Generation of ECG synthetic signal using coupled ordinary differential equations and Hopf bifurcation

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***Abstract*— In this study, we present a modeling technique for producing a synthetic ECG signal with realistic PQRST morphology and prescribed heart rate dynamics including various cardiac arrhythmias. The proposed model is based on the use of three coupled ordinary differential equations, where the two first equations depend on Hopf bifurcation. The fourth-order Runge–Kutta method is used to solve numerically the model.**

The results of numerical simulation show that, a wide variety of synthetic ECG can be generated, including normal and arithmetic heartbeat; a visual examination of arithmetic ECG signals taken from the MIT database was utilized to recommend acceptable values for the model's parameters.

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Keywords*— *Dynamical model, Hopf bifurcation, synthetic electrocardiogram signals, cardiac arrhythmias

I. Introduction

The ECG (electrocardiogram) is a diagram which records the heart's electrical activity. Electrodes on the surface of the body are inserted in the ECG to capture these electrical cardiac currents [1]. A typical ECG consists of five major waves, which are typically referred to as (P, QRS and T). When the atrial depolarizes, the P wave is produced, followed by the QRS complex and the T wave [2].

Signal modeling is one of the most commonly utilized methods, ECG modeling can be used in a variety of ways, depending on the application [3], There are several studies in literature on synthetizing ECG signals by using mathematical modeling. McSharry et al.[4] use dynamic model to generate synthetic ECG signals based on coupled ordinary differential equations, both are responsible for introducing the cycle limit, the three equations have a push up and down as the P, Q, R, ,S and T waves. Li and Ma [5] proposed a data flow graph method based on piecewise curve to model ECG signals. Sameni et al. [6] proposed a 3-D dynamic model to generate ECG and ECG noise model., Clifford et al. in [7] proposed a method based on a three-dimensional vectorcardiogram (VCG) formulation to generate the normal and abnormal heartbeats. Roonizi et al. [8] proposed an approach for modeling the morphology of electrocardiogram (ECG) signals. Pascalin et al [9] generate several cardiac arrhythmias based on McSharry mathematical model and compared with a real cardiac arrhythmias.

In this study, we propose to modify the model of McSharry et al by using a novel Hopf bifurcation in the first two equations to create a new model for synthetic ECG. We believe that, such approach leads to generate a wide range of synthetic ECG signals that can provide us with some clinically significant information about human heart health.

II. Mathematical modeling of synthetic ECG

A. McSharry model

The dynamical model of McSharry is given by:

$$\begin{cases} \dot{x}_1 = \alpha x_1 - \omega x_2 \\ \dot{x}_2 = \alpha x_1 + \omega x_2 \\ \dot{x}_3 = \sum_{i\in\{P,Q,R,S,T\}} a_i \, \Delta\theta_i \, \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - (x_3 - z_0) \end{cases} \quad (1)$$

with $\alpha = 1 - \sqrt{{x_1}^2 + {x_2}^2}$, $\Delta\theta_i = (\theta - \theta_i) \bmod 2\pi$, $\theta = \operatorname{atan2}(x_1, x_2)$, ω is the angular velocity as it moves around the cycle limit. The baseline wander was introduced by coupling the baseline value in (1) to the respiratory frequency using

$$z_0(t) = A \sin(2\pi f2\, t) \quad (2)$$

Where A = 0.15 mV.

McSharry et al used for generate healthy ECG signal the typical parameters as shown in Table I.

TABLE I: PARAMETERS OF THE ECG MODEL USED IN (1)

Index(i)	P	Q	R	S	T
Time (secs)	-0.2	-0.05	0	0.05	0.3
θ_i	-1/3 π	-1/12 π	0	1/12 π	1/2 π
a_i	1.2	-5.0	30.0	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

B. The proposed model

To create our model, we propose to replace the two first equation of the McSharry model by the Hopf bifurcation [10], defined in equation (3)

$$\begin{cases} \frac{dx_1}{dt} = -(g + {x_1}^2 + {x_2}^2)x_1 - \omega x_2 \\ \frac{dx_2}{dt} = -(g + {x_1}^2 + {x_2}^2)x_2 - \omega x_1 \end{cases} \quad (3)$$

The new model is defined as:

$$\begin{cases} \dot{x_1} = -(g + {x_1}^2 + {x_2}^2)x_1 - \boldsymbol{\omega} x_2 \\ \dot{x_2} = -(g + {x_1}^2 + {x_2}^2)x_2 - \boldsymbol{\omega} x_1 \\ \dot{x_3} = \sum_{\imath \in \{P,Q,R,S,T\}} a_\imath \, \Delta\,\theta_i \; \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - (x_3 - z_0) \end{cases} \quad (4)$$

with $\Delta\theta_i = (\theta - \theta_i) \bmod 2\pi$, $\theta = \mathrm{atan2}(x_1, x_2)$,

We fixe values of the parameters g = -1.

ω is the angular velocity as it moves around the cycle limit.

The most widely used methods of integration for ordinary differential equations is the Runge-Kutta method, we use these algorithm (fourth-order Runge–Kutta) for solving the equations (4).

The equations (4) can be written as :

$$\frac{d\vec{X}}{dt} = \vec{f}\,(t, x) \quad , x(t_0) = x_0 \quad (5)$$

Where $\vec{X}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and

$$\vec{f}\,(t,x) = \begin{pmatrix} \dot{x_1} = x_1 - \boldsymbol{\omega} x_2 - (x_1^2 + x_2^2)x_1 \\ \dot{x_2} = \boldsymbol{\omega} x_1 + x_2 - (x_1^2 + x_2^2)x_2 \\ \dot{x_3} = \sum_{\imath \in \{P,Q,R,S,T\}} a_\imath \, \Delta\,\theta_i \exp\left(-\frac{\Delta\theta_i^2}{2b_i^2}\right) - (x_3 - z_0) \end{pmatrix}$$

Runge-kutta 4th order formula :

$$\vec{x}_{n+1} = \vec{x}_n + \frac{h}{6}\left[\vec{k}_1 + 2\vec{k}_2 + 3\vec{k}_3 + \vec{k}_4\right] \quad (6)$$

$$t_{n+1} = t_n + h$$

with h the time step , h=1/f_s and f_s is the sampling frequency .

The $\overrightarrow{k_1, k_2}, \vec{k}_3$ and $\vec{k}_4$ are defined as :

$$\vec{k}_1 = \vec{f}\,(t_n, \vec{x}_n) \quad (7)$$

$$\vec{k}_2 = \vec{f}\left(t_n + \frac{h}{2}, \vec{x}_n + h\frac{\vec{k}_1}{2}\right) \quad (8)$$

$$\vec{k}_3 = \vec{f}\left(t_n + \frac{h}{2}, \vec{x}_n + h\frac{\vec{k}_2}{2}\right) \quad (9)$$

$$\vec{k}_4 = \vec{f}\left(t_n + h, \vec{x}_n + h\vec{k}_3\right) \quad (10)$$

The flow chart of the Runge-Kutta method and processes in ECG synthetic are shown in fig1

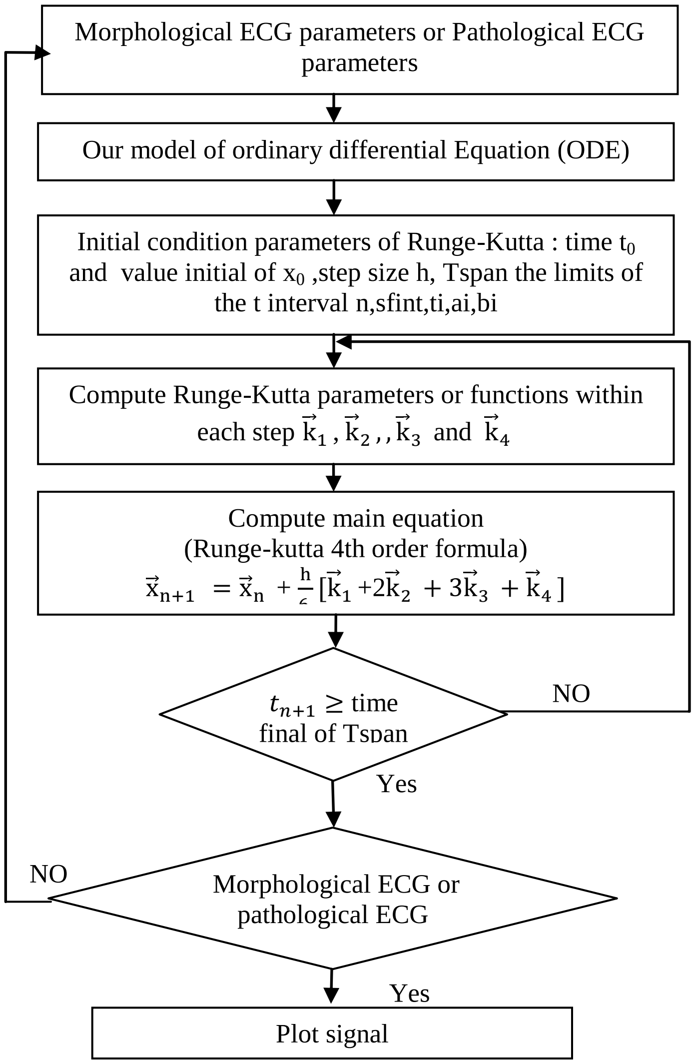

Fig1 ECG sybthetic (morphological and pathological) flow chart of the Runge-kutta 4th order

III. Results and discussion

To resolve equation (4) and as mentioned above, the fourth-order Runge–Kutta is use with fixed step size of $\Delta t = \frac{1}{f_s} = 0.0039$ s and an initial starting point equal to x_0 = [1, 0, 0.04].

In order to evaluate the ability of our model to generate reel synthetic ECG signal, a healthy and unhealthy ECG signal has been taken from the MIT database [11]. We compared generated signals by our model with those obtained from the MIT database.

To this end, visual analysis is used to suggest suitable values for our mathematical model parameters are suggested for each type of arrhythmia.. Fig 2.a shows the normal ECG signal generated with our model. and Fig 2.b shows the normal ECG signal taken from MIT database [11]. Tables 2,3,4,5,6,7,8 and 9 summarize the difference between the model created using parameter values and those taken from the MIT database.

Table 2: parameters values for the proposed model for producing sinus bradycardia ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	1.2	-5.0	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4
ω	$\boldsymbol{\frac{15\,\pi}{10}}$				

Table 3: parameters values for the proposed model for producing junctional bradycardia ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\boldsymbol{\frac{-4\,\pi}{5}}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	1.2	**1.0**	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4
ω	$\boldsymbol{\frac{4\,\pi}{3}}$				

Table 4: parameters values for the proposed model for producing tachycardia ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	1.2	-5.0	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4
ω	$\boldsymbol{4\,\pi}$				

Table 5: parameters values for the proposed model for producing flutter ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	1.2	**1**	**-10**	**-7.5**	**0.075**
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	**5.0**	**15.0**	15.0	**15**	**5.0**
b_i	0.25	0.1	0.1	0.1	0.4

Table 6: parameters values for the proposed model for producing atrial extrasystole ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\boldsymbol{\frac{-3\,\pi}{4}}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	**-1.2**	-5.0	30	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

Table 7: parameters values for the proposed model for producing ventricular extrasystole ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	**0.5**	**1.0**	**-30.0**	**0.2**	**1**
b_i	0.25	0.1	0.1	0.1	0.4

Table 8: parameters values for the proposed model for producing left branch ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\frac{\pi}{12}$	$\frac{\pi}{2}$
a_i	**-1.2**	**0.0**	**-30**	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

Table 9: parameters values for the proposed model for producing right branch ECG signal.

Index(i)	P	Q	R	S	T
Time(s)	-0.2	-0.05	0	0.05	0.3
$\theta_i\,(rad)$	$\frac{-\pi}{3}$	$\frac{-\pi}{12}$	0	$\boldsymbol{-0.4\pi}$	$\boldsymbol{0.4\pi}$
a_i	1.2	-5.0	30	**0**	**-0.3**
b_i	0.25	0.1	0.1	0.1	0.4

DISCUSSION

For all the studied cases, we observe that by using suitable parameters, the proposed model permits to generate an excellent ECG for cardiac arrhythmias similar to those taken from the MIT database.

For example, in fig.3 our model allows the generation of sinus bradycardia with a low heart rate (the rate is below than 50 Bpm); and in fig.5 the model can generate sinus tachycardia with heart rate upper than 100 Bpm.

In fig.4 we observe that, we can control the presence/absence of P or superimposed P wave with QRS complex (junctional bradycardia). The fig.6 illustrate flutter ECG signal when the frequency is greater than or equal to 300 Bpm ($\omega > 10\pi$). Atrial extrasystole is generated by the proposed model and illustrated in fig.7, where a negative P-wave is created. fig8 illustrate the ventricular extrasystole and fig.9 and fig.11 illustrate the Left branch block.

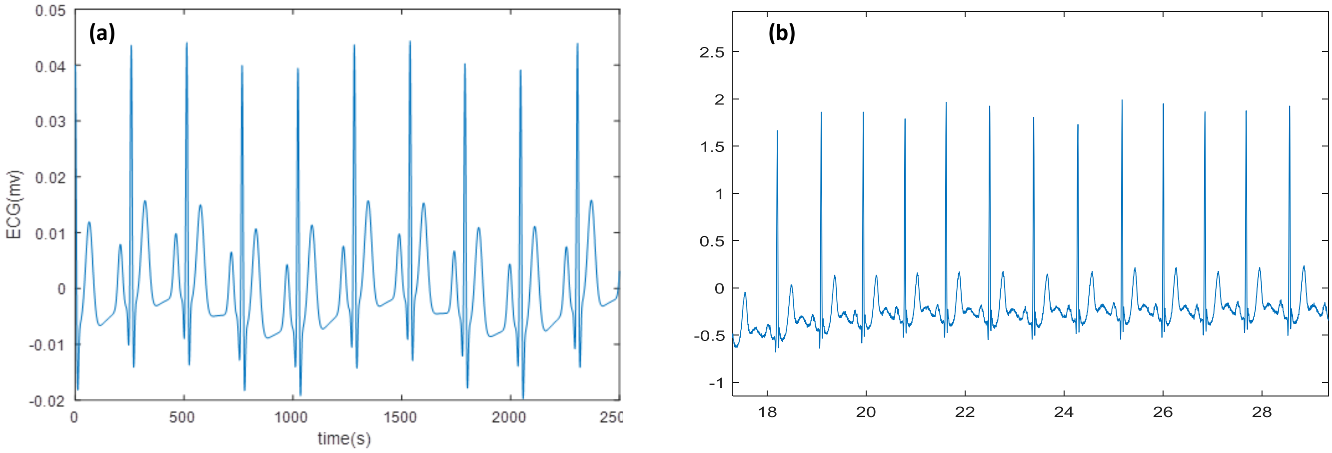

Fig. 2 Normal ECG signal: (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

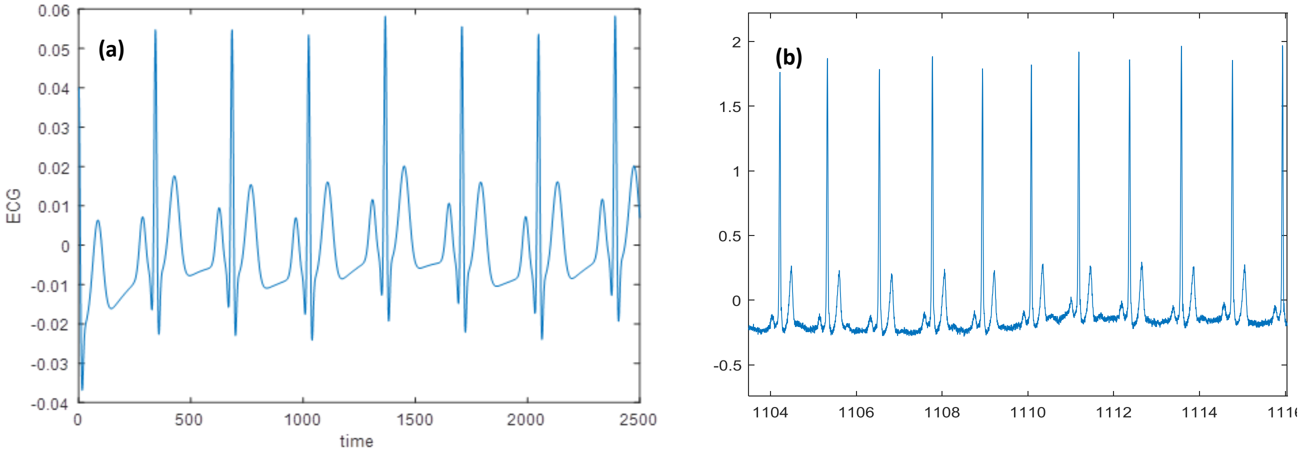

Fig. 3 Sinus bradycardia ECG signal :(**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

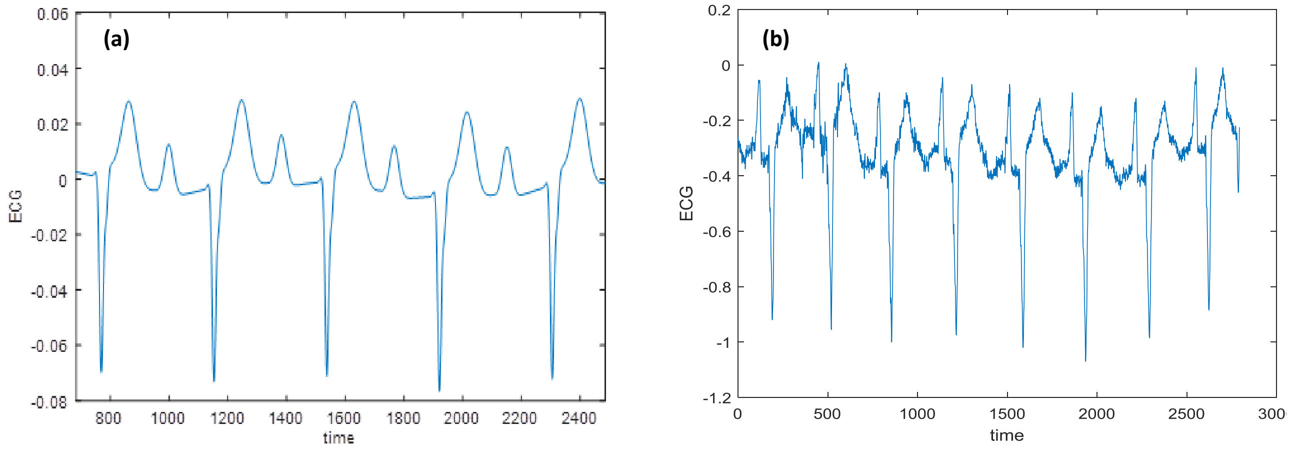

Fig. 4 Junctional bradycardia ECG signal **:** (a) generated by our model and (b) taken from MIT database (lead MLII) .

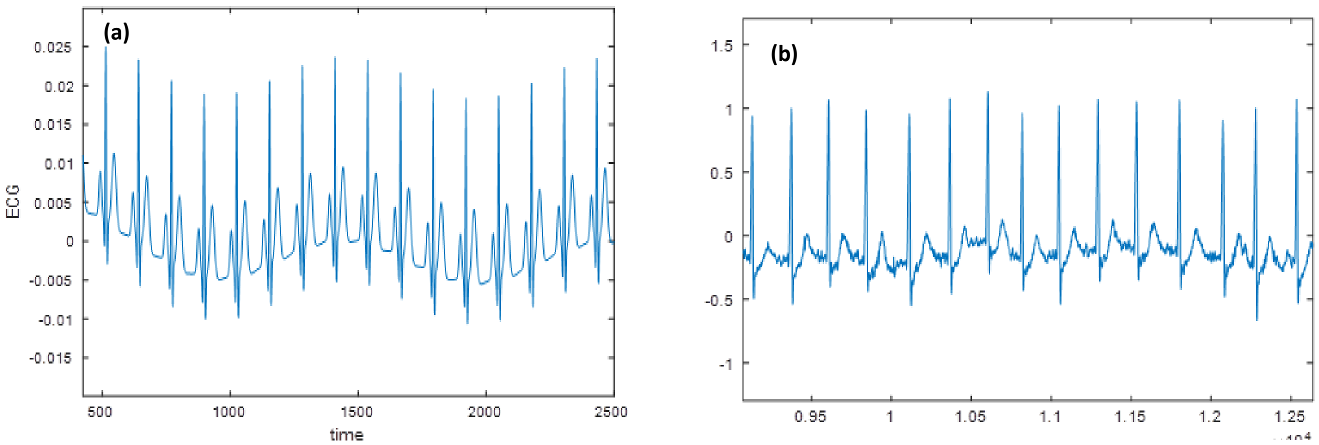

Fig. 5 Tachycardia ECG signal: (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

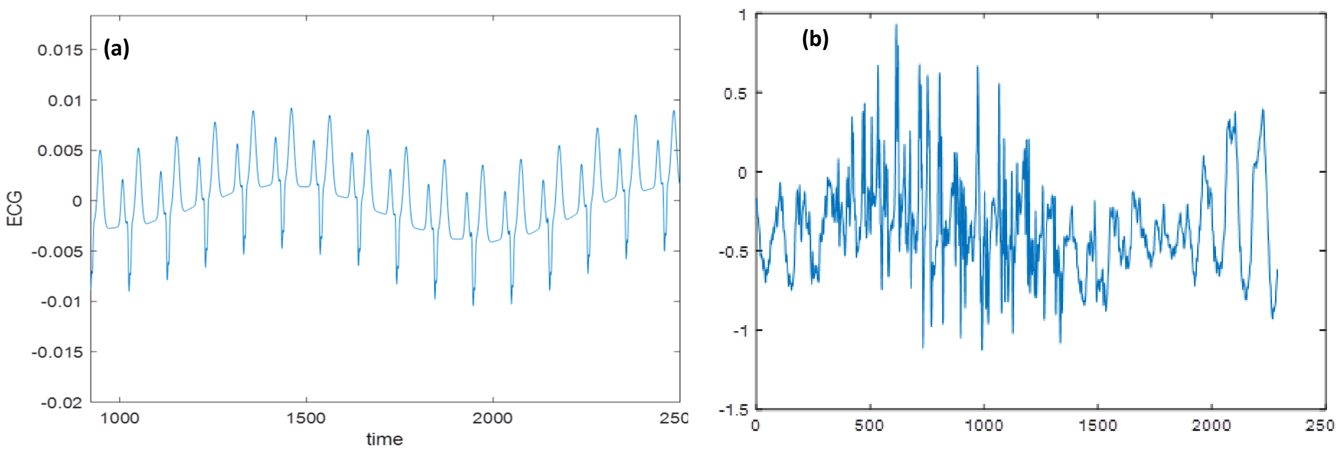

Fig.6 Flutter ECG signal **:** (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

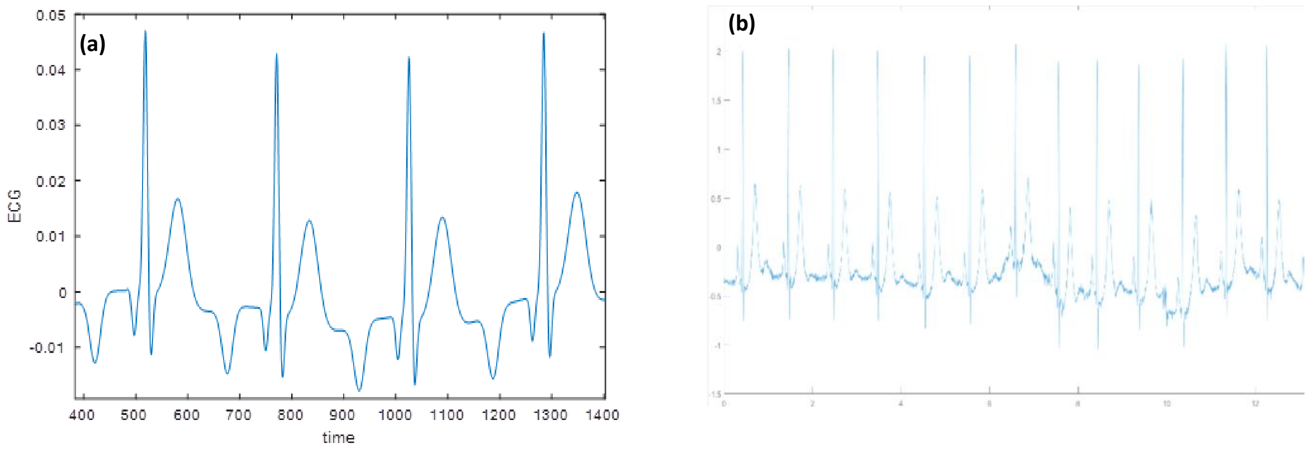

Fig. 7 Atrial extrasystole ECG signal : (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

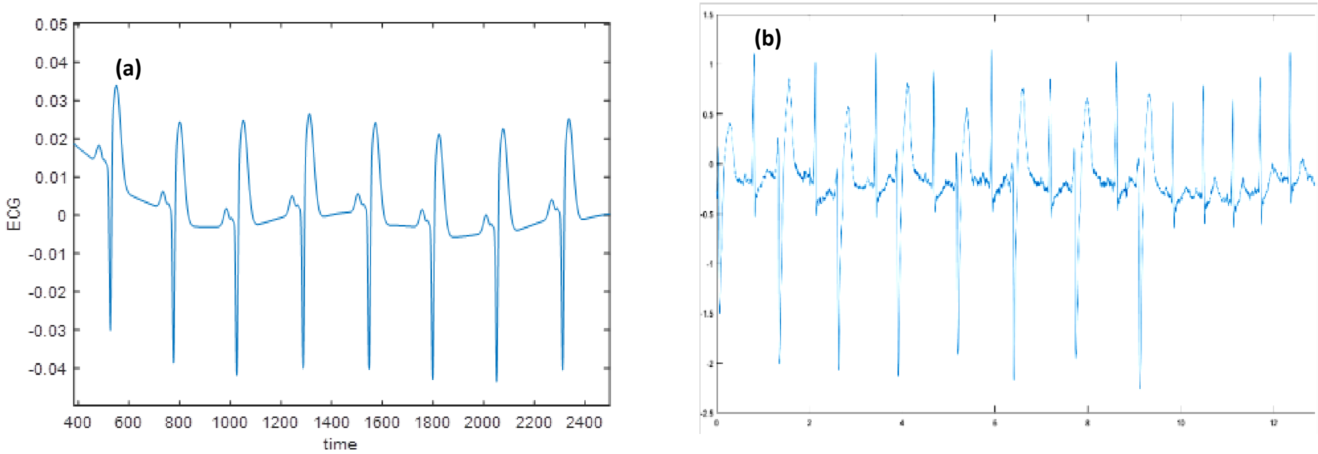

Fig. 8 Ventricular extrasystole ECG signal : (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

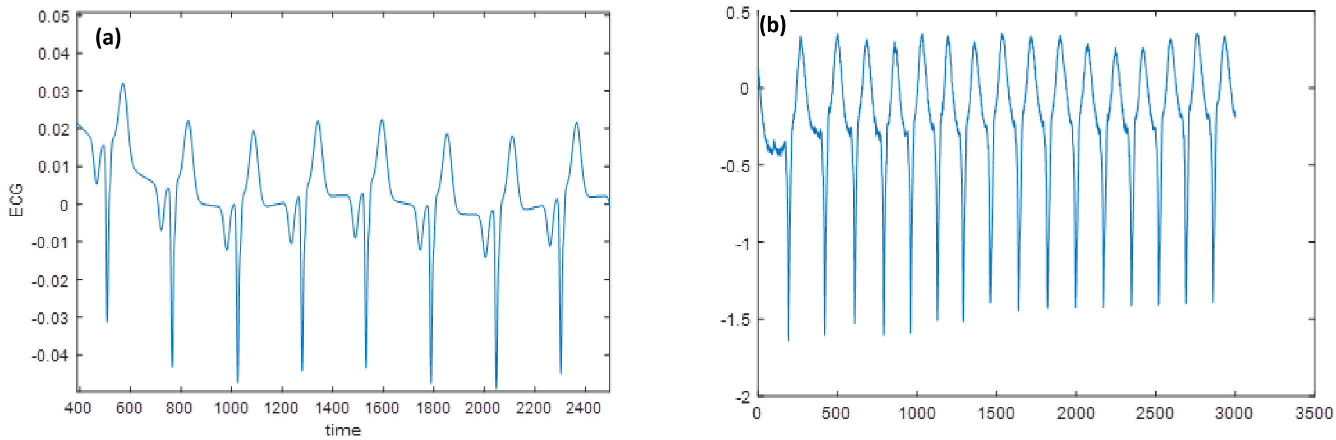

Fig. 9 Left branch block ECG signal : (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

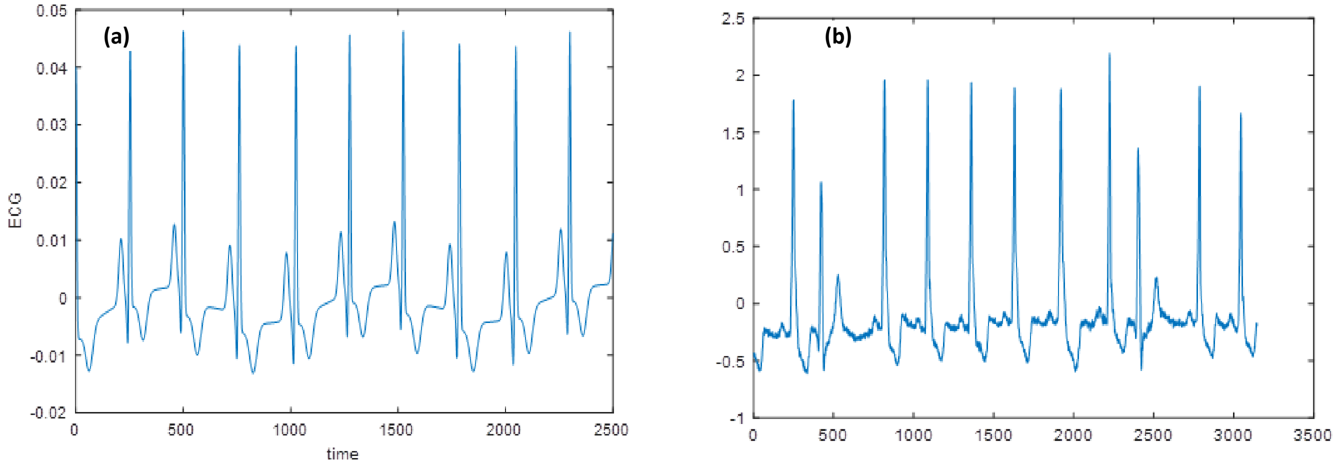

Fig. 10 Right branch block ECG signal : (**a**) generated by our model and (**b**) taken from MIT database (lead MLII) .

IV. CONCLUSION

In this paper, we propose a new model for synthetic ECG generation, this model is based on the McSharry and the use of Hopf bifurcation in the first two equations. The proposed model allows the generation of normal and pathological ECG signal.

The results from numerical simulation show that there is a great similitude between the generated ECG signal and the reel ECG taken from the MIT.

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