Multivariable Fuzzy Adaptive Control of Nonlinear Systems

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Abstract— **Based on Takagi-Seguno (TS) fuzzy systems, we present a direct fuzzy model-following adaptive control for multivariable (MIMO) nonlinear systems. The use of the TS fuzzy systems allows the inclusion of a priori information in terms of qualitative knowledge about the plant operating points or analytical conventional linear regulators. It is proven, using Lyapunov stability, that this adaptive scheme is robust against external disturbance, approximation error and input gain variation, and achieves asymptotic tracking of a stable reference model. The effectiveness of the proposed fuzzy approach is demonstrated, by simulation, on a two-link robot model.**

Keywords— **Fuzzy systems, Adaptive control, Reference model, Stability.**

I. Introduction

Recently, there has been a surge of interest in the adaptive control of nonlinear systems. Some adaptive control schemes via feedback linearization have been reported in [1]-[2]. Since fuzzy systems are universal approximators [3]-[6], the Adaptive control schemes that incorporate fuzzy systems have grown rapidly [3]-[4] and [7]-[14]. Fuzzy systems are used to approximate the plant's unknown nonlinearities or a desired nonlinear ideal controller. In recent years, fuzzy systems have been applied to model-following adaptive control [9]-[14]. In [9] a good performance is shown through Lyapunov stability approach. In [10] and [11] an indirect model-following control, using fuzzy linguistic model and neural networks, is proposed. In [12] a model-reference control is developed based on fuzzy basis functions of Wang [3], and in [13] a direct approach, based on Takagi-Sugeno [15] system, is proposed and the stability is analyzed in the hyperstability framework. However, the major research effort has been focused on SISO nonlinear systems, and very few results exist for MIMO systems case [14].

This work develops a new stable fuzzy direct model-following adaptive control for disturbed MIMO nonlinear continuous-time systems. The proposed Fuzzy Control system uses adaptive TS fuzzy systems to approximate the nonlinear ideal control inputs. This adaptive scheme presents the following advantages: i) the qualitative information about the plant operating points can be used to design the fuzzy controller antecedents, ii) for some operating points, if linear regulators are available (e.g., state-feedback), they can be directly incorporated into the fuzzy controllers rules consequences, and iii) Since the few rules are used, this allows fast control update, which is limit factor for some applications. Lyapunov approach is used to establish the stability and robustness properties of the proposed fuzzy adaptive scheme, in presence of approximation error, external disturbance and input gain variation. The simulation results, for a two-link robot model, show that, this fuzzy adaptive control realizes a consistent tracking performance under approximation error and external disturbance. The tracking errors are shown to converge rapidly, with very acceptable transient dynamic.

II. Problem Statement

Consider the multivariable nonlinear system given

$$\dot{x}_i = A_i x_i + b_i \left[a_i(x) + \sum_{j=1}^{p} b_{ij}(x) u_j + \eta_i \right] \qquad (1)$$

where $x_i \in R^{n_i}$ is the ith subsystem state vector, $a_i(x)$ and $b_{ij}(x)$, $i, j = 1..p$ are smooth unknown functions, η_i are unknown bounded external disturbances, u_j are the plant inputs, and $x^T = \left[\begin{array}{cccc} x_1^T & x_2^T & \ldots & x_p^T \end{array} \right] \in R^n$ is the state vector assumed to be available, with $n = n_1 + n_2 + \ldots + n_p$. A_i and b_i are given by

$$A_i = \left[\begin{array}{cc} 0 & I_{n_i - 1} \\ & 0 \end{array} \right]_{n_i \times n_i}, b_i^T = \left[\begin{array}{c} 0 \\ \vdots \\ 0 \\ 1 \end{array} \right]_{n_i \times 1}$$

where I_{n_i} is the $n_i \times n_i$ identity matrix.

The stable reference models are given by the following set of state equation

$$\dot{x}_{im} = A_{im} x_{im} + b_{im} r_i \qquad (2)$$

where $x_{im} \in R^{n_i}$ is the state vector of the ith reference model, r_i is a bounded reference input, A_{im}, b_{im} are given by

$$A_{im} = \left[\begin{array}{cc} 0 & I_{n_i - 1} \\ & -a_{im} \end{array} \right]_{n_i \times n_i}$$

and

$$b_{im}^T = \left[\begin{array}{cccc} 0 & \ldots & 0 & b_{n_i m} \end{array} \right]_{1 \times n_i}$$

with $a_{im} \in R^{n_i}$.

The tracking error dynamic is given by

$$\dot{e}_i = A_{im}e_i - b_i\left[a_i(x) + \sum_{j=1}^{p} b_{ij}(x)u_j + a_{im}x_i - b_{n_i m}r_i + \eta_i\right] \quad (3)$$

where $e_i = x_{im} - x_i$ are the tracking errors.

If the functions $a_i(x)$ and $b_{ii}(x)$ are known, then, the ideal control inputs

$$u_i^* = \frac{1}{b_{ii}(x)}\left[b_{n_i m}r_i - a_i(x) - a_{im}x_i\right] \quad (4)$$

reduces the error dynamic to

$$\dot{e}_i = A_{im}e_i - b_i\left[\sum_{j=1,j\neq i}^{p} b_{ij}(x)u_j + \eta_i\right] \quad (5)$$

Since the nonlinear functions are not known, the control inputs (4) will be approximated, in the following, with adaptive TS fuzzy systems.

III. Fuzzy Adaptive System

The controllers to be designed are Multi-input single-output TS fuzzy system [15], constituted by a set of If-Then fuzzy rules of the form

$$R_k^i : If \;\; v^i \; is \; V_k^i \; Then \; u_{f_i} = \theta_{ik}z_i \;, k = 1..m_i$$

where $\theta_{ik} \in R^{n+1}$ is the consequence vector parameters, $z_i^T = \begin{bmatrix} x^T & r_i \end{bmatrix}$, and $v^i \in R^{q_i}$ is the fuzzy controller input vector. The fuzzy sets V_k^i operate a fuzzy partition of the fuzzy controller input space.

The final output of the ith fuzzy controller is inferred as follows

$$u_{f_i} = \sum_{k=1}^{m_i} \varphi_{ik}\theta_{ik}z_i \quad (6)$$

where φ_{ik} is the normalized ring strength of the rule k, given by

$$\varphi_{ik} = \frac{\mu_{ik}(v^i)}{\sum_{j=1}^{m_i}\mu_{ik}(v^i)} \quad (7)$$

and $\mu_{ik}(v^i)$ is the grade of membership of v_i in V_k^i (i.e., the ring strength of the rule k). In this paper, it assumed that there exist always at least one active rule, i.e. $\sum_{j=1}^{m_i}\mu_{ik}(v^i) > 0$.

Consider the input control

$$u_i = u_{f_i} + u_{s_i} \quad (8)$$

where u_{s_i} is a switching control term yet to be defined.

Using (8), the error dynamic becomes

$$\begin{aligned}\dot{e}_i = \; & A_{im}e_i - b_i\Bigg[a_i(x) + b_{ii}(x)\sum_{k=1}^{m_i}\varphi_{ik}\theta_{ik}z_i \\ & + a_{im}x_i - b_{n_i m}r_i + b_{ii}(x)u_{s_i} \\ & + \sum_{j=1,j\neq i}^{p} b_{ij}(x)\left(\sum_{k=1}^{m_j}\varphi_{jk}\theta_{jk}z_j + u_{s_j}\right) + \eta_i\Bigg] \end{aligned} \quad (9)$$

Then, adding and substracting $b_{ii}(x)u_i^*$ from the right side of (9) yields

$$\begin{aligned}\dot{e}_i = \; & A_{im}e_i - b_i\Bigg[b_{ii}(x)\left(\sum_{k=1}^{m_i}\varphi_{ik}\theta_{ik}z_i - u_i^*\right) + b_{ii}(x)u_{s_i} \\ & + \sum_{j=1,j\neq i}^{p} b_{ij}(x)\left(\sum_{k=1}^{m_j}\varphi_{jk}\theta_{jk}z_j + u_{s_j}\right) + \eta_i\Bigg] \end{aligned} \quad (10)$$

Following the universal approximation results [3-6], the fuzzy controller (6) can approximate the ideal controller u_i^* on a compact operating space X_c to any degree of accuracy. Next, we define the fuzzy controller optimal parameters θ_{ik}^* be such as

$$\sup_{x,r\in X_c}\left|\sum_{k=1}^{m_i}\varphi_{ik}\theta_{ik}^*z - u_i^*\right| < \epsilon_i \quad (11)$$

where ϵ_i is arbitrary positive constant. It follows that, the control input (4) can be represented as

$$u_i^* = \sum_{k=1}^{m_i}\varphi_{ik}\theta_{ik}^*z_i + \omega_i \quad (12)$$

where ω_i is the minimum approximation error achieved by the fuzzy controller with the optimal parameters.

Using (12) in (10) yields

$$\begin{aligned}\dot{e}_i = \; & A_{im}e_i - b_i\Bigg[b_{ii}(x)\sum_{k=1}^{m_i}\varphi_{ik}\widetilde{\theta}_{ik}z_i + b_{ii}(x)u_{s_i} \\ & + \sum_{j=1,j\neq i}^{p} b_{ij}(x)\left(\sum_{k=1}^{m_j}\varphi_{jk}\theta_{jk}z_j + u_{s_j}\right) + \eta_i\Bigg] \end{aligned} \quad (13)$$

where $\widetilde{\theta}_{ik} = \theta_{ik} - \theta_{ik}^*$ is the parameters error.

IV. Stability and Robustness

To establish the stability of the feedback system given by (13), the following assumptions are needed.

Assumption 1: The input gains are bounded by $0 < \underline{b}_{ii} < |b_{ii}(x)| < \overline{b}_{ii}$, where $\underline{b}_{ii}$ and $\overline{b}_{ii}$ are positive constants, and their variations are bounded by $\left|\dot{b}_{ii}(x)\right| < \beta_{ii}(x)$ where $\beta_{ii}(x)$ are known functions.

Assumption 2: The approximation errors and the external disturbances are upper bounded by $|\eta_i| < \overline{\eta}_i$ and $|\omega_i| < \overline{\omega}_i$, respectively.

Consider the Lyapunov function

$$V = \sum_{i=1}^{p} V_i \quad (14)$$

with

$$V_i = \frac{\gamma_i}{2b_{ii}(x)}e_i^T P_i e_i + \frac{1}{2}\sum_{k=1}^{m_i}\widetilde{\theta}_{ik}\widetilde{\theta}_{ik}^T \quad (15)$$

where γ_i is positive constant and P_i is symmetric definite positive matrix solution of the following Lyapunov equation

$$A_{im}^T P_i + P_i A_{im} = Q_i \qquad (16)$$

Since A_{im} is a Hurwitz matrix, a symmetric positive definite matrix P_i always exists for any symmetric positive definite matrix Q_i [16].

The differentiation of (15) along the trajectory of (13) yields

$$\begin{aligned} \dot{V}_i = & -\frac{\gamma_i}{2b_{ii}(x)} e_i^T Q_i e_i - \frac{\gamma_i \dot{b}_{ii}(x)}{2b_{ii}^2(x)} e_i^T P_i e_i \\ & + \gamma_i \sum_{j=1, j\neq i}^{p} b_{ij}(x) \left(\sum_{k=1}^{m_j} \varphi_{jk} \theta_{jk} z_j + u_{s_j} \right) e_i^T P_i b_i \\ & + \gamma_i \left[b_{ii}(x) u_{s_i} + \eta_i - b_{ii}(x) \omega_i \right] e_i^T P_i b_i \\ & + \sum_{k=1}^{m_i} \widetilde{\theta}_{ik} \left(\dot{\widetilde{\theta}}_{ik}^T - \gamma_i \varphi_{ik} z_i e_i^T P_i b_i \right) \end{aligned} \qquad (17)$$

Using the update law

$$\dot{\theta}_{ik} = \gamma_i \varphi_{ik} z_i^T b_i^T P_i e_i \qquad (18)$$

in (17) yields

$$\begin{aligned} \dot{V}_i = & -\frac{\gamma_i}{2b_{ii}(x)} e_i^T Q_i e_i - \frac{\gamma_i \dot{b}_{ii}(x)}{2b_{ii}^2(x)} e_i^T P_i e_i \\ & + \sum_{j=1, j\neq i}^{p} b_{ij}(x) \left(\sum_{k=1}^{m_j} \varphi_{jk} \theta_{jk} z_j + u_{s_j} \right) e_i^T P_i b_i \\ & + \left[b_{ii}(x) u_{s_i} + \eta_i - b_{ii}(x) \omega_i \right] e_i^T P_i b_i \end{aligned} \qquad (19)$$

Then, to overcome the uncertainties, the switching terms u_{s_i} are chosen as

$$u_{s_i} = \left(\sum_{j=1, j\neq i}^{p} \frac{\overline{b}_{ij}}{\underline{b}_{ii}} u_{j\max} + \kappa_i \right) \operatorname{sgn}\left(e_i^T P_i b_i \right) \qquad (20)$$

where

$$\kappa_i = \sum_{j=1, j\neq i}^{p} \frac{\overline{b}_{ij}}{\underline{b}_{ii}} \sum_{k=1}^{m_i} \left| \varphi_{jk} \theta_{jk} z_j \right| + \overline{\eta}_i + \frac{\overline{\omega}_i}{\underline{b}_{ii}} \qquad (21)$$

the terms $u_{j\max}$ are the maximum on the magnitude of the control terms u_{s_j}. Using the fact that $u_{s_i} < u_{i\max}$ for $i = 1..p$, (20) can be arranged as

$$\Lambda u_{\max} \geq K \qquad (22)$$

where

$$u_{\max} = \left[\begin{array}{cccc} u_{1\max} & u_{2\max} & \cdots & u_{p\max} \end{array} \right]^T$$

$$\Lambda = \begin{bmatrix} 1 & -\frac{\overline{b}_{12}}{\underline{b}_{11}} & \cdots & -\frac{\overline{b}_{1p}}{\underline{b}_{11}} \\ -\frac{\overline{b}_{21}}{\underline{b}_{22}} & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & -\frac{\overline{b}_{p-1p}}{\underline{b}_{p-1p-1}} \\ -\frac{\overline{b}_{p1}}{\underline{b}_{pp}} & \cdots & -\frac{\overline{b}_{pp-1}}{\underline{b}_{pp}} & 1 \end{bmatrix}$$

and

$$K^T = \left[\frac{\beta_{ii}(x)}{2\underline{b}_{ii}^2} \left| e_i^T P_i b_i \right| + \kappa_i, i = 1..p \right]$$

Then, the vector $u_{\max}$ is given by

$$u_{\max} \geq \Lambda^{-1} K \qquad (23)$$

A sufficient condition for Λ to be invertible is that

$$\underline{b}_{ii} \geq \sum_{j=1, j\neq i}^{p} \overline{b}_{ij} \qquad (24)$$

Finally, introducing (20) in (19) yields

$$\dot{V}_i \leq -\frac{\gamma_i}{2b_{ii}(x)} e_i^T Q_i e_i \qquad (25)$$

which results in

$$\dot{V}_i \leq -\frac{\gamma_i}{2\overline{b}_{ii}} e_i^T Q_i e_i \qquad (26)$$

Hence e_i and $\widetilde{\theta}_{ik}$ $(i = 1..p)$ are bounded, which implies that V is bounded, and consequently $\dot{V}$ is uniformly continuous. A standard application of Barbalat's lemma [16] implies that $\dot{V} \rightarrow 0$, and hence $e_i \rightarrow 0$ for $i = 1..p$.

Remark:

Since the switching control (20) is discontinuous, the control inputs may oscillate (i.e., chattering control may occurred). In practice, chattering is undesirable, because it involves high control organ solicitation, and may excite the plant unmodeled dynamics. To avoid this problem, the signum function in (20) is replaced by the smooth nonlinearity

$$\delta(\sigma_i, \beta_i) = \left(1 + \frac{\sigma_i}{\beta_i} \right) \frac{e_i^T P_i b_i}{\left| e_i^T P_i b_i \right| + \sigma_i} \qquad (27)$$

where σ_i and β_i are positive constants. The constant σ_i is chosen based on engineering considerations to achieve an admissible tracking error. It can be shown that the tracking error converges to the bounded region given by

$$\Omega(e_i) = \left\{ e_i / \left| e_i \right| \leq \left(\frac{2\sigma_i \left(\overline{\eta}_i + \frac{\overline{\omega}_i}{\underline{b}_{ii}} \right)}{\lambda_{\min}(Q_i)} \right)^{1/2} \right\} \qquad (28)$$

In this case, the parameters error is not guaranteed to remain bounded. To prevent this situation, various modification of the update law (18) are proposed [17]; one possible solution is

$$\dot{\theta}_{ik} = \begin{cases} \gamma_i \varphi_{ik} z_i^T b_i^T P_i e_i & if \quad e_i \notin \Omega(e_i) \\ 0 & if \quad e_i \in \Omega(e_i) \end{cases} \qquad (29)$$

Hence, using the same stability arguments, one can conclude that the fuzzy controller parameters are guaranteed to remain bounded.

V. Simulation Results

The fuzzy adaptive control is evaluated on the two-link robot model given by the following equation

$$D(q)\ddot{q} + C(q,\dot{q}) + G(q) = u \tag{30}$$

where q, $\dot{q}$ and $\ddot{q}$ are the 2×1 vectors of joint positions, velocities, and accelerations, respectively. $D(q)$ is the 2×2 inertia matrix, $C(q,\dot{q})$ is the 2×1 Coriolis and centrifugal vector, $G(q)$ is the 2×1 gravitational torque vector, $F(\dot{q})$ is the 2×1 vector of the friction terms and u is the 2×1 vector of control torques. The detailed expressions of the different terms in (33) can be found in [18].The robot (see fig. 1) physical parameters are: $l_1 = l_2 = 0.5$ m, $m_1 = 2$ kg and $m_2 = 3$ kg.

The reference model is the same for both the two links and is given by $a_{1m} = a_{2m} = \begin{bmatrix} 16 & 8 \end{bmatrix}$ and $b_{1m} = b_{2m} = 16$. The TS fuzzy controller for each link is composed of three fuzzy sets (fig. 2) of the form

$$\begin{cases} \mathrm{R}_1^k : \text{If } q_1 \text{ is } V_1^k \text{ Then } u_1 = \theta_{1k} z_1 \\ \mathrm{R}_2^k : \text{If } q_2 \text{ is } V_2^k \text{ Then } u_2 = \theta_{2k} z_2 \end{cases} \quad k = 1..3$$

where $\theta_{1k}(\theta_{2k})$ is 1×5 vector of adjustable parameters, $z_1^T = \begin{bmatrix} x^T & r_1 \end{bmatrix}$ and $z_2^T = \begin{bmatrix} x^T & r_2 \end{bmatrix}$ with $x^T = \begin{bmatrix} q^T & \dot{q}^T \end{bmatrix}$. No a priori knowledge is assumed in this simulation, and the parameters are initialized to zero. The matrix Q is chosen as $Q = diag[10]$ and the solution of (16) yields P. The switching control term is chosen as in (20) with $\sigma_1 = \sigma_2 = 0.01$ and $\beta_1 = \beta_2 = 0.1$. The controllers parameters are updated using (32) with $\gamma_1 = \gamma_2 = 0.1$. The fuzzy systems approximation errors are estimated to $\overline{\omega}_1 = \overline{\omega}_2 = 0.2$.

Both trajectory tracking and step response tests are performed. In the simulation the reference inputs are chosen as $r_1 = r_2 = 3\sin(\pi t)$ (rad). The fuzzy adaptive controller performance are depicted on g. 3(a-c) for the first link and on fig. 4(a-c) for the second link. It is clear that the positions and velocities converge rapidly to their respective references, the controller achieves full rejection of the viscous friction disturbance and the approximation errors and the control inputs are seen to be smooth. As shown, the tracking errors transient dynamics are very acceptable and that the convergence to a small values around zero is fast. The step response performance is depicted in fig. 5(a-c) for the link 1 and fig. 6(a-c) for the second link. From those figures, it is apparent that the regulation task is achieved efficiently with little overshot and small steady-state error.

VI. Conclusion

Fuzzy direct adaptive control for MIMO nonlinear continuous systems, is developed. The stability and Robustness of the proposed fuzzy adaptive scheme, against approximation error, external disturbance and input gain variation are demonstrated using Lyapunov theory. Simulation results, for the two-link robot, show that good performance is achieved and effective attenuation of the uncertainties is achieved. Major features of this approach are: low computation cost, effective perturbations rejection, and fast tracking performance.

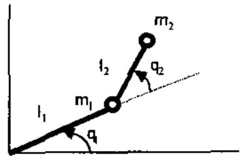

Fig .1 Tow-link Robot.

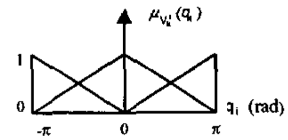

Fig .2 Membership Functions.

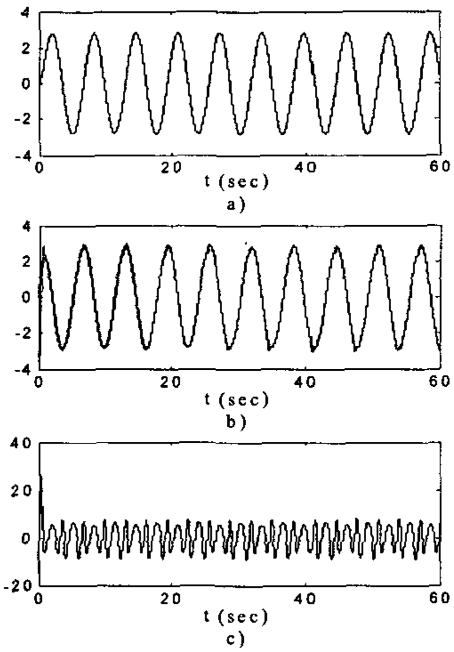

Fig. 2. Link 1 tracking: a) position, b) velocity, c) torque.

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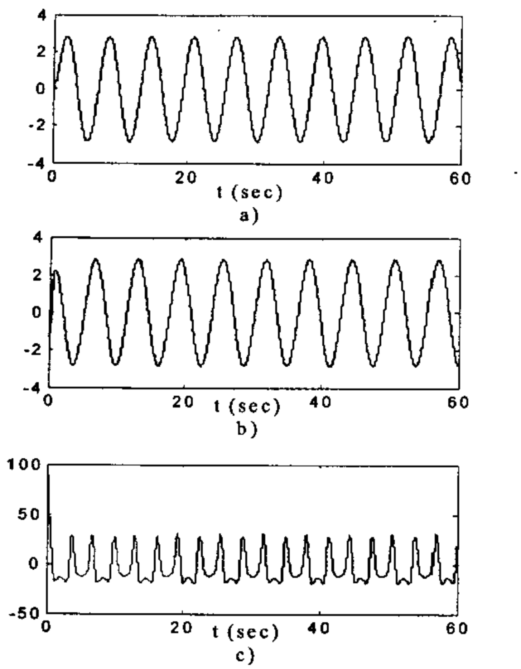

Fig. 3. Link 2 tracking: a) position, b) velocity, c) torque.

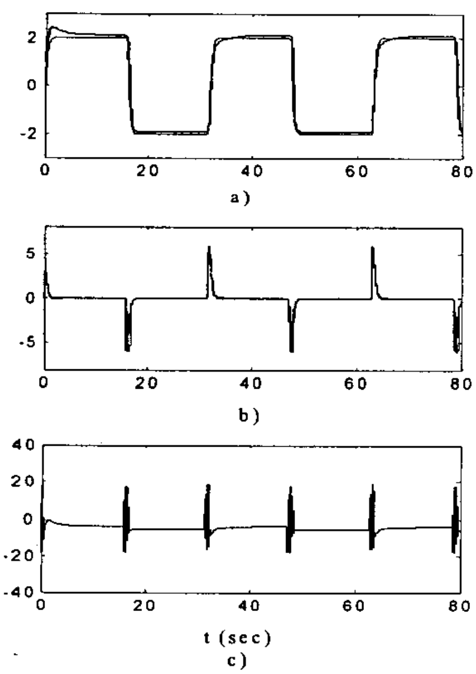

Fig. 4. Link 1 step response: a) position, b) velocity, c) torque.

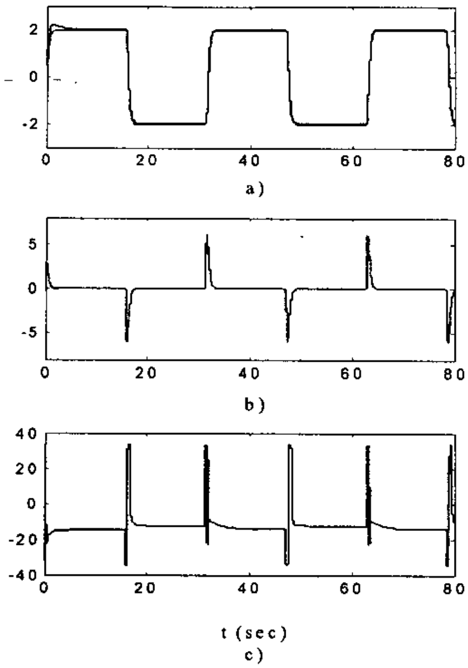

Fig. 5. Link 2 step response: a) postion, b) velocity, c) torque.

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