and* (14) *verifying* Assumptions 1–3, *guarantees the following*:

1. $|\Theta_f| \leq f_M$ *and* $g_m \leq |\Theta_g| \leq g_M$
2. $|\mathrm{e}| \in L_\infty$
3. $|u| \in L_\infty$

Proof.

1. To prove that $g_m \leq |\Theta_g| \leq g_M$, let $V_g = \frac{1}{2\gamma_1} \text{tr}[\Theta_g^T \Theta_g]$, then $\dot{V}_g = \text{tr}[\dot{\Theta}_g^T \Theta_g]$. If the first line of (12) is true, we have either $|\Theta_g| < g_M$ or $\dot{V}_g = -\text{tr}[W_g^T \Theta_g] \leq 0$ when $|\Theta_g| = g_M$, that is, we get always $|\Theta_g| \leq g_M$. If the second line of (12) is true, we have $|\Theta_g| = g_M$ and

$$\dot{V}_g = \text{tr}\left[-W_g^T \Theta_g + \text{tr}[W_g \Theta_g^T] \left(\frac{1 + |\Theta_g|}{g_M} \right)^2 \Theta_g^T \Theta_g \right]$$
$$= -\text{tr}[W_g^T \Theta_g] + \text{tr}[W_g \Theta_g^T] \left(\frac{1 + |\Theta_g|}{g_M} \right)^2 \text{tr}[\Theta_g^T \Theta_g] \tag{15}$$

since $|\Theta_g| = g_M$, we get

$$\dot{V}_g = \text{tr}[W_g \Theta_g^T] g_M^2 \leq 0 \tag{16}$$

that is, $|\Theta_g| \leq g_M$. Therefore, we have $|\Theta_g| \leq g_M$, $\forall t \geq 0$.

From (14) we see that if $|\Theta_g|_{ij} = g_m$, then $[\dot{\Theta}_g]_{ij} \geq 0$; that is, we have that $|\Theta_g| \geq g_m$. Using the same analysis, we can prove that $|\Theta_f| \leq f_M$, $\forall t \geq 0$.

2. Consider the Lyapunov function:

$$V = \frac{1}{2} \mathrm{e}^T P \mathrm{e} + \frac{1}{2\gamma_1} \text{tr}[\bar{\Theta}_f^T \bar{\Theta}_f] + \frac{1}{2\gamma_2} \text{tr}[\bar{\Theta}_g^T \bar{\Theta}_g] \tag{17}$$

The differentiation of (17) along (10) yields

$$\dot{V} = -\frac{1}{2} \mathrm{e}^T Q \mathrm{e} - \mathrm{e}^T P B [\bar{\Theta}_f \Phi(q, \dot{q}) + \bar{\Theta}_g \Psi(q) u + H(q) u$$
$$+ \omega(q, \dot{q})] + \frac{1}{\gamma_1} \text{tr}\,[\dot{\bar{\Theta}}_f^T \bar{\Theta}_f] + \frac{1}{\gamma_2} \text{tr}[\dot{\bar{\Theta}}_g^T \bar{\Theta}_g] \tag{18}$$

which, using matrices traces properties, can be arranged as

$$\dot{V} = -\frac{1}{2} \mathrm{e}^T Q \mathrm{e} - \mathrm{e}^T P B [H(q) u + \omega(q, \dot{q})]$$
$$+ \frac{1}{\gamma_1} \text{tr}[(\dot{\bar{\Theta}}_f^T - \gamma_1 \Phi(q, \dot{q}) \mathrm{e}^T P B) \bar{\Theta}_f]$$
$$+ \frac{1}{\gamma_2} \text{tr}[(\dot{\bar{\Theta}}_g^T - \gamma_2 \Psi(q) u \mathrm{e}^T P B) \bar{\Theta}_g] \tag{19}$$

Then, using (11) and (12) and the fact that $\dot{\bar{\Theta}}_f = -\dot{\Theta}_f$ ($\dot{\bar{\Theta}}_g = -\dot{\Theta}_g$), (19) can be written as

$$\dot{V} = -\frac{1}{2} \mathrm{e}^T Q \mathrm{e} - \mathrm{e}^T P B [H(q) u + \omega(q, \dot{q})]$$
$$- \frac{1}{\gamma_1} \text{tr}[(\dot{\Theta}_f^T + \gamma_1 W_f^T) \bar{\Theta}_f] - \frac{1}{\gamma_2} \text{tr}[(\dot{\Theta}_g^T + \gamma_2 W_g^T) \bar{\Theta}_g] \tag{20}$$

We now prove that the last two terms in (20) are always ≤ 0. If the first lines of (11) and (12) are true the result is trivial. If the second lines are true, then we get

$$\dot{V} = -\frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB[H(q)u + \omega(q,\dot{q})] - \mathrm{tr}[W_f \Theta_f^T]\left(\frac{1+|\Theta_f|}{f_M}\right)^2 \mathrm{tr}[\Theta_f^T \bar{\Theta}_f] - \mathrm{tr}[W_g \Theta_g^T]\left(\frac{1+|\Theta_g|}{g_M}\right)^2 \mathrm{tr}[\Theta_g^T \bar{\Theta}_g] \quad (21)$$

On the other hand, we have

$$\mathrm{tr}[\Theta_f^T \bar{\Theta}_f] = \frac{1}{2}\mathrm{tr}[\Theta_f^{*T}\Theta_f^*] - \frac{1}{2}\mathrm{tr}[\Theta_f^T\Theta_f] - \frac{1}{2}\mathrm{tr}[\bar{\Theta}_f^T\bar{\Theta}_f] \quad (22)$$

Since $\mathrm{tr}[\Theta_f^{*T}\Theta_f^*] \le f_M^2$, $\mathrm{tr}[\Theta_f^T\Theta_f] = f_M^2$ and $\mathrm{tr}[\bar{\Theta}_f^T\bar{\Theta}_f] \ge 0$ we get $\mathrm{tr}[\Theta_f^T\bar{\Theta}_f] \le 0$, which, with $\mathrm{tr}[W_f\Theta_f^T] < 0$, means that third term in (21) is ≤ 0. The same analysis can be used to show that last term in (21) is also ≤ 0. Then, (21) can be arranged as

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB[H(q)u + \omega(q,\dot{q})] \quad (23)$$

Further, introducing the control law (9) in (23) yields

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB\zeta_1\mathrm{e} - \mathrm{e}^T PB\zeta_2 \quad (24)$$

with the bounded terms:

$$\zeta_1 = H(q)[\Theta_g\Psi(q)]^{-1}K$$

$$\zeta_2 = H(q)[\Theta_g\Psi(q)]^{-1}[-\Theta_f\Phi(q,\dot{q}) + \ddot{q}_r] + \omega(q,\dot{q})$$

More, (24) can be upper bounded by

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB\zeta_1\mathrm{e} + \frac{1}{2}\mathrm{e}^T PBB^T P\mathrm{e} + \frac{1}{2}|\zeta_2|^2 \quad (25)$$

which can be arranged as

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T (Q + 2PB\zeta_1 - PBB^T P)\mathrm{e} + \frac{1}{2}|\zeta_2|^2 \quad (26)$$

Then, if Q and P are chosen such that the following inequality

$$Q + 2PB\zeta_1 - PBB^T P \ge Q_1 \quad (27)$$

holds for some positive definite matrix Q_1, then we get

$$\dot{V} \le -\frac{1}{2}\mathrm{e}^T Q_1\mathrm{e} + \frac{1}{2}|\zeta_2|^2 \quad (28)$$

which can be upper bounded by

$$\dot{V} \le -\frac{1}{2}\lambda_{\min}(Q_1)|\mathrm{e}|^2 + +\frac{1}{2}|\zeta_2|^2 \quad (29)$$

where $\lambda_{\min}(Q_1)$ is the smallest eigenvalue of Q_1.

Then $\dot{V} \le 0$ whenever the tracking error is outside the region given by

$$|\mathrm{e}| \le \frac{|\zeta_2|}{\sqrt{\lambda_{\min}(Q_1)}} \quad (30)$$

which implies that $|\mathrm{e}| \in L_\infty$.

3. Consider that (9) can be upper bounded by

$$u \le |[\Theta_g\Psi(q)]^{-1}|[|\Theta_f\Phi(q,\dot{q})| + |\ddot{q}_r| + |K||\mathrm{e}|] \quad (31)$$

and the results 1, 2 and (31) yields

$$|u| \le \frac{1}{g_m}\left(f_M + q_0 + |K|\frac{|\zeta_2|}{\sqrt{\lambda_{\min}(Q_1)}}\right) \quad (32)$$

this implies that $|u| \in L_\infty$. □

4. Observer-based fuzzy adaptive control

In Section 3, the joint velocities were supposed to be available for feedback. This assumption limits the application of the proposed approach, because, in many practical situations only the joint positions are measured. In this section, fuzzy adaptive output feedback is investigated by introducing an observer to reconstruct the robot state vector.

Let's define the following linear state observer:

$$\dot{\hat{x}} = A\hat{x} + L\bar{q} \quad (33)$$

$$\hat{q} = C\hat{x} \quad (34)$$

where $\hat{x}^T = [\hat{q}^T \quad \dot{\hat{q}}^T]$ are the estimated positions and velocities, $\bar{q} = q - \hat{q}$ are the positions estimation errors. $L^T = \mathrm{diag}[L_1, \ldots, L_n]$, with $L_i \in R^2$, is the observer gain vector, and $C = \mathrm{diag}[c_1, \ldots, c_n]$ with $c_i = [1 \quad 0]$.

The state and output estimation errors, using (8), (33) and (34), are given by

$$\dot{\bar{x}} = A\bar{x} + B[\Theta_f^*\Phi(q,\dot{q}) + \Theta_g^*\Psi(q)u + H(q)u + \omega(q,\dot{q})] - L\bar{q} \quad (35)$$

$$\bar{q} = C\bar{x} \quad (36)$$

where $\bar{x} = x - \hat{x}$ is the state estimation error.

Based on robot estimated state vector, the control input (9) is redefined as

$$u = [\Theta_g\Psi(q)]^{-1}[-\Theta_f\Phi(q,\dot{\hat{q}}) + \ddot{q}_r + K\hat{\mathrm{e}}] \quad (37)$$

where $\hat{\mathrm{e}}$ is the estimated tracking error.

Then, introducing (37) in (35) yields

$$\dot{\bar{x}} = A\bar{x} + B[\Theta_f^*\Phi(q,\dot{q}) - \Theta_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u + \ddot{q}_r + K\hat{\mathrm{e}} + H(q)u + d(q,\dot{q})] - L\bar{q} \quad (38)$$

Using (36) and the fact that $\hat{\mathrm{e}} = \mathrm{e} + \bar{x}$, (38) can be arranged as

$$\dot{\bar{x}} = A_0\bar{x} + B[\Theta_f^*\Phi(q,\dot{q}) - \Theta_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u + \ddot{q}_r + H(q)u + d(q,\dot{q})] + BK\mathrm{e} \quad (39)$$

where $A_0 = (A + BK - LC)$.

From the definition of the membership functions (6), $\Phi(q,\dot{q})$ can be decomposed as

$$\Phi(q,\dot{q}) = \Phi(q,\dot{\hat{q}}) + S(q,\dot{\tilde{q}}) \tag{40}$$

where $S(q,\dot{q})$ represents the high order terms of Taylor development of $\Phi(q,\dot{q})$ in the neighborhood of $\dot{\hat{q}}$. Note, that since $|\Phi(q,\dot{q})|, |\Phi(q,\dot{\hat{q}})| \leq 1$ then $S(q,\dot{\tilde{q}})$ is also bounded by $|S(q,\dot{\tilde{q}})| \leq 1$.

Substituting (40) in (39) yields

$$\dot{\bar{x}} = A_0\bar{x} + B[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u + \ddot{q}_r + H(q)u + \varsigma(q,\dot{q})] + BK\mathrm{e} \tag{41}$$

where $\varsigma(q,\dot{q}) = \Theta_f^* S(q,\dot{q}) + \omega(q,\dot{q})$.

Then, using the control input (37), the tracking error dynamic is given by

$$\dot{\mathrm{e}} = A_c\mathrm{e} - B[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u + H(q)u + \varsigma(q,\dot{q})] - BK\bar{x} \tag{42}$$

In order to constraint the parameters Θ_f and Θ_g within the sets Ω_f and Ω_g, respectively, the same update laws (11), (12) and (14) are used with $W_f = B^T P\hat{\mathrm{e}}\Phi^T(q,\dot{\hat{q}})$ and $W_g = B^T P\hat{\mathrm{e}}u^T\Psi^T(q)$.

Theorem 2. *The observer-based fuzzy adaptive robot control, composed by the robot dynamic (2), the observer (33) and (34), the control input (37) and the update laws (11), (12) and (14) (with the indicated changes), guarantees the following*:
1. $|\Theta_f| \leq f_M$ *and* $g_m \leq |\Theta_g| \leq g_M$
2. $|\mathrm{e}| \in L_\infty, |\bar{x}| \in L_\infty$
3. $|u| \in L_\infty$

Proof.

1. The proof follows the same line as in Theorem 1
2. Consider the Lyapunov function:

$$V = \frac{1}{2}\mathrm{e}^T P\mathrm{e} + \frac{1}{2}\bar{x}^T P_0\bar{x} + \frac{1}{2\gamma_1}tr[\bar{\Theta}_f^T\bar{\Theta}_f] + \frac{1}{2\gamma_2}tr[\bar{\Theta}_g^T\bar{\Theta}_g] \tag{43}$$

where $P_0 = P_0^T > 0$ is the solution, for a given $Q_0 = Q_0^T > 0$, of the Lyapunov equation

$$A_0^T P_0 + P_0 A_0 = -Q_0 \tag{44}$$

The differentiation of (43) along (41) and (42) yields

$$\begin{aligned}\dot{V} = &-\frac{1}{2}\bar{x}^T Q_0\bar{x} + \bar{x}^T P_0 B[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u \\ &+ \ddot{q}_r + H(q)u + \varsigma(q,\dot{q})] - \frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) \\ &+ \bar{\Theta}_g\Psi(q)u + H(q)u + \varsigma(q,\dot{q})] + \bar{x}^T P_0 BK\mathrm{e} \\ &- \mathrm{e}^T PBK\bar{x} + \frac{1}{\gamma_1}\mathrm{tr}[\dot{\bar{\Theta}}_f^T\bar{\Theta}_f] + \frac{1}{\gamma_2}\mathrm{tr}[\dot{\bar{\Theta}}_g^T\bar{\Theta}_g]\end{aligned} \tag{45}$$

Then, using the fact that $\mathrm{e} = \hat{\mathrm{e}} - \bar{x}$ in (45) becomes

$$\begin{aligned}\dot{V} = &-\frac{1}{2}\bar{x}^T Q_0\bar{x} - \frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \hat{\mathrm{e}}^T PB[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u] \\ &- \mathrm{e}^T PB[H(q)u + \varsigma(q,\dot{q})] \\ &+ \bar{x}^T P_0 B[H(q)u + \ddot{q}_r + \varsigma(q,\dot{q})] \\ &+ \bar{x}^T P_0 B[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u] \\ &+ \bar{x}^T PB[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u] + \bar{x}^T P_0 BK\mathrm{e} \\ &- \mathrm{e}^T PBK\bar{x} + \frac{1}{\gamma_1}\mathrm{tr}[\dot{\bar{\Theta}}_f^T\bar{\Theta}_f] + \frac{1}{\gamma_2}\mathrm{tr}[\dot{\bar{\Theta}}_g^T\bar{\Theta}_g]\end{aligned} \tag{46}$$

Further, using the update laws (11) and (12)(with the indicated changes) in (46) yields

$$\begin{aligned}\dot{V} \leq &-\frac{1}{2}\bar{x}^T Q_0\bar{x} - \frac{1}{2}\mathrm{e}^T Q\mathrm{e} - \mathrm{e}^T PB[H(q)u + \varsigma(q,\dot{q})] \\ &+ \bar{x}^T P_0 B[H(q)u + \ddot{q}_r + \varsigma(q,\dot{q})] \\ &+ \bar{x}^T P_0 B[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u] \\ &+ \bar{x}^T PB[\bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)u] + \bar{x}^T P_0 BK\mathrm{e} \\ &- \mathrm{e}^T PBK\bar{x}\end{aligned} \tag{47}$$

which, using the control input (37), can be arranged as

$$\begin{aligned}\dot{V} \leq &-\frac{1}{2}\bar{x}^T Q_0\bar{x} - \frac{1}{2}\mathrm{e}^T Q\mathrm{e} + \bar{x}^T[2P_0 + P]B\Lambda_1 K\bar{x} \\ &- \mathrm{e}^T PB\Lambda_1 K\mathrm{e} - \mathrm{e}^T PB[\Lambda_1 - I_n]K\bar{x} \\ &+ \bar{x}^T[2P_0 B(\Lambda_1 + I_n) + PB\Lambda_1]K\mathrm{e} \\ &+ \bar{x}^T[P_0 B(\Lambda_3 + \Lambda_4) + PB\Lambda_4] - \mathrm{e}^T PB\Lambda_2\end{aligned} \tag{48}$$

where $\Lambda_1 = H(q)[\Theta_g\Psi(q)]^{-1}$, $\Lambda_2 = \Lambda_1[-\Theta_f\Phi(q,\dot{\hat{q}}) + \ddot{q}_r] + \varsigma(q,\dot{q})$, $\Lambda_3 = \Lambda_2 + \ddot{q}_r$, $\Lambda_4 = \bar{\Theta}_f\Phi(q,\dot{\hat{q}}) + \bar{\Theta}_g\Psi(q)[\Theta_g\Psi(q)]^{-1}[-\Theta_f\Phi(q,\dot{\hat{q}}) + \ddot{q}_r]$.

Further, adopting the following notation:

$$z = \begin{bmatrix}\bar{x} \\ e\end{bmatrix},$$

$$\mathcal{Q} = \begin{bmatrix} Q_0 - [2P_0 + P]B\Lambda_1 K & -[2P_0 B(\Lambda_1 + I_n) + PB\Lambda_1]K \\ PB[\Lambda_1 - I_n]K & Q + PB\Lambda_1 K\end{bmatrix},$$

$$\Gamma = \begin{bmatrix}[P_0 B(\Lambda_3 + \Lambda_4) + PB\Lambda_4] \\ -PB\Lambda_2\end{bmatrix}$$

Eq. (48) can be rewritten as

$$\dot{V} \leq -z^T\mathcal{Q}z + z^T\Gamma \tag{49}$$

Hence, if there exist definite matrices P and P_0 such that $\mathcal{Q}$ is positive definite matrix, then (49) can be upper bounded by

$$\dot{V} \leq -\lambda_{\min}(\mathcal{Q})|z|^2 + |z||\Gamma| \tag{50}$$

where $\lambda_{\min}(\mathcal{Q})$ is the smallest eigenvalue of $\mathcal{Q}$.

Hence, $\dot{V} \leq 0$ whenever $|z|$ is outside the bounded region defined by

$$|z| \leq \frac{|\Gamma|}{\lambda_{\min}(\mathcal{Q})} \tag{51}$$

which implies that $|z| \in L_\infty$ and by the same that $|e|, |\bar{x}| \in L_\infty$.

3. To prove part 3, using results 1 and 2, and the fact that $|\hat{e}| \leq |z|$ together with (37) yields the following upper bound for the control inputs:

$$|u| = \frac{1}{g_m}[f_M + q_0 + |K|\frac{|\Gamma|}{\lambda_{\min}(\mathcal{Q})}] \tag{52}$$

which implies that $|u| \in L_\infty$. □

Remarks.

1. Only the diagonal elements of $G(q)$ are estimated and used in the control inputs design. By doing this, we avoid the estimation of the coupling terms (considered here as disturbances) and the need to compute the inverse of the estimation of $G(q)$.
2. Although, the control torques (9) and (37) are presented in vector form, they can be, in practice, computed independently, since $\Theta_g\Psi(q)$ and K are diagonal matrices and no information is needed from the other inputs.
3. The proposed observer (33) and (34) does not take in account the nonlinear terms of the robot dynamic, since they are, in part, cancelled by the control loop. This simplifies the observer design and allows for various choices, following the selection of the observer gains $L_i, i = 1, \ldots, n$, such as Lunberguer observer of high gain observer.
4. The stability conditions (13), (27) arising in state feedback and (13), (44) and (49) arising in observer-based design can be formulated as LMI problems, whose feasibility can be determined using dedicated optimization software, e.g., matlab LMI toolboox.

   That is, conditions (13) and (27) involved in the state feedback design can be organized as the following LMIs:

$$P > 0 \tag{53}$$

$$-A_c^T P - PA_c + 2PB\zeta_1 - PBB^T P > 0 \tag{54}$$

and observer-based design stability conditions (13), (44) and (49) can be organized as

$$P > 0 \tag{55}$$

$$P_0 > 0 \tag{56}$$

$$\begin{bmatrix} -A_0^T P_0 - P_0 A_0 - [2P_0 + P]B\Lambda_1 K & -PB[\Lambda_1 - I_n]K \\ -[2P_0 B(\Lambda_1 + I_n) + PB\Lambda_1]K & -A_c^T P - PA_c + PB\Lambda_1 K \end{bmatrix} > 0 \tag{57}$$

The LMIs (53), (55) and (56) are used to force the positivity of P and P_0. Note that LMIs (54) and (57) are LPV since they depend on bounded varying parameters. The feasibility of these LMIs depends on the choice of the feedback gain K for (54), and the of both the feedback gain K and the observer gain L for (57).

5. Simulation results

In this section, we test the proposed observer-based adaptive fuzzy control design on the tracking control of a two-link robot. Consider a two-link manipulator described in Fig. 1. The parameters for the equation of motion (1) are

$$M = \begin{bmatrix} (m_1 + m_2)l_1^2 & m_2 l_1 l_2(s_1 s_2 + c_1 c_2) \\ m_2 l_1 l_2(s_1 s_2 + c_1 c_2) & m_2 l_2^2 \end{bmatrix},$$

$$c = \begin{bmatrix} -m_2 l_1 l_2(c_1 s_2 - s_1 c_2)\dot{q}_2^2 \\ -m_2 l_1 l_2(c_1 s_2 - s_1 c_2)\dot{q}_1^2 \end{bmatrix},$$

$$g = \begin{bmatrix} -g(m_1 + m_2)s_1 l_1 \\ -g m_2 l_2 s_2 \end{bmatrix}$$

where $c_1 = \cos(q_1)$, $c_2 = \cos(q_2)$, $s_1 = \sin(q_1)$ and $s_2 = \sin(q_2)$. The robot physical parameters are: $l_1 = l_2 = 1$ m, $m_1 = m_2 = 1$ kg, and $g = 9.81$ m/s^2.

The uncertainties terms in (1) are given by

$$\tau_c = \begin{bmatrix} \dot{q}_1 + 10\sin(3q_1) \\ 1.2\dot{q}_2 + 5\sin(2q_2) \end{bmatrix}, \qquad \tau_d = \begin{bmatrix} 2\,\mathrm{sign}(\dot{q}_1) \\ \mathrm{sign}(\dot{q}_2) \end{bmatrix}$$

The observer-based fuzzy adaptive control design for the two-link robot is as follows:

1. Since the joint positions are the only measurable quantities, the required joint velocities are estimated using the observer described in (33) and (34), with $L = \text{block-diag}[L_1, L_2]$ where $L_1 = L_2 = [50 \quad 2500]$.
2. Design the fuzzy systems to approximate $g_{11}(q)$, $g_{22}(q)$, $f_1(q, \dot{q}_2)$ and $f_2(q, \dot{q}_1)$ with $q^T = [q_1 \quad q_2]$. For this purpose the positions and velocities spaces are fuzzified using three fuzzy sets as depicted in Fig. 2. Then, the corresponding fuzzy systems rules bases are designed as follows:

$$g_{11}: \text{if } q_1 \text{ is } Z_1^j \text{ and } q_2 \text{ is } Z_2^j \quad \text{then } \hat{g}_{11}(q) \text{ is } Y_1^j, \quad j = 1, \ldots, 9 \tag{58}$$

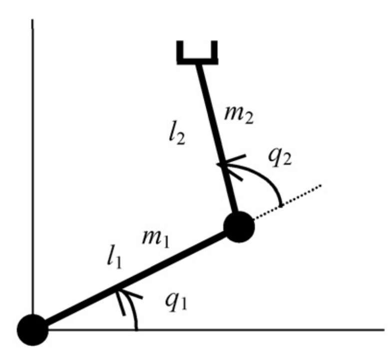

Fig. 1. A two-link robot.

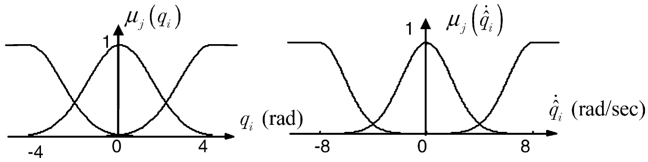

Fig. 2. Fuzzy controller membership functions.

$$g_{22}: \text{if } q_1 \text{ is } Z_1^j \text{ and } q_2 \text{ is } Z_2^j \quad \text{then } \hat{g}_{22}(q) \text{ is } Y_2^j, \quad j = 1, \ldots, 9 \tag{59}$$

$$f_1: \text{if } q_1 \text{ is } Z_1^j \text{ and } q_2 \text{ is } Z_2^j \text{ and } \dot{\hat{q}}_2 \text{ is } Z_3^j \quad \text{then } \hat{f}_1(q, \dot{\hat{q}}_2) \text{ is } Y_1^j, \quad j = 1, \ldots, 27 \tag{60}$$

$$f_2: \text{if } q_1 \text{ is } Z_1^j \text{ and } q_2 \text{ is } Z_2^j \text{ and } \dot{\hat{q}}_1 \text{ is } Z_4^j \quad \text{then } \hat{f}_2(q, \dot{\hat{q}}_1) \text{ is } Y_1^j, \quad j = 1, \ldots, 27 \tag{61}$$

where the fuzzy sets Z_l^j, $l = 1, \ldots, 4$ are fuzzy sets with the membership functions depicted in Fig. 2.

Then, the outputs of the fuzzy systems (58) and (61) are respectively, as follows:

$$\hat{g}_{11}(q) = \theta_{g_1}^T \psi_1(q) \tag{62}$$

$$\hat{g}_{22}(q) = \theta_{g_2}^T \psi_2(q) \tag{63}$$

$$\hat{f}_1(q, \dot{\hat{q}}_2) = \theta_{f_1}^T \phi_1(q, \dot{\hat{q}}_2) \tag{64}$$

$$\hat{f}_2(q, \dot{\hat{q}}_1) = \theta_{f_2}^T \phi_2(q, \dot{\hat{q}}_1) \tag{65}$$

where $\theta_{g_1}, \theta_{g_2} \in R^{9\times 1}$ and $\theta_{f_1}, \theta_{f_2} \in R^{27\times 1}$ are the fuzzy systems adjustable parameters.

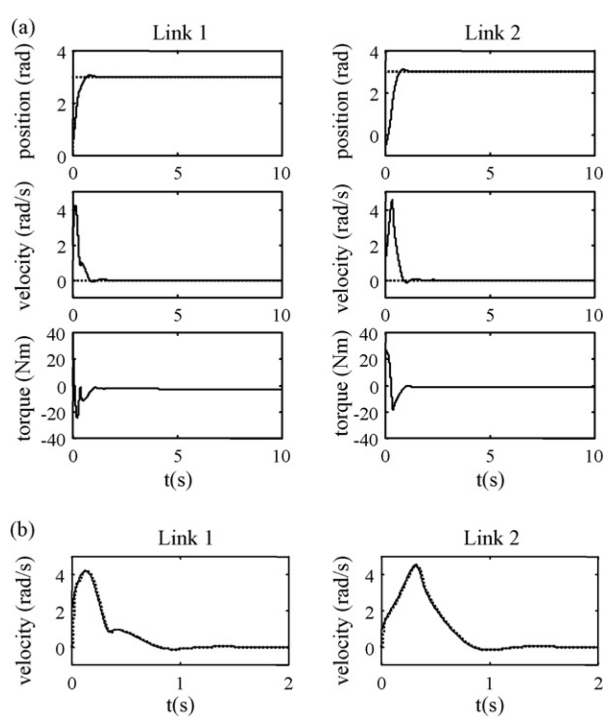

Fig. 3. Regulation in nominal case: (a) regulation performance and (b) observer performance.

3. The control input is designed as in (37) with the adaptive fuzzy systems defined in (62)–(65). The PD gain is defined as $K = \mathrm{diag}[K_1, K_2]$ with $K_1 = K_2 = [16 \quad 8]$.
4. For the choice of $Q = Q_0 = 100I_4$ and the solutions of (55)–(57) we get P and P_0.

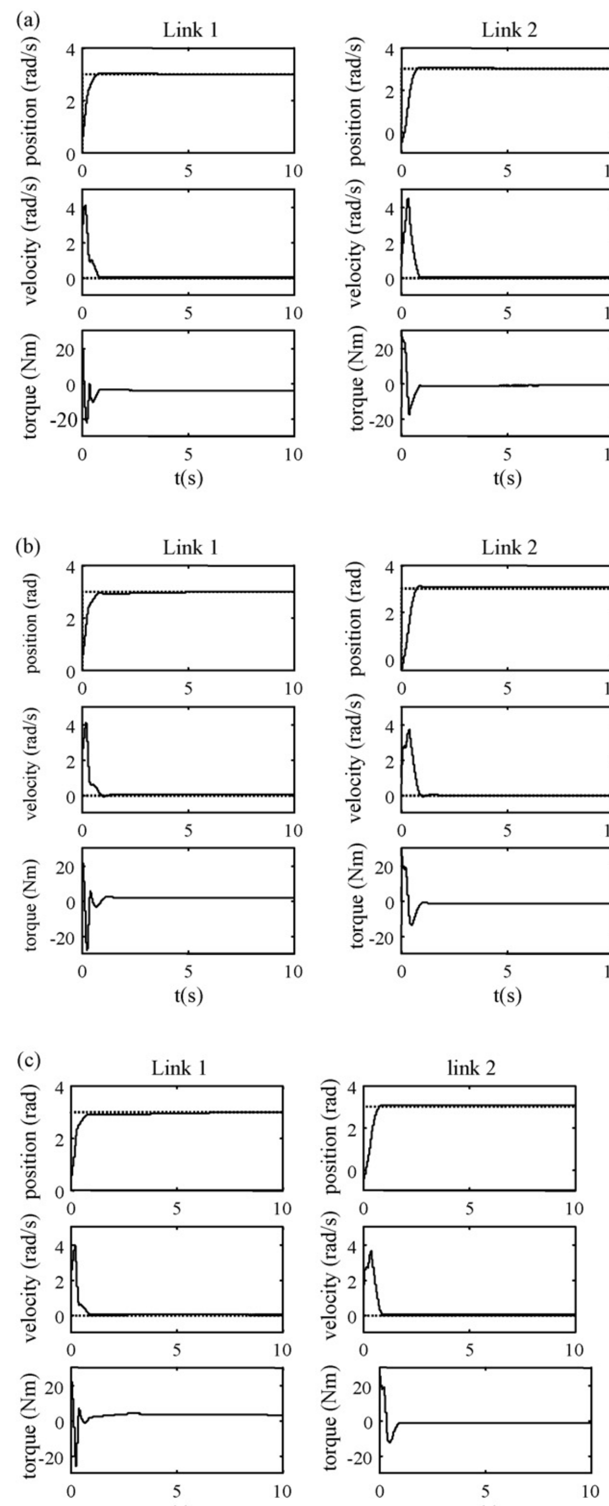

Fig. 4. Regulation under uncertainties effects: (a) τ_d effect, (b) τ_c effect and (c) $\tau_d + \tau_c$ effects.

5. By analyzing the dynamic of the robot, the following bounds are fixed $g_M = 2.5$, $g_m = 0.5$ and $f_M = 20$. The adjustable parameters are updated using (11), (12) and (14) with $\gamma_1 = 1$ and $\gamma_2 = 0.1$.

Apart from the bounds used in the step 5, no a priori knowledge on the robot dynamic is required. In the simulation, the fuzzy systems parameters are initialized as $|\theta_{g_1}(0)| = |\theta_{g_2}(0)| = 1$ and $|\theta_{f_1}(0)| = |\theta_{f_2}(0)| = 0$. The initial states, in all simulations, are $x^T(0) = [0.5 \quad 2 \quad -0.5 \quad 1]$ and $\hat{x}^T(0) = [0.5 \quad 0 \quad -0.5 \quad 0]$.

The first simulation test concerns the regulation of the joint positions to fixed values. As depicted in Fig. 3 a, the joint positions exhibit a good transient performance, and small error is remarked in steady-state regime. Fig. 3 b shows the real and estimated joint velocities. It is clear that the observer achieves fast and accurate estimation of the real joint velocities. Fig. 4 a–c shows the regulation performance for various uncertainties effects. From those figures it is seen that these uncertainties affect little the regulation performance and small steady state error is introduced and the fuzzy control achieves good compensation of the uncertainties effects.

The second test concerns the tracking performance for the reference $q_r^T = [2\sin(3t) \quad 2\sin(3t)]$. Fig. 5 a shows the tracking performance under nominal conditions, and Fig. 5 b shows the observer performance. As can be remarked, the tracking of the desired trajectories is fast and only small errors subsist after few seconds. As can seen from Fig. 6 a–c, the uncertainties effects is accounted for, and the tracking performance is little affected.

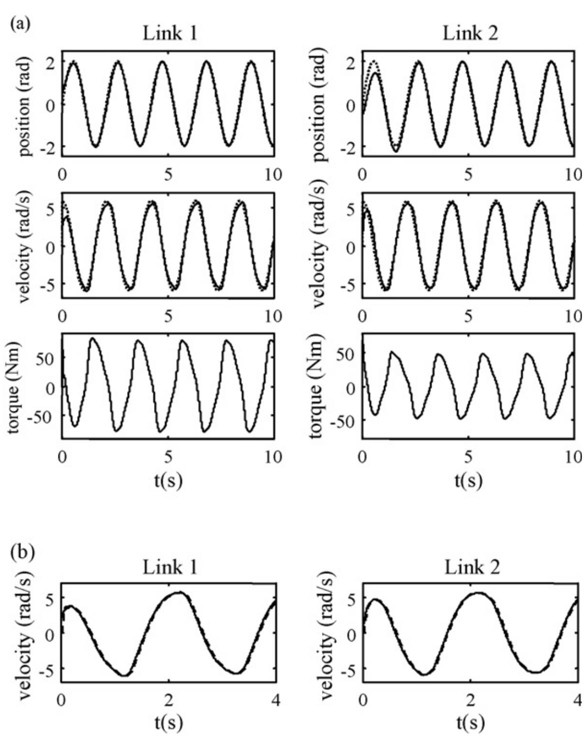

Fig. 5. Tracking in nominal case: (a) tracking performance and (b) observer performance.

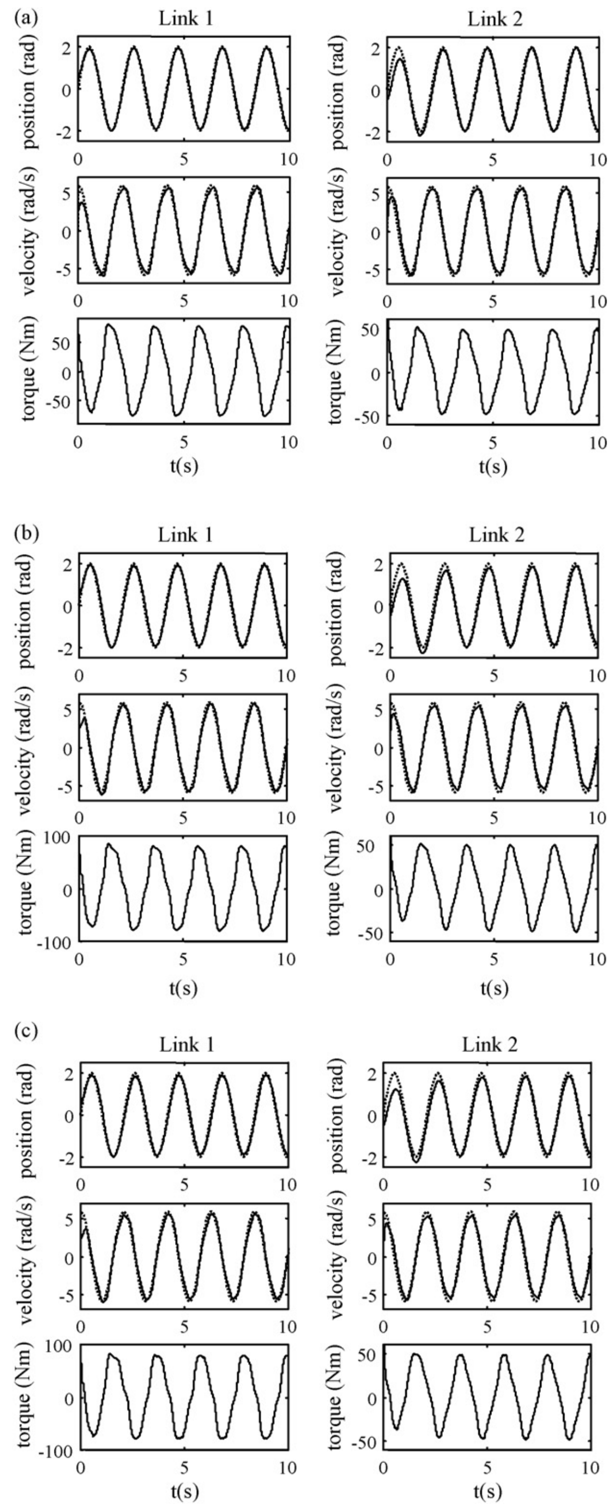

Fig. 6. Tracking under uncertainties effects: (a) τ_d effect, (b) τ_c effect and (c) $\tau_d+\tau_c$ effects.

6. Conclusion

In this paper, an adaptive fuzzy control scheme for rigid robot control, was proposed. The adaptive capability of handling modeling errors and external disturbances was demonstrated. The error convergence rate with the fuzzy adaptive approach was found to be fast. Asymptotic stability of the control system is established using the Lyapunov approach. Both cases of full state information and observer-based approach were developed to account for the unavailable joint velocities. Simulation studies for a two-link robot verify the flexibility, adaptation and tracking performance of the proposed approach. The major contributions of the paper are as follows: reduction of the fuzzy algorithm complexity and rules number compared to the cited works; no robustifying control is required to achieve the stability or to enhance the control performance; the joint velocities are not required to be available, since an observer is introduced to estimated these quantities. Finally, stability conditions can be efficiently checked using dedicated optimization software.

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