Simple Criterions to Achieve Hybrid Dislocated Synchronization of Two Identical Chua's Circuits

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Abstract—**This paper presents simple criterions to achieve synchronization scheme between two chaotic systems combining tow classical synchronizations i.e the hybrid synchronization (HS) and the dislocated synchronization(DS) , the novelty of the proposed scheme named hybrid dislocated synchronization (HDS synchronization for short) is that it is much more simple to perform. Two mathematical theorems present the necessary and sufficient conditions under which (HDS) synchronization can be achieved. To show the effectiveness of our approach, numerical example of the (HDS) synchronization of two identical Chua's circuits is executed**.

Keywords— **Chua circuit, synchronization, chaos, Lyapunov stability, simple criterion**.

I. INTRODUCTION

The concept of synchronization of chaotic systems was presented to the scientific community first by Yamada and Fujisaka [1] followed by the work of Pecora and Carroll [2],[3] . Synchronization chaos is a way to explain the sensitive dependence on initial conditions [4]-[6]. It was established that the synchronization of two or more chaotic systems shows the tendency of these systems coupled together to closely follow the same trajectory. The output of the slave system is asymptotically the output of the master system i.e. the output of the master system controls the output of the slave system.

Synchronization of chaotic systems was generalized by the discovery of several types such as generalized synchronization [7],[8], phase synchronization [9], projective synchronization [10], generalized projective synchronization [11]-[14] and anti-synchronization [15],[16]. When synchronization and anti synchronization coexist at the same time, in chaotic systems, the synchronization is called hybrid synchronization [17]-[20]. Several schemes to ensure the control and synchronization of chaotic systems were demonstrated on the basis of their potential applications in areas such as design chaos generators, secure communication [21]-[23]. So far, a variety of impressive approaches have been proposed for the synchronization of chaotic systems as the method of OGY [24] , sampled feedback synchronization method [25], the method of time delay feedback [26], adaptive design method [27], the sliding mode control method [28], the active control method [29] and back-stepping control design [30]. The paper is organized as follows. In Section 2, a presentation of general form of the dynamical chaotic systems used followed by some useful definitions relevant to classical synchronization and the detailed theoretical study of the presented synchronization scheme. In Section 3, after a brief presentation of Chua's circuit and a reinvestigation of its sensibility to the initial conditions changes as a chaotic circuit, a numerical application of the proposed scheme of two identical chaotic Chua's circuits is executed. In Section 4, conclusion concludes the paper.

II. Main results

A. Systems Presentation

In this section we present the synchronization scheme named hybrid dislocated synchronization involving two types of synchronization: the hybrid synchronization and the dislocated synchronization. We use a general form of 3D autonomous continuous dynamical chaotic system which can be applied to present a wide class of well-known chaotic systems.

Consider two 3D autonomous continuous dynamical chaotic systems (1) and (2) as master system and slave system respectively.

$$\begin{cases}\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + d_1\\ \dot{x}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + d_2\\ \dot{x}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + d_3\end{cases} \quad (1)$$

$$\begin{cases}\dot{y}_1 = a_{11}y_1 + a_{12}y_2 + a_{13}y_3 + \acute{d}_1 + u_1\\ \dot{y}_2 = a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + \acute{d}_2 + u_2\\ \dot{y}_3 = a_{31}y_1 + a_{32}y_2 + a_{33}y_3 + \acute{d}_3 + u_3\end{cases} \quad (2)$$

where $(x_1, x_2, x_3)^T$ and $(y_1, y_2, y_3)^T$ are the states of the master system and the slave system, respectively, $a_{ij} \in \mathbb{R}^{3\times3}$ are constants determined by the linear part of the autonomous continuous dynamical chaotic systems , $(d_i)_{1\le i\le 3}$, $(\acute{d}_i)_{1\le i\le 3}$ are non linear functions and u_i; i= 1; 2; 3; are synchronization controllers to be determined.

B. Hybrid Synchronization (synchronization and anti synchronization):

The error system of hybrid synchronization is defined as follows

$$e_{hi} = x_i \pm y_i \ ; \ i = 1, 2, 3 \quad (3)$$

One can see in (3) that there are several configurations in our study we take only one configuration the following

$$\begin{cases}e_{h1} = x_1 - y_1\\ e_{h2} = x_2 + y_2\\ e_{h3} = x_3 - y_3\end{cases} \quad (4)$$

To estimate the control functions, we can easily subtract (2) to (1).

C. Dislocated Synchronization:

The error system of dislocated synchronization is defined as follows

$$e_{di} = x_i \pm y_j, i = 1, 2, 3; j = 1,2,3 \quad (5)$$

With at least one configuration $i \neq j$.

In (5) there are several configurations in our study we take only the following configuration

$$\begin{cases}e_{d1} = x_1 - y_3\\ e_{d2} = x_2 - y_2\\ e_{d3} = x_3 - y_1\end{cases} \quad (6)$$

D. Hybrid Dislocated Synchronization (HDS)

The errors system of the hybrid dislocated synchronization (HDS) is defined as follows

$$\begin{cases}e_1 = x_1 - y_3\\ e_2 = x_2 + y_2\\ e_3 = x_3 - y_1\end{cases} \quad (7)$$

In order to define the threshold for chaos synchronization, one needs to define controllers u_i; i =1; 2; 3; which stabilize the synchronization errors (7), then the aim of synchronization is to make $\lim_{n\to\infty}\|e_i(t)\| = 0 \ \ i = 1; 2; 3$ where $\|.\|$ is Euclidean norm.

The error system (7) between the master system (1) and the slave system (2), can be derived to obtain error dynamical as

$$\begin{cases}\dot{e}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + d_1 - a_{31}y_1 - a_{32}y_2 - a_{33}y_3 - \acute{d}_3 - u_2\\ \dot{e}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + d_2 + a_{21}y_1 + a_{22}y_2 + a_{23}y_3 + \acute{d}_2 + u_2\\ \dot{e}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + d_3 - a_{11}y_1 - a_{12}y_2 - a_{13}y_3 - \acute{d}_1 - u_3\end{cases} \quad (8)$$

The error dynamical system (8) can be written as

$$\begin{cases}\dot{e}_1 = a_{11}e_1 + a_{12}e_2 + a_{13}e_3 + Q_1 - u_3\\ \dot{e}_2 = a_{21}e_1 + a_{22}e_2 + a_{23}e_3 + Q_2 + u_2\\ \dot{e}_3 = a_{31}e_1 + a_{32}e_2 + a_{33}e_3 + Q_3 - u_1\end{cases} \quad (9)$$

With

$$\begin{cases}Q_1 = (a_{13} - a_{31})y_1 - (a_{12} - a_{32})y_2 + (a_{11} - a_{33})y_3 + d_1 - \acute{d}_3\\ Q_2 = (a_{21} + a_{23})(y_1 + y_3) + d_2 + \acute{d}_2\\ Q_3 = (a_{33} - a_{11})y_1 - (a_{32} + a_{12})y_2 + (a_{31} - a_{13})y_3 + d_3 - \acute{d}_1\end{cases} \quad (10)$$

Theorem 1 The master system (1) and the slave system (2) are globally hybrid dislocated synchronized under the following controllers

$$\begin{cases}u_1 = Q_3 + a_{31}e_1 + a_{32}e_2 + (a_{33} + \|a_{33}\|)e_3\\ u_2 = -Q_2 - a_{21}e_1 - (a_{22} + \|a_{22}\|)e_2 - a_{13}e_3\\ u_3 = Q_1 + (a_{11} + \|a_{11}\|)e_1 + a_{12}e_2 + a_{13}e_3\end{cases} \quad (11)$$

Proof 1 the error dynamical system can be described as

$$\begin{cases}\dot{e}_1 = -\|a_{11}\|e_1\\ \dot{e}_2 = -\|a_{22}\|e_2\\ \dot{e}_3 = -\|a_{33}\|e_3\end{cases} \quad (12)$$

We construct the candidate Lyapunov function in the form

$$V(e(t)) = \frac{1}{2}\sum_{i=1}^{3} e_i^2(t) \quad (13)$$

We obtain

$$\dot{V}(e(t)) = -\|a_{11}\|e_1^2 - \|a_{22}\|e_2^2 - \|a_{33}\|e_3^2 \le 0$$

Thus, from the Lyapunov stability theory that is the zero solution of the errors system (7) is globally asymptotically stable and therefore, systems (1) and (2) are globally dislocated hybrid synchronized. That completes the proof of theorem 1.

Theorem 2 There exist a suitable control matrix $L \in \mathbb{R}^{3\times3}$ to realize the dislocated hybrid synchronization between the master system (1) and the slave system (2).

Proof 2 The error dynamical system can be written in the compact form as

$$\dot{e}(t) = Ae(t) + Q + U \quad (14)$$

Where

e (t) = $(e_1(t),e_2(t),e_3(t))^T$; $A = (a_{ij})_{3\times3}$; Q = $(Q_1;Q_2;Q_3)$ et U = $(-u_3;\ u_2;-u_1)$

Let the vector controller be

$$U = -Le(t) - Q \quad (15)$$

where L is the control matrix.

The error dynamics becomes

$$\dot{e}(t) = (A - L)e(t) \quad (16)$$

Construct the Lyapunov function candidate in the following form

With $V(e\ (t)) = e^{T}(t)\ e(t)$

We obtain

$$\dot{V}(t) = \dot{e}^T(t)e(t) + e^T(t)\dot{e}(t)$$

$$\dot{V}(t) = e^T(t)(A - L)^T e(t) + e^T(t)(A - L)e(t)$$

$$\dot{V}(t) = e^T(t)[(A - L)^T + (A - L)]e(t)$$

If the control matrix L is chosen such that $(A - L)T + (A - L)$ is negative definite matrix, then we get $\dot{V}(t) < 0$. Thus, from the Lyapunov stability theory, that is the zero solution of the error system (7) is globally asymptotically stable and therefore, systems (1) and (2) are globally dislocated hybrid synchronized.

III. Numerical examples

A. Chua Circuit Presentation

In this section we present Chua's circuit as introduced by [31]. The famous chaotic circuit consists of two linear capacitors C_1 and C_2, a linear inductor L, two linear resistors R and R_0, and a piecewise-linear negative resistor called Chua's diode. Chua's circuit is described by the following dynamical state system:

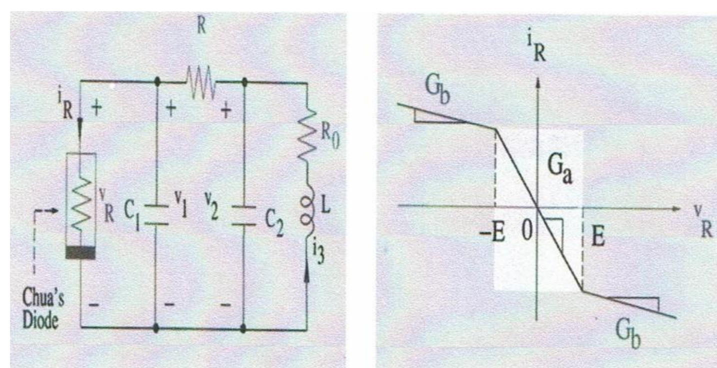
$$\begin{cases} C_1 \frac{dv_1}{dt} = G(v_2 - v_1) - Gg(v_1) \\ C_2 \frac{dv_2}{dt} = G(v_1 - v_2) + i_3 \\ L \frac{di_L}{dt} = -v_2 - R_0 i_3 \end{cases} \quad (17)$$

where v_1, v_2 and i_L are voltage across C_1, voltage across C_2 and current through L respectively with $G = 1/R$

and

$$g(v_1) = -G_b v_1 - \frac{1}{2}(G_a - G_b)\{|v_1 + E| - |v_2 - E|\}. \quad (18)$$

$g(v_1)$ is the voltage-current characteristic of the non linear resistor (Chua diode),E is the breakpoint voltage of the Chua's diode and G_a , G_b are slops in the inner and outer regions of the voltage-current characteristic as shown in fig.1(b) with corresponding circuit shown in Fig.1(a).



(a) ***(b)***

Fig.1 (a) Chua's circuit , (b) voltage-current characteristic of Chua's diode.

Let

$$\begin{cases} x_1 = v_1 \\ x_2 = v_2 \\ x_3 = i_3 \end{cases} \quad (19)$$

After simple transformations and by taking $R_0 = 0$ and $E = 1$, the system (17) can be written in its compact form as,

$$\begin{cases} \dot{x_1} = \alpha(x_2 - x_1 - g(x_1)) \\ \dot{x_2} = x_1 - x_2 + x_3 \\ \dot{x_3} = -\beta x_2 \end{cases} \quad (20)$$

with $g(x_1) = -G_b x_1 - \frac{1}{2}(G_a - G_b)(|x_1 + 1| - |x_1 - 1|)$

In spite of its extreme sensitivity to initial conditions, Chua's system (20) exhibits chaotic behavior when the parameters are

selected as α= 9,β=100/7, G_a =8/7 , G_b =5/7 and with initial conditions given by *($x_1(0);x_2(0);x_3(0)$)=(-1.6; 1; 1.6).*
The simulation results are shown in Fig.2 which displays the chaotic attractors of the Chua circuit.

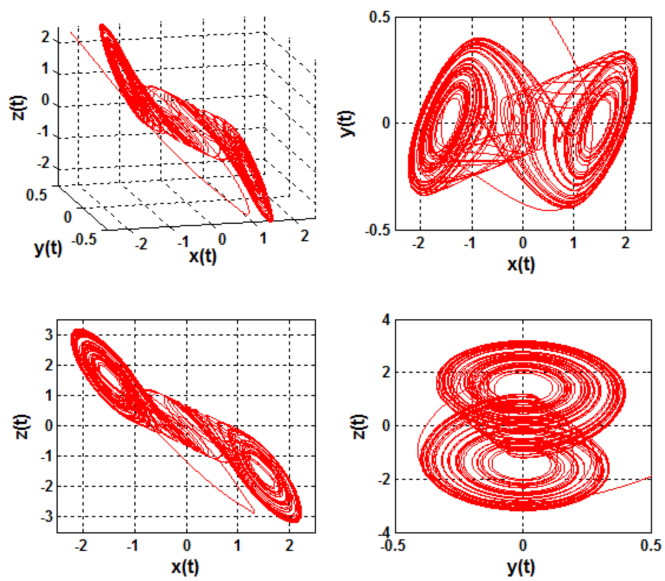


Fig.2 Chua chaotic attractors : (a) 3D attractor; (b)(c) and (d) 2D attractors with initial conditions ($x_1(0);x_2(0);x_3(0)$)=(-1.6; 1; 1.6).

To apply the hybrid dislocated synchronization (HDS) of two identical chaotic Chua systems, the master system is presented by system (20) and the slave system is taken as

$$\begin{cases} \dot{y}_1 = \alpha(y_2 - y_1 - g(y_1)) + u_1 \\ \dot{y}_2 = y_1 - y_2 - y_3 + u_2 \\ \dot{y}_3 = -\beta x_2 + u_3 \end{cases} \quad (21)$$

where $(u_1, u_2, u_3)^T$ is the vector controller.

B. Sensitivity to initial conditions (ICs) changes in Chua circuit.

To show the sensitivity to the changes in initial conditions (ICs) for Chua's chaotic system, we have plotted trajectories, shown in fig.4, of the master system (20) and the slave system (21) without controllers with the same initial conditions and with minimal changes. We have chosen four cases:

- *$x_1(0);x_2(0);x_3(0)$)*= ($y_1(0);y_2(0);y_3(0)$)= (-1; 1; 1)
- *$x_1(0);x_2(0);x_3(0)$)*= (-1; 1; 1) *and* ($y_1(0);y_2(0);y_3(0)$)= (-1.001; 1.001; 1.001)
- *$x_1(0);x_2(0);x_3(0)$)*= (-1; 1; 1) *and* ($y_1(0);y_2(0);y_3(0)$)= (-1.01; 1.01; 1.01)
- *$x_1(0);x_2(0);x_3(0)$)*= (-1; 1; 1) *and* ($y_1(0);y_2(0);y_3(0)$)= (-1.1; 1.1; 1.1).

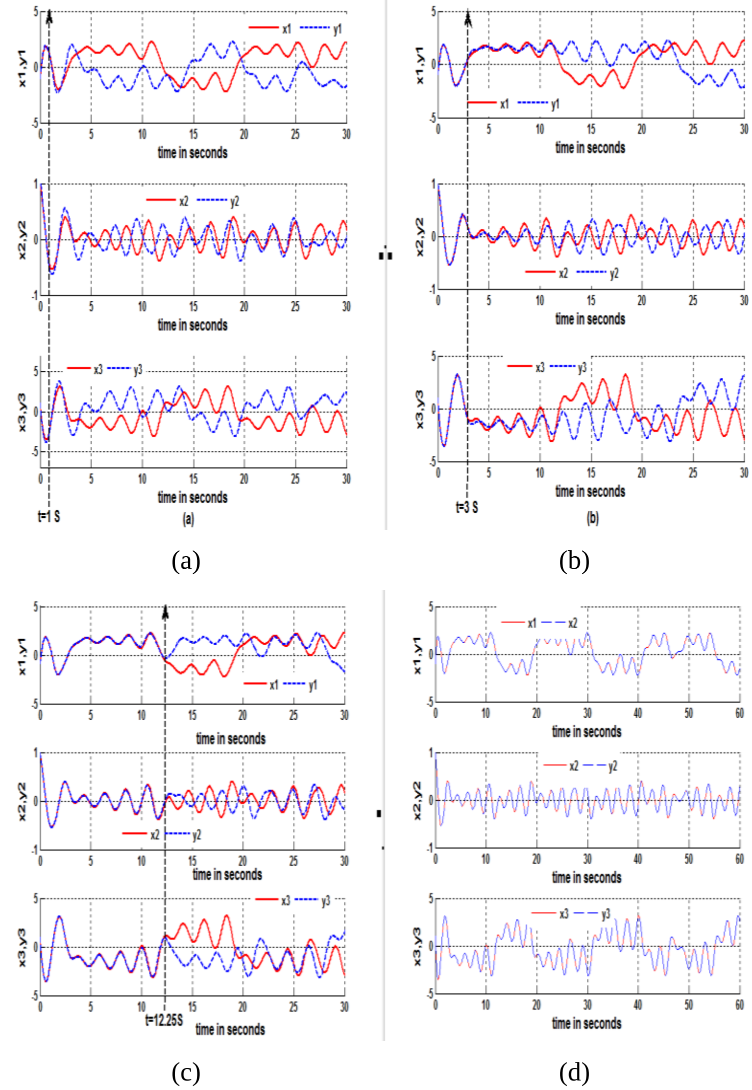


Fig.3 Temporal evolutions of master system and slave system trajectories without controllers with initial conditions as follow:
(a) *$x_1(0);x_2(0);x_3(0)$)= (-1; 1; 1) and ($y_1(0);y_2(0);y_3(0)$)= (-1.1; 1.1; 1.1).*
(b) *$x_1(0);x_2(0);x_3(0)$)= (-1; 1; 1) and ($y_1(0);y_2(0);y_3(0)$)= (-1.01; 1.01; 1.01).*
(c) *$x_1(0);x_2(0);x_3(0)$)= (-1; 1; 1) and ($y_1(0);y_2(0);y_3(0)$)= (-1.001; 1.001; 1.001).*
(d) *$x_1(0);x_2(0);x_3(0)$)= (-1; 1; 1) and ($y_1(0);y_2(0);y_3(0)$)= (-1; 1; 1).*

In fig.3, we notice that a minimal changes in the initial conditions leads to large divergences between the same trajectories. The TABLE I recapitulates the results of the simulation concerning the sensitivity to changes in the initial conditions.

TABLE I

Sensitivity to changes in Initial conditions for Chua's System

Rate of change in ICs	Time delay before the first divergence of trajectories (in seconds)
10 %	1
1 %	3
0. 1%	12.5
0	∞

In TABLE I, one can see that the decrease of the rate of change in the initial conditions leads to an increase in the time delay before the first divergence of trajectories.

C. The Hybrid Dislocated Synchronization (HDS) of Two Identical Chua's Systems

In fig.4 and fig.5 and with the help of MATLAB, we get the numerical results showing the temporal evolution of master system and slave system trajectories before and after adding controllers respectively. One can see, clearly, the hybrid dislocated synchronized trajectories of the master system (20) and the slave systems (21) displayed in fig.5.

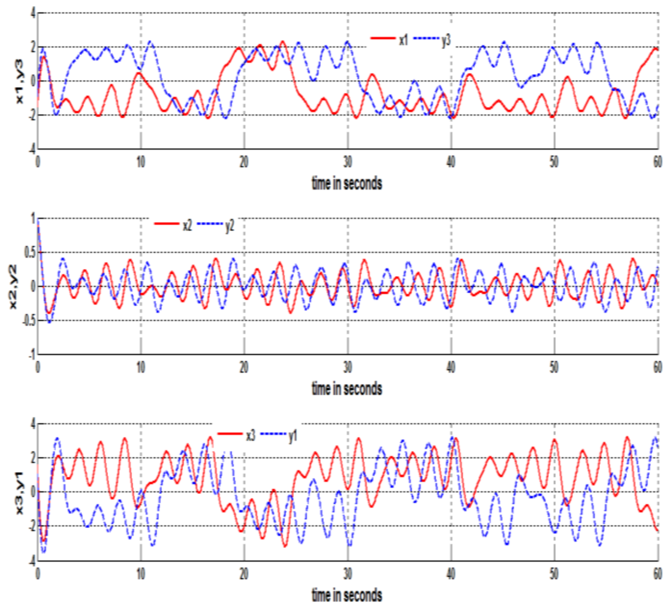


Fig.4 Temporal evolutions of master system and slave system trajectories without controllers

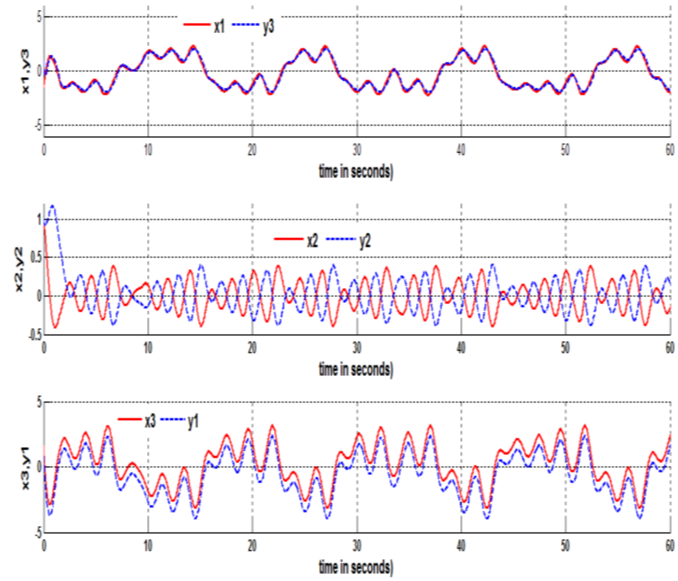


Fig.5 Temporal evolutions of master system and slave system trajectories after adding controllers

The synchronization errors between the master system and the slave system are defined for hybrid dislocated synchronization as in system (7) and according to our approach in Theorem 02, one can choose the matrix A as

$$A = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \tag{22}$$

and the control matrix L is selected as

$$L = \begin{pmatrix} 0 & \alpha & 0 \\ 1 & 0 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \tag{23}$$

We can simply show that $(A-L)^T + (A-L)$ is a negative definite matrix. Then, according to Theorem 2, the systems (20) and (21) are globally dislocated hybrid synchronized under the chosen controllers. The error functions can be written as

$$\begin{cases} \dot{e_1} = -\alpha e_1 \\ \dot{e_2} = -e_2 \\ \dot{e_3} = -2e_3 \end{cases} \tag{24}$$

Finally, fig.6 shows temporal evolution of synchronization errors .

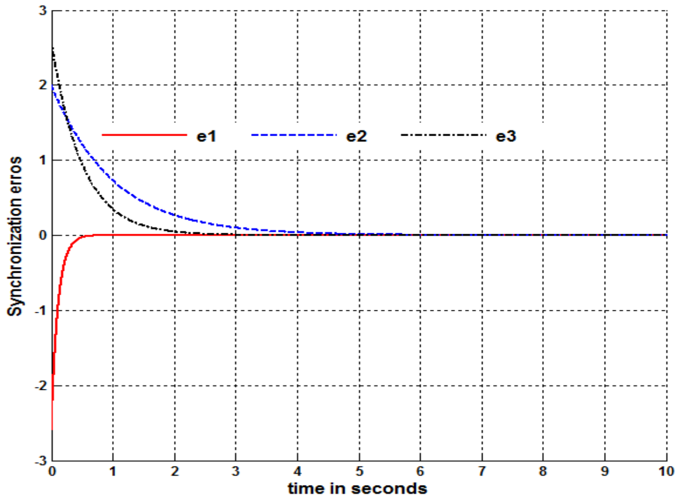


Fig.6 Temporal evolutions of synchronization errors e_1, e_2 and e_3 .

IV. CONCLUSION

In this paper, we present theoretical and simulation study results on a synchronization scheme named Hybrid Dislocated Synchronization (HDS) which can be executed between two identical systems of a large family of tridimensional autonomous continuous chaotic systems. As an example and to show the effectiveness of our approach, we use Chua's chaotic circuit to demonstrate the validity of the theoretical results. Using numerical simulations, we have showed the sensitivity to initials conditions changes of Chua's system and finally we have executed successfully the (HDS) synchronization between two identical Chua's chaotic systems. Our approach is rigorous because it's based, only, on two mathematical theorems presenting the necessary and sufficient conditions under which (HDS) synchronization can be achieved.

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