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Particles of Spin Zero and 1/2 in Electromagnetic Field with Confining Scalar Potential in Modified Heisenberg Algebra

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Abstract In this paper, we propose to solve the relativistic Klein Gordon and Dirac equations subjected to the action of a uniform electomagnetic field confining scalar potential yin the presence of a minimal length in the momentum space. In both cases, the energy eigenvalues and their corresponding eigenfunctions are obtained. The limiting cases is then deduced for a small parameter of deformation.

1 Introduction

Motivated by string theory [1–3], quantum gravity [4], noncommutative geometry [5], and black hole physics [6,7], the quantum mechanics in the presence of a minimal length has attracted much attention. The concept of this minimal length consists to take into account the effects of quantum fluctuations of the gravitational field in order to incorporate gravity into quantum mechanics. A significant consequence deduced from this unification is the existence of a minimal observable distance on the order of the Planck length [8–10]. This feature leads to a modification of Heisenberg uncertainty principle and plays a significant role in the regularization of certain classes of singular potentials in quantum mechanics and in the physical processes characterized by anomalies due to singularities at short distances. Futhermore, it could be considered as an effective tool to eliminate infinite quantities and divergences emerged in gauge theories.

Within this framework, big problems were solved; for example, the Schrodinger equation with the harmonic oscillator in [11–17]. The coulomb potential by [18–21], the inverse square [22], potential and the time-dependant linear potential [23], a nonminimal Woods–Saxon interaction [24], the potential well and the step potential was treated in [25], the case of a Cusp potential which includes the exponential interaction in [26], an external magnetic field in noncommutative phase space with an explicit minimal length relation was studied in [27] also in the relativistic extension of this problem, we found, the case of harmonic oscillator which has been recently considered by [28–31], the Klein–Gordon equation with harmonic oscillator in three spatial dimensions [32], the construction of the generalized Dirac equation [10], Dirac equation in the linear potential [33], Dirac equation by adding a harmonic oscillator potential [34], particles scattering [35,36], path integral for Dirac oscillator with generalized uncertainty principle [37], Dirac equation with square well and step potentials [38] and the number of publications is increasing.

On the other hand, in literature, an interaction of great importance has known many applications in various domain of physics, which is the interaction of electromagnetic fields. For example, we find the solution of the Dirac equation in orthogonal electric and magnetic fields [39,40], relativistic particles in orthogonal electric

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and magnetic fields confining scalar potentials [41], oblique electric and magnetic fields case by [42], the Klein–Gordon and Dirac equations for a particle in a constant electric field and a plan electromagnetic wave propagating along the field in [43], the motion of an electron in electromagnetic fields [44], Dirac equation in the presence of a spatially-periodic magnetic field [45], creation of spin 1/2 particles by an electric field in the sitter space [46], creation of Dirac particles in the presence of a constant electric field in an anisotropic Bianchi I Universe [47], Dirac particles in a rotating magnetic field [48], Green functions of the Dirac equation with magnetic-solenoid field [49], quantum mechanics of Klein–Gordon Fields II: relativistic coherent states [50], electron in crossed constant electromagnetic fields and a plane wave field [51], equivalent sets of solutions of the Klein–Gordon equation with a constant electric field [52]. In all these cases and to our knowledge the models have been studied with in the context of point particles.

The main purpose of this paper is to solve exactly the Klein–Gordon (K–G) and Dirac equations in momentum space in the presence of minimal length, where a particle is subjected to an external electromagnetic with confining scalar potentials in the framework of quantum mechanics with the presence of minimal length, and it will be shown that the problem admits exact analytical solutions.

The introduction of the scalar potential can be justified in different ways. Firstly, the gravitionnal field of general relativity decomposes in a scalar field and a field of spin 2 and in this case the scalar component plays an important role in the cosmological scalar field and secondly the Kaluza–Klein theory [53,54], unifying gravity with electromagnetism, incorporates it naturally in the five dimensional space-time description. In nuclear physics, the covariant interactions permits its introduction according to the experimental and theoretical considerations [55–57]. Moreover, under the laws of special relativity, it is well known that the invariance of the rest mass allows its manifestation according to the type of interaction [58].

The presence of a minimal observable length on the scale of the Planck length is one of the most important predictions of some models related to quantum gravity. In this context, the usual Heisenberg algebra was renewed and it is changed to the so-called "generalized uncertainty principle" (GUP) which obeys the following canonical commutation rules [11–13]:

$$\left[\hat{X}_i, \hat{P}_j\right] = i\hbar\delta_{ij}\left(1 + \beta P^2\right), \tag{1}$$

where β is a very small positive deforming parameter. This constant reflects the effect of classical gravitation on the Heiseinberg uncertainty principle. It limits the measurement position (and also on the momentum). Initially, it was introduced to contain divergences of quantum field theory. It is naturally found in modern theories such as string and M-theory, noncommutative geometry and quantum gravity theories in their effective forms, and it is fundamentally related to the Planck length [59].

According to algebra (1), which implement the minimum length, we have the deformed the uncertainty relation which appears in perturbation string theory and in line with the proposed UV/IR mixing

$$(\Delta X_i)(\Delta P_i) \geq \frac{\hbar}{2}\left[1 + 3\beta(\Delta P_i)^2\right], \quad i = 1, 2, 3. \tag{2}$$

where we choose the states for which $\langle P_i \rangle = 0$. For the sake of simplicity, we assume isotropic uncertainties $\Delta P_i = \Delta P$. We, therefore, arrive at a minimal uncertainty in the position given by $(\Delta X_i)_{\min} = \hbar\sqrt{3\beta}$, since from (1), the operators $\hat{X}$ and $\hat{P}$ are realized in momentum space by

$$\hat{X}_i = i\hbar\left(1 + \beta p^2\right)\frac{\partial}{\partial p_i}, \hat{P}_i = p_i, \quad \text{and} \int \frac{d^3p}{\left(1 + \beta p^2\right)} |p\rangle\langle p| = 1. \tag{3}$$

2 Resolution of Klein–Gordon Equation

The standard Klein–Gordon equation for a scalar particle of mass m_0 and charge q subjected to the action of a uniform electromagnetic field A^μ and with confining scalar potential S, which is given by ($\hbar = c = 1$):

$$\left[\left(p_\mu - qA_\mu\right)\left(p^\mu - qA^\mu\right) + (m_0 + S)^2\right]\varphi\left(t, \overrightarrow{x}\right) = 0, \tag{4}$$

where $A^\mu = \left(V, \overrightarrow{\mathbf{A}}\right)$ is the Lorentz vector potential and S denotes the Lorentz scalar potential.

In what follows, we are interested in the following choice, where V and S vary linearly in x,

$$qV = V_0 x, \quad S = S_0 x, \tag{5}$$

and $\overrightarrow{\mathbf{A}}$ describes a uniform magnetic field $\overrightarrow{\mathbf{B}}_0$ in the z direction, defined in the gauge as :

$$q\overrightarrow{\mathbf{A}} = (0, B_0 x, 0)\,. \tag{6}$$

By replacing the electromagnetic field (5) and (6) in Eq. (4), we obtain the equation written below in the momentum space and depends only one variable p_x

$$\left[\Lambda^2\left(i\left[1+\beta p^2\right]\frac{d}{dp_x}\right)^2 + 2\left(EV_0 + m_0 S_0 - Bp_y\right)\left(i\left[1+\beta p^2\right]\frac{d}{dp_x}\right) + \left(p^2 + m_0^2 - E^2\right)\right]\Psi\left(p_x\right) = 0, \tag{7}$$

where we have used the sationary solution form $\varphi\left(t, \overrightarrow{x}\right) = \exp(-iEt)\,\Psi\left(\overrightarrow{x}\right)$ and (3) with Λ is a constant given by $\Lambda = \sqrt{B_0^2 + S_0^2 - V_0^2}$.

Now introducing the following change of variable:

$$i\left[1+\alpha p_x^2\right]\frac{d}{dp_x} = i\frac{d}{dk}, \text{ with } \alpha = \frac{\beta}{1+\beta\left(p_y^2 + p_z^2\right)}, \tag{8}$$

where

$$k = \frac{1}{\sqrt{\alpha}}\arctan\sqrt{\alpha}\,p_x \text{ and } p_x = \frac{1}{\sqrt{\alpha}}\tan\sqrt{\alpha}k, \tag{9}$$

the equation (7) becomes:

$$\left[-\frac{\beta^2\Lambda^2}{\alpha^2}\frac{d^2}{dk^2} + \frac{2i\beta}{\alpha}\left(EV_0 + m_0 S_0 - B_0 p_y\right)\frac{d}{dk} + \left(p_y^2 + p_z^2 + m_0^2 - E^2\right) + \frac{1}{\alpha}\tan^2\sqrt{\alpha}k\right]\Psi\left(k\right) = 0. \tag{10}$$

Thereafter, we are interested in considering the following case where Λ is a real, in the case where the value of Λ is zero, the equation (10) reduces to a differential equation of the first order which admits the solutions devoid of bound states:

$$\Psi\left(p_x\right) = \Psi\left(0\right)\exp\left[\frac{i}{2\beta\left(EV_0 + m_0 S_0 - B_0 p_y\right)}\left(p_x + \frac{\left[\beta\left(m_0^2 - E^2\right) - 1\right]}{\sqrt{\beta\left(1+\beta\left(p_y^2+p_z^2\right)\right)}}\arctan\frac{\sqrt{\beta}p_x}{\sqrt{(1+\beta\left(p_y^2+p_z^2\right)}}\right)\right]. \tag{11}$$

Now, in order to solve Eq (10), we use the ansatz

$$\Psi\left(k\right) = \exp\left\{i\left(\frac{\left(EV_0 + m_0 S_0 - B_0 p_y\right)}{\Lambda^2\left(1+\beta\left(p_y^2+p_z^2\right)\right)}k\right)\right\}\left(\cos\sqrt{\alpha}k\right)^{\lambda} f(\sin^2\sqrt{\alpha}k). \tag{12}$$

Then, the new form of Eq.(10) will be rewritten as

$$\left[y\left(1-y\right)f'' + \left(\frac{1}{2} - \left(\lambda+1\right)y\right)f' + \frac{1}{4}\left[\frac{\alpha}{\beta^2\Lambda^2}\left(\frac{\left(EV_0 + m_0 S_0 - B_0 p_y\right)^2}{\Lambda^2} + \frac{1}{\alpha} - \left(p_y^2 + p_z^2 + m_0^2 - E^2\right)\right) - \lambda^2\right]\right]f\left(y\right) = 0, \tag{13}$$

where we have used the following change:

$$y = \sin^2\sqrt{\alpha}k, \tag{14}$$

with λ is fixed as

$$\lambda = \frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{\beta^2\Lambda^2}}. \tag{15}$$

The solution of this differential equation (13) is nothing but the hypergeometric function

$$f(y) = C_1 F(a, b, c; y) + C_2 y^{1-c} F(a+1-c, b+1-c, 2-c; y), \tag{16}$$

with

$$a = \frac{1}{2}\left[\lambda + \frac{\sqrt{\alpha}}{\beta\Lambda}\left(\frac{(EV_0+m_0S_0-B_0p_y)^2}{\Lambda^2} + \frac{1}{\alpha} - \left(p_y^2 + p_z^2 + m_0^2 - E^2\right)\right)^{\frac{1}{2}}\right], \tag{17}$$

$$b = \frac{1}{2}\left[\lambda - \frac{\sqrt{\alpha}}{\beta\Lambda}\left(\frac{(EV_0+m_0S_0-B_0p_y)^2}{\Lambda^2} + \frac{1}{\alpha} - \left(p_y^2 + p_z^2 + m_0^2 - E^2\right)\right)^{\frac{1}{2}}\right] \text{ and } c = \frac{1}{2}. \tag{18}$$

Considering the boundary condition that $(y \longrightarrow 0)$ and $(y \longrightarrow 1)$ leads $f(y)$ tending to finite, the hypergeometric function reduced to a polynomial with the following restriction

$$b = -n, \tag{19}$$

then the solution can be written in the following form

$$f(y) = C_1 F(n+\lambda, -n, \frac{1}{2}; y). \tag{20}$$

Applying Eqs (19) and (20), it is straightforward to show that the energy spectrum is:

$$E^{\beta}_{n,\pm} = -\frac{V_0\left(m_0S_0 - B_0p_y\right)}{\left(B_0^2 + S_0^2\right)} \pm \frac{\Lambda}{\left(B_0^2 + S_0^2\right)}\left\{\left(m_0B_0 + p_yS_0\right)^2 + p_z^2\left(B_0^2 + S_0^2\right)\right.$$
$$\left. + \left[1 + \beta\left(p_y^2 + p_z^2\right)\right]\left(B_0^2 + S_0^2\right)\Lambda^2\left[4n^2\beta + (2n+1/2)\beta\left(1 + \sqrt{1 + \frac{4}{\beta^2\Lambda^2}}\right)\right]\right\}^{1/2}. \tag{21}$$

However, the shape of the energy spectrum can be tested; using the limit $\beta \to 0$. We obtain the following spectrum:

$$E^{\beta=0}_{n,\pm} = -\frac{V_0\left(m_0S_0 - B_0p_y\right)}{\left(B_0^2 + S_0^2\right)} \pm \frac{\Lambda}{\left(B_0^2 + S_0^2\right)}\left\{\left(m_0B_0 + p_yS_0\right)^2 + p_z^2\left(B_0^2 + S_0^2\right) + (4n+1)\Lambda\left(B_0^2 + S_0^2\right)\right\}^{1/2} \tag{22}$$

which is the result obtained by Dominguez [41].

Also in the particular case where $B_0 = 0$ (without magnetic field), the energy spectrum (21) takes the following expression:

$$E^{\beta}_{n,\pm} = -\frac{V_0m_0}{S_0} \pm \frac{\Lambda_0}{S_0^2}\left\{\left(p_y^2 + p_z^2\right)S_0^2 + \left[1 + \beta\left(p_y^2 + p_z^2\right)\right]S_0^2\Lambda_0^2\left[4n^2\beta\right.\right.$$
$$\left.\left. + (2n+1/2)\beta\left(1 + \sqrt{1 + \frac{4}{\beta^2\Lambda_0^2}}\right)\right]\right\}^{1/2} \tag{23}$$

with $\Lambda_0 = \sqrt{S_0^2 - V_0^2}$.This gives exactly the spectrum of the one-dimensional case [60–62], wherein it is supposed that $p_y, p_z \sim 0$ because of isotropic form of the relation (1)

$$E^{\beta}_{n,\pm} = -\frac{m_0V_0}{S_0} \pm \frac{\left(S_0^2 - V_0^2\right)}{S_0}\sqrt{\beta\left(4n^2 + 2n + \frac{1}{2}\right) + \beta(2n + \frac{1}{2})\sqrt{1 + \frac{4}{\beta^2\left(S_0^2 - V_0^2\right)}}} \tag{24}$$

and now if we add the limit $\beta \to 0$, we obtain the following spectrum

$$E^{(\beta=0)}_{n,\pm} = -\frac{m_0V_0}{S_0} \pm \frac{\left(S_0^2 - V_0^2\right)^{3/4}}{S_0}\sqrt{4n+1}, \tag{25}$$

which is the same as in [62].

Now, by returning to the old variable $y = \dfrac{\beta p_x^2}{1+\beta\left(p_y^2+p_z^2\right)+\beta p_x^2}$, the final result of the solution of Eq (7) is expressed as

$$\Psi\left(p_x\right) = C_1\left(\frac{1+\beta\left(p_y^2+p_z^2\right)}{1+\beta\left(p_x^2+p_y^2+p_z^2\right)}\right)^{1/4\left(1+\sqrt{1+\frac{4}{\beta^2\Lambda^2}}\right)} \times \exp i\left[\frac{\left(EV_0+m_0S_0-B_0p_y\right)}{\sqrt{\beta}\Lambda^2\sqrt{1+\beta\left(p_y^2+p_z^2\right)}}\arctan\left(\frac{\sqrt{\beta}}{\sqrt{1+\beta\left(p_y^2+p_z^2\right)}}p_x\right)\right] F\left[n+\lambda,-n,\frac{1}{2};\frac{\beta p_x^2}{1+\beta\left(p_x^2+p_y^2+p_z^2\right)}\right]. \tag{26}$$

3 Resolution of the Dirac Equation

In this section we are interested to the case of relativistic particles with spin $\frac{1}{2}$, let us consider the Dirac equation for a Lorentz potential $\left(V,\overrightarrow{\mathbf{A}}\right)$ in addition to the scalar potential reads as follows:

$$\left[\overrightarrow{\gamma}.\left(\overrightarrow{\mathbf{p}}-q\overrightarrow{\mathbf{A}}\right)+\gamma^0\left(m_0+S\right)-\left(E-qV\right)\right]\Psi\left(\overrightarrow{x}\right)=0, \tag{27}$$

where the matrices $\overrightarrow{\gamma}$ and γ^0 are given by

$$\overrightarrow{\gamma} = \begin{bmatrix} 0 & \overrightarrow{\sigma} \\ \overrightarrow{\sigma} & 0 \end{bmatrix}, \gamma^0 = \begin{bmatrix} I_2 & 0 \\ 0 & -I_2 \end{bmatrix}, \tag{28}$$

with $\overrightarrow{\sigma} = \left(\sigma_x,\sigma_y,\sigma_z\right)$ are Pauli matrices and I_2 is the unitary matrix 2×2.

To solve the Dirac equation, we use the following ansatz:

$$\Psi\left(\overrightarrow{x}\right)=\left[\overrightarrow{\gamma}.\left(\overrightarrow{\mathbf{p}}-q\overrightarrow{\mathbf{A}}\right)+\gamma^0\left(m_0+S\right)+\left(E-qV\right)\right]\chi\left(\overrightarrow{x}\right). \tag{29}$$

By a direct calculation, from (27) and (29) we find the Dirac equation with its quadratic form in the momentum space following

$$\left[\Pi^2+(1+\beta p^2)\overline{M}\right]\chi\left(p\right)=0, \tag{30}$$

where $\overline{M}$ is given by:

$$\overline{M} = \begin{bmatrix} -B_0 & 0 & 0 & i\left(S_0+V_0\right) \\ 0 & B_0 & i\left(S_0+V_0\right) & 0 \\ 0 & -i\left(S_0-V_0\right) & -B_0 & 0 \\ -i\left(S_0-V_0\right) & 0 & 0 & B_0 \end{bmatrix}, \tag{31}$$

with

$$\Pi^2 = \left[\left(\overrightarrow{\mathbf{p}}-q\overrightarrow{\mathbf{A}}\right)^2+\left(m_0+S\right)^2-\left(E-qV\right)^2\right], \tag{32}$$

is the operator present in the Klein–Gordon equation.

To resolve (30), we diagonalize the matrix $\overline{M}$, whose eigenvalues ν_s are given by

$$\nu_s = s\Lambda \quad \text{with} \quad s=\pm 1. \tag{33}$$

where Λ is the same constant given in the previous case of spin 0.

Therefore, the solutions of the equation (30) may be constructed as follows:

$$\chi\left(p\right)=u_s\zeta_s\left(p\right), \tag{34}$$

where u_s is an eigenvector associated with the eigenvalues ν_s,

$$u_{1,s} = \begin{bmatrix} -\frac{i}{S_0 - V_0} \left(B_0 - s\Lambda\right) \chi_+ \\ \chi_- \end{bmatrix}, \quad u_{2,s} = \begin{bmatrix} \frac{i}{S_0 - V_0} \left(B_0 + s\Lambda\right) \chi_- \\ \chi_+ \end{bmatrix}, \tag{35}$$

and

$$\chi_+ = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \chi_- = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{36}$$

Now, replacing $\chi(p)$ by (35) in (30) yields the following equation:

$$\left[\Pi^2 + s\Lambda(1+\beta p^2)\right] \zeta_s(p_x) = 0. \tag{37}$$

It is remarkable that Eq. (37) converted to the Klein–Gordon equation with an additional spin field term.

Let us proceed in the same way as in the case of Klein–Gordon equation in the previous section, then using the relations (3), (5) and (6), Eq. (37) can be written as,

$$\left[\Lambda^2 \left(i\hbar\left[1+\beta p^2\right]\frac{d}{dp_x}\right)^2 + 2\left(EV_0 + m_0 S_0 - B_0 p_y\right)\left(i\hbar\left[1+\beta p^2\right]\frac{d}{dp_x}\right) + \left(p^2 + m_0^2 - E^2\right) + s\Lambda\left(1+\beta p^2\right)\right]\zeta_s(p_x) = 0, \tag{38}$$

which can be written in this form

$$\left(-\Lambda^2\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2 \frac{d^2}{dk^2} + 2i\left[1+\beta\left(p_y^2+p_z^2\right)\right]\left(EV_0 + m_0 S_0 - B_0 p_y\right)\frac{d}{dk} + \left(s\Lambda + m_0^2 - E^2\right) + (1+s\beta\Lambda)\left(p_y^2+p_z^2\right) + (1+s\beta\Lambda)\left(\frac{1+\beta\left(p_y^2+p_z^2\right)}{\beta}\right)\tan^2\sqrt{\frac{\beta}{1+\beta\left(p_y^2+p_z^2\right)}}k\right)\zeta_s(k) = 0, \tag{39}$$

where we have used the same change of variables (8) as in the previous section.

In order to solve Eq. (39), we use the ansatz

$$\zeta_s(k) = \exp i\left\{\frac{\left(EV_0 + m_0 S_0 - B_0 p_y\right)}{\Lambda^2\left[1+\beta\left(p_y^2+p_z^2\right)\right]}k\right\}\left(\cos\sqrt{\alpha}k\right)^{\lambda} g_s(\sin^2\sqrt{\alpha}k). \tag{40}$$

We obtain

$$\left[y(y-1)\frac{d^2}{dy^2} + \left[\frac{1}{2} - (\lambda+1)y\right]\frac{d}{dy} + +\frac{1}{4}\left(\frac{[1+s\beta\Lambda]}{\alpha^2\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2} - \frac{s\Lambda + m_0^2 - E^2 + \left(p_y^2+p_z^2\right)[1+s\beta\Lambda]}{\alpha\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2} - \frac{\left(EV_0 + m_0 S_0 - B_0 p_y\right)^2}{\alpha\Lambda^4} - \lambda^2\right)\right]g_s(y) = 0, \tag{41}$$

where y is given in (14) and

$$\lambda = 1 + \frac{s}{\beta\Lambda}. \tag{42}$$

The general solution of this differential equation is expressed to the hypergeometric functions in the following form,

$$g_s(y) = C_1 F(a,b,c;y) + C_2 y^{1-c} F(a+1-c, b+1-c, 2-c; y), \tag{43}$$

with

$$a=\frac{1}{2}\left[\lambda+\left(\frac{[1+s\beta\Lambda]}{\alpha^2\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2}-\frac{s\Lambda+m_0^2-E^2+\left(p_y^2+p_z^2\right)[1+s\beta\Lambda]}{\alpha\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2}-\frac{(EV_0+m_0S_0-B_0p_y)^2}{\alpha\Lambda^4}\right)^{\frac{1}{2}}\right],\ c=\frac{1}{2} \quad (44)$$

and

$$b=\frac{1}{2}\left[\lambda-\left(\frac{[1+s\beta\Lambda]}{\alpha^2\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2}-\frac{s\Lambda+m_0^2-E^2+\left(p_y^2+p_z^2\right)[1+s\beta\Lambda]}{\alpha\left[1+\beta\left(p_y^2+p_z^2\right)\right]^2\Lambda^2}-\frac{(EV_0+m_0S_0-B_0p_y)^2}{\alpha\Lambda^4}\right)^{\frac{1}{2}}\right]. \quad (45)$$

In order to determine energy spectra, we follow the same steps as in the case of spin 0. We obtain the following equation for the energy E:

$$E_{n,\pm}^{\beta}=-\frac{V_0\left(m_0S_0-B_0p_y\right)}{\left(B_0^2+S_0^2\right)}\pm\frac{\Lambda}{\left(B_0^2+S_0^2\right)}\left(\left(m_0B_0+p_yS_0\right)^2+p_z^2\left(B_0^2+S_0^2\right)\right.$$
$$\left.+\left[1+\beta\left(p_y^2+p_z^2\right)\right]\left(B_0^2+S_0^2\right)\Lambda^2\left[4n^2\beta+(4n+1)\beta\left(1+\frac{s}{\beta\Lambda}\right)+\frac{s}{\Lambda}\right]\right)^{1/2} \quad (46)$$

In the limit $\beta\to 0$, equation (46) becomes:

$$E_{n,\pm}^{\beta=0}=-\frac{V_0\left(m_0S_0-B_0p_y\right)}{\left(B_0^2+S_0^2\right)}\pm\frac{\Lambda}{\left(B_0^2+S_0^2\right)}\left[\left(m_0B_0+p_yS_0\right)^2+p_z^2\left(B_0^2+S_0^2\right)+2\left(2n+1\right)s\left(B_0^2+S_0^2\right)\Lambda\right]^{1/2} \quad (47)$$

it is the same result obtained by [41].

Via a similar calculation to that of spin 0, it is easy to obtain the following final solution:

$$\Psi_{1,s}(p_x)=\begin{bmatrix}\left[s\Lambda\left(1+\alpha p_x^2\right)\frac{\partial}{\partial p_x}-p_x+ip_y+i\frac{(B_0-s\Lambda)}{S_0-V_0}\left(E+m_0\right)\right]\\ p_z\\ \frac{i}{S_0-V_0}\left(B_0-s\Lambda\right)p_z\\ \frac{i}{S_0-V_0}\left(B_0-s\Lambda\right)\left[s\Lambda\left(1+\alpha p_x^2\right)\frac{\partial}{\partial p_x}+p_x+ip_y-i\frac{B_0+s\Lambda}{S_0+V_0}\left(E-m_0\right)\right]\end{bmatrix}\zeta\left(p_x\right), \quad (48)$$

$$\Psi_{2,s}(p_x)=\begin{bmatrix}-p_z\\ \left[s\Lambda\left(1+\alpha p_x^2\right)\frac{\partial}{\partial p_x}-p_x-ip_y-i\left(E+m_0\right)\frac{B_0+s\Lambda}{S_0-V_0}\right]\\ \frac{i}{S_0-V_0}\left(B_0+s\Lambda\right)\left[s\Lambda\left(1+\alpha p_x^2\right)\frac{\partial}{\partial p_x}-p_x+ip_y+i\frac{B_0+s\Lambda}{S_0-V_0}\left(E-m_0\right)\right]\\ \frac{i}{S_0-V_0}\left(B_0+s\Lambda\right)p_z\end{bmatrix}\zeta\left(p_x\right), \quad (49)$$

where $\zeta\left(p_x\right)$ is defined as:

$$\zeta\left(p_x\right)=C_1\left(\frac{\left(1+\beta\left(p_y^2+p_z^2\right)\right)}{1+\beta\left(p_x^2+p_y^2+p_z^2\right)}\right)^{\frac{1}{2}\left(1+\frac{s}{\beta\Lambda}\right)}\exp i\left(\frac{\left(EV_0+m_0S_0-B_0p_y\right)}{\sqrt{\beta}\Lambda^2\sqrt{1+\beta\left(p_y^2+p_z^2\right)}}\arctan\frac{\sqrt{\beta}p_x}{\sqrt{\left(1+\beta\left(p_y^2+p_z^2\right)\right)}}\right)$$
$$F\left(n+\lambda,-n,1/2;\frac{\beta p_x^2}{1+\beta\left(p_x^2+p_y^2+p_z^2\right)}\right) \quad (50)$$

4 Conclusion

In this paper, we have exposed an explicit calculation of the relativistic Klein Gordon and Dirac equations subjected to the action of a uniform electromagnetic field with confining scalar potential in the context of a minimal length in the momentum space. In both cases, the exact solution is obtained, where the wave functions are expressed by hypergeometric function. The exact energy spectrum deduced contains an additional correction, which depends on the deformation parameter, and its deviation grows quickly with on n^2 as the energy levels of a particle confined in a potential as well as in the noncommutative case. We note in the case where $\Lambda = \sqrt{B_0^2 + S_0^2 - V_0^2} = 0$, there is no bound state and no confining. From the results of this paper, if we let the parameter $\beta \to 0$, the result of this paper will be reduced to the result as [41].

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