Non-coherent Radar CFAR Detection Based on Ratios of Consecutive Order Statistics for Pareto Distributed Clutter

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***Abstract*—The Pareto distribution has been validated recently as a new statistical model for radar clutter backscattered from the sea surface for X-band high resolution maritime surveillance radar. Consequently, the derivation of the corresponding detection schemes has become of much interest, for operation under such clutter model, with the constant false alarm rate (CFAR) property. In this paper, a novel order statistic based CFAR detector operating in Pareto distributed clutter is introduced. This detector, which will be referred to us TGMR-CFAR, consists on using the trimmed geometric mean of ratios of consecutive order statistics obtained from samples of the sliding window. It is designed essentially to operate in a more realistic case, where nonhomogeneous clutter is considered. The author provides, here, a complete performance analysis for the TGMR-CFAR processor in homogenous and when interfering targets are present within the reference window. The exact expression of the false alarm probability of the proposed algorithms in a homogenous background is also presented. The detection performance of the TGMR-CFAR detector is investigated and compared with some exciting detectors by means of simulations.**

Keywords—Radar Signal Processing, Noncoherent CFAR Detection, Pareto Distributed Clutter, Order Statistics.

I. INTRODUCTION

The signal returns from radar targets are typically embedded in thermal noise and clutter, which is an unwanted backscatter from buildings, clouds, the sea, etc. Since the power of clutter and thermal noise is unknown at any given location, fixed thresholding techniques cannot be applied because they may result in an excessive number of false alarms. An attractive class of schemes that can be used to overcome the problem of clutter are the Noncoherent *constant false alarm rate* (CFAR) processing schemes which set the threshold adaptively based on local information of total noise power. Noncoherent CFAR detection is one of the main problems in practical radar signal processing systems. It was first examined by Finn and Johnson [1], where they proved that detection process could be formulated by maintaining the probability of false alarm (P_{FA}) constant in ideal situations. In this class of process, the received signal is passed into a serial shift register. The result of this is then passed again through a square-law envelope detector which converts the complex returns to amplitude or intensity measurements. For a typical CFAR detector, the square-law detected video range samples are then sent serially into a shift register (or a sliding window) of $M+1$ resolution cells. Hence, the detection threshold is set on a cell-by-cell basis using estimated local clutter power by processing a group of M reference cells surrounding a specific cell commonly called, the cell under test (CUT) (or simply test cell). We shall assume that the CUT is collocated in the middle of the sliding window, a customary assumption made in the literature. The cells surrounding the CUT are known as the clutter range profile (CRP) or reference range cells which comprise the reference window. The estimated clutter level in the reference window is then normalized by a constant, producing an adaptive threshold. The signal in the CUT is then compared to this threshold, and a target detection decision can be made. The same process is then repeated and applied to subsequent groups of data, which will be used to produce the updated estimate of the clutter power level [2, 3]. This procedure can be formulated mathematically in a statistical hypothesis testing problem. Suppose we denote by H_0 the hypothesis that the CUT does not contain a target (it instead contains only clutter). This is referred to as the null hypothesis. Let H_1 be the hypothesis that the CUT does contain a target signature plus clutter, which is known as the alternative hypothesis. Here, the target signal is combined coherently with clutter in the complex domain, and the modulus of the resultant complex measurement is taken. As the size of the reference window increases, the detection performance of a given CFAR scheme approaches that of the optimum detector which is based on a fixed threshold [3].

The statistical modeling of radar clutter was the key to any development of CFAR detection algorithms. As a result, modeling radar clutter statistics has become a very rich scientific subject over the past four decades. A variety of distributions have been proposed and reported in the literature in order to model clutter statistics, from Gaussian [3, 4], Lognormal [5] and Weibull [6], through to modern models such as the *K*, *K* plus Noise and Pareto distributions [7-10]. Recently, it has been demonstrated that the class of Pareto distributions is an excellent fit for X-Band high resolution radar clutter returns [11, 12]. Similar to the Gaussian model, the simplicity of the Pareto distribution has facilitated greatly

the development of a rich mathematical framework for the study of CFAR detection schemes [13-16]. In the same context, the present paper examines the development of a new CFAR detector operating in such distributed clutter.

Recent studies in the literature have shown that it is possible to produce CFAR detectors for Pareto distributed clutter by transforming the Gaussian intensity CFAR detectors. This can be accomplished by using the natural duality between the Exponential and Pareto distributions. The transformed schemes are significantly different in form compared to the traditional Gaussian processors extensively analyzed in the literature [2-4]. For each transformed scheme, the false alarm probability and the adaptive threshold are related through simple relationships which are linked inherently to the Gaussian detection case [13].

Furthermore, detection performance analysis reported in [13] showed that the existing detectors experienced serious shortcomings in nonhomogeneous (heterogeneous) clutter. Mainly, the geometric mean (GM) CFAR detector was shown to have the most consistent performance in homogenous clutter. However, it also experienced serious shortcomings when subjected to interference. Hence there is a need to investigate the construction of CFAR detectors that can accommodate interfering targets.

In this paper, we introduce and assess a new CFAR detector, which will be referred to us TGMR-CFAR. It consists on taking the product of *k*–1 ratios of any two consecutive ordered statistics of the reference cells to obtain the local clutter power estimate. A significant advantage of this contribution is that the proposed TGM-CFAR scheme is designed specifically to manage nonhomogeneous Pareto distributed background including multiple targets situation.

II. Pareto Distributed Clutter

The Pareto distribution has been validated as a new clutter intensity model for X-band high resolution maritime surveillance radar reverberation data [11, 12]. The validation of the Pareto model will not be provided here, since this has already been reported in the open literature. In addition to its appropriate statistical descriptions for the observed returns, the analytical simplicity of this intensity clutter model makes it more practical, especially for detection purposes, than other modern non-Gaussian distributions such as lognormal, Weibull and *K* models. Hence, the development and analysis of such detection processes has been facilitated by the elegant nature of the Pareto distributional model. As a result of the model's simplicity it has been possible to develop a rich mathematical framework for the study of CFAR detection schemes. A random variable, r.v., X is said to have a Pareto distribution with shape parameter $\alpha > 0$ and a scale factor $\beta > 0$, if its probability density function (PDF) is given by

$$f_X(x) = \frac{\alpha \beta^{\alpha}}{x^{\alpha+1}} \tag{1}$$

for $x \geq \beta$ and zero otherwise. The shape parameter governs the distribution's shape, while the scale parameter determines where its support begins. Hence such a random variable has support $[\beta, \infty[$. By integration of (1), the corresponding cumulative distribution function is given by

$$F_X(x) = \Pr(X \leq x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha} \tag{2}$$

where $\Pr(\cdot)$ denotes probability.

To denote that the random variable X has a PDF similar to (1), we sometimes use the short notation $X \sim \mathrm{Pa}(\alpha, \beta)$, throughout this paper.

The *r*th moments of $X \sim \mathrm{Pa}(\alpha, \beta)$ are given by

$$\mathrm{E}(X^r) = \alpha \beta^{\alpha} \int_{\beta}^{\infty} x^{-\alpha-1} dx = \frac{\alpha \beta^r}{\alpha - r} \tag{3}$$

provided that $\alpha > r$. Hence the mean is expressed as follow

$$\mathrm{E}(X) = \frac{\alpha \beta}{\alpha - 1} \tag{4}$$

while its variance is

$$\mathrm{Var}(\mathrm{X}) = \mathrm{E}(X^2) - (\mathrm{E}(X))^2 = \frac{\alpha \beta^2}{(\alpha-2)(\alpha-1)^2} \tag{5}$$

showing that for finite first and second order moments, one requires $\alpha > 2$. This has been found to be the case for the real data sets acquired by the DSTO Ingara Radar [12].

It is worth observing that if Y has a standard Exponential distribution with unity mean (i.e, $f_Y(y) = e^{-y}$), then one can write the following relationship

$$X = \beta\, e^{Y/\alpha} \tag{6}$$

showing that Pareto and Exponential distributions have a natural association.

We now turn to the construction of CFAR schemes for the scenario under consideration. The key to this will be the application of (6).

III. Trimmed Geometric Mean of Ratios CFAR Detector

The trimmed geometric mean of ratios of consecutive order statistics (TGMR) CFAR processor is now introduced mathematically. Suppose we have a series of M independent and identically (iid) distributed Pareto clutter measurements, $X_1, X_2, \ldots, X_M$ and the statistic of the test cell is X_0 , which is considered independent of these measurements as well. Furthermore, let $X_{(1)}, X_{(2)}, \ldots, X_{(M)}$ be the corresponding order statistics (OS) and let $R_i = X_{(i)} / X_{(i-1)}$, $1 < i \leq M$, be the ratio of the two adjacent order samples $X_{(i-1)}$ and $X_{(i)}$. Hence, for the proposed TGMR-CFAR detector, the decision about the presence (hypothesis H_1) or the absence (hypothesis H_0) of a target in the CUT is made according to

$$X_0 \underset{H_0}{\overset{H_1}{\gtrless}} \beta \prod_{i=2}^{k} \left(\frac{X_{(i)}}{X_{(i-1)}} \right)^{(M-i+1)T} \tag{7}$$

where, X_0 is the statistic of the CUT, β is the scale parameter of the Pareto distribution, $X_{(i)}$ is the *i*-th largest statistic,

$k \le M$ is an OS index which is a scalar number and T is a fixed positive scale factor.

By application of the useful duality defined in (6) to each one of the random variables appeared in (7), it is straightforward to prove that the decision rule (7) is CFAR with respect to the shape parameter α. It may be noted also that the trimmed product of ratios $\prod_{i=2}^{k}(X_{(i)}/X_{(i-1)})^{M-i+1}$ is a geometric mean variant, hence the name Trimmed Geometric Mean of Ratios (TGMR) CFAR.

It is well known that the performance of different CFAR schemes in various environments may be evaluated with either the closed-form approach or the Monte-Carlo approach using probability distribution functions. The first approach, when mathematically tractable, is generally preferred since it is less time-consuming and yields more precise results. It is used here to analyze false alarm regulation for the TGMR-CFAR algorithm since it is possible to derive a simple closed-form expression for the P_{FA}. The key input to this closed-form approach is the elegant nature and the simplicity of the Pareto distribution function used to model radar clutter. As a result of that, it has been possible to develop a rich mathematical framework for the study of many CFAR detection schemes under the assumption of this model [13-15]. The probability of false alarm of the TGMR-CFAR is then derived as follow

$$P_{FA} = \Pr\left(X_0 > \beta \prod_{i=2}^{k} (X_{(i)}/X_{(i-1)})^{(M-i+1)T} \,\middle|\, H_0 \right) \tag{8}$$

By application of the useful duality defined in (6) to the random variables appeared in (8) and then by introducing the natural logarithm function to both sides of the inner inequality of (8), it follows straightforwardly that

$$\begin{aligned} P_{FA} &= \Pr\left(Y_0 > T \sum_{i=2}^{k} (M-i+1)(Y_{(i)} - Y_{(i-1)}) \,\middle|\, H_0 \right) \\ &= \Pr\left(Y_0 > TQ \,\middle|\, H_0 \right) \end{aligned} \tag{9}$$

which can be expressed in integral form as follow

$$P_{FA} = \int_0^{\infty} \left[\int_{Tq}^{\infty} f_{Y_0|H_0}(t \mid H_0) dt \right] f_Q(q) dq \tag{10}$$

where $Q = \sum_{i=2}^{k} (M-i+1)(Y_{(i)} - Y_{(i-1)})$ and $f_Q(q)$ is its corresponding PDF.

In addition, by using relationship (6), it can be easily shown that the conditional PDF of the modified test statistic Y_0 under hypothesis H_0 is

$$f_{Y_0|H_0}(t \mid H_0) = e^{-t} \tag{11}$$

Hence, it remains only to define the PDF of the statistic Q (i.e, $f_Q(q)$) in order to solve (10).

Lemma: *Let $Y_{(1)}, Y_{(2)}, ..., Y_{(M)}$ be the order statistics of iid random sample of size M from an Exponential distribution with unity mean. Moreover, let $V_i = Y_{(i)} - Y_{(i-1)}$, i = 1,2,...,M, be the difference between the two adjacent order statistics $Y_{(i)}$ and $Y_{(i-1)}$ with the convention $Y_{(0)} \triangleq 0$. Then the random variables V_i are independent and exponentially distributed with parameter $(M-i+1)$ for each $i = 1,2,...,M$ respectively. Hence, the PDF of V_i is given by*

$$f_{V_i}(t) = (M-i+1) e^{-(M-i+1)t} \tag{12}$$

for $t > 0$.

The proof of this lemma can be done in a similar way to that presented in [3], and hence is omitted here for brevity.

Corollary: *Let $Y_{(1)}, Y_{(2)}, ..., Y_{(M)}$ be the order statistics of an iid random variable distributed according to an Exponential law with unity mean. Moreover, let W_i, i = 1,2,...,M, be another random variable, defined by $W_i = (M-i+1) V_i$, then the random variables $W_1, W_2, ..., W_M$ are iid with standard Exponential distribution with unity mean. In other words, the PDF of W_i is given by*

$$f_{W_i}(t) = e^{-t} \tag{13}$$

for $t > 0$. This corollary can be proved straightforwardly by using random variables transformation rules. It is also argued in [3] that the random variables W_i's are independent since V_i's are independent.

Hence, it follows that the statistic Q, which is the sum of $k-1$ iid standard Exponential random variables, has a Gamma distribution with parameters $k-1$ and 1. Thus

$$f_Q(q) = \frac{1}{\Gamma(k-1)} q^{k-2} e^{-q} \tag{14}$$

By resolving (10), it follows that the P_{FA} of the TGMR-CFAR processor simplifies to

$$P_{FA} = (1+T)^{-k+1} \tag{15}$$

showing that the new decision rule defined in (7) is in fact a CFAR process. Hence, CFAR control can be achieved using (15), together with the detector (7).

The corresponding expression for the probability of detection P_D is given by

$$P_D = \int_0^{\infty} \left[\int_{Tq}^{\infty} f_{Y_0|H_1}(t \mid H_1) dt \right] f_Q(q) dq \tag{16}$$

Unlike the case of P_{FA} where closed-form approach was used; the calculation of the probability of detection P_D will be carried out by using Monte-Carlo approach. This is due to lack of closed-form expression for the distribution $f_{Y_0|H_1}(t \mid H_1)$ of the statistic of the test cell, Y_0, under the alternative hypothesis H_1.

IV. Simulation Results

It is now of interest to investigate the performance of the new TGMR-CFAR detector relative to that of the Ideal, GM-CFAR and OS-CFAR detectors [13-15]. This will be accomplished by generating simulated Pareto distributed

clutter, with shape parameter $\alpha = 4.7241$ and scale parameter $\beta = 0.0446$. These values have been selected because they have arisen from Pareto fit to X-band high-resolution high-grazing angle clutter data set [13, 14]. For consistency, the design P_{FA} is set to 10^{-4}, which allows for 10^6 Monte Carlo runs to be sufficient for estimation of the resultant probabilities of false alarm. A Swerling 1 fluctuation model is used throughout, to produce a synthetic target in the test cell and also to generate interfering targets in the reference cells [17]. The OS index selected to be $k = M - 2$ for the two considered processors OS and TGMR-CFAR. In order to provide an indication of detector performance, the P_D is plotted as a function of the signal-to-clutter ratio (SCR), which has been estimated using 10^6 Monte-Carlo runs.

Figures 1-3 show detection performance as a function of SCR, in the case of homogeneous clutter for different window size (M = 16, 24 and 32 respectively). Comparing these figures, we can conclude that the GM-CFAR detector has almost the best performance followed by the proposed TGMR-CFAR processor, while the OS-CFAR exhibits an extra detection loss compared to the TGMR-CFAR. For $M = 16$ the detection performance loss of all schemes is quite large compared with the ideal decision rule, but decreases considerably for increasing M (See Figures 2 and 3).

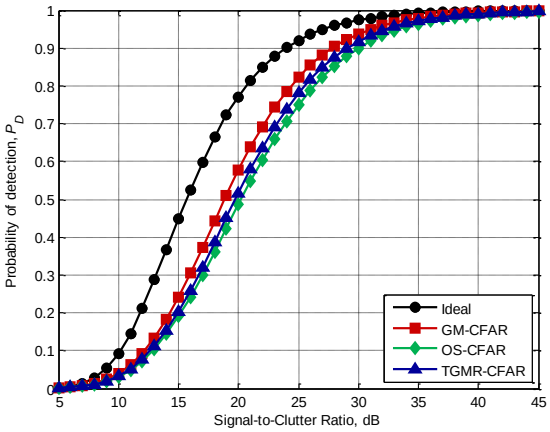

Fig. 1. Detection performance of Ideal, GM, OS and TGMR processors in homogenous clutter for M = 16.

The performance of the studied detectors becomes much more interesting when independent interfering targets are inserted into the reference cells (CRPs). In this case, we define the interference-to-clutter ratio (ICR) of the interfering target, and hence the remaining reference cells are supposed to have identical total power clutter. Next, the effect of these interfering target is investigated. In order to simulate such effects to test the robustness of the proposed detector, interfering targets are generated, as mentioned before, according to Swerling 1 model, with a fixed ICR and then inserted into a cell in the CRP and combined with the clutter in the complex domain. Detection performance is illustrated when each detector is subjected to only one interfering target, and then when each one of them is subjected to two interfering targets simultaneously. Here we observe a significant change in the performance when compared to the case of no interference discussed above. In Fig. 4, for a single interfering target, with an *ICR* = 20dB, the GM-CFAR performs very poorly, while the OS-CFAR experiences a slight CFAR loss. By contrast the proposed TGMR detector performs very well in this situation. When subjected to two interfering targets, the GM detector saturates as shown in Fig. 5, while the OS is slightly worse than the TGMR. Based upon these numerical simulations, it has been found that the proposed TGMR detector provides robust solution to interference, with acceptable CFAR loss relative to the ideal detector.

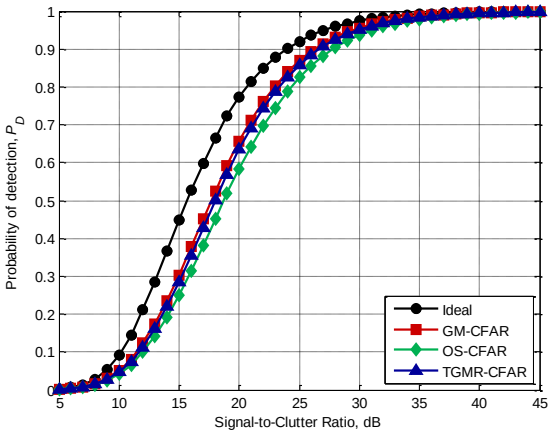

Fig. 2. Detection performance of Ideal, GM, OS and TGMR processors in homogenous clutter for M = 24.

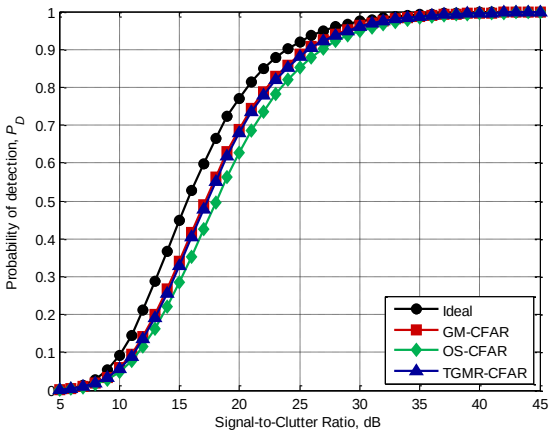

Fig. 3. Detection performance of Ideal, GM, OS and TGMR processors in homogenous clutter for M = 32.

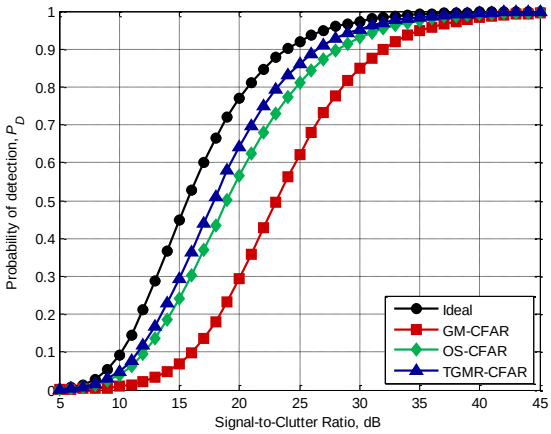

Fig. 4. Detection performance of Ideal, GM, OS and TGMR processors in the presence of one interferring target with *ICR* = 20dB and for M = 24.

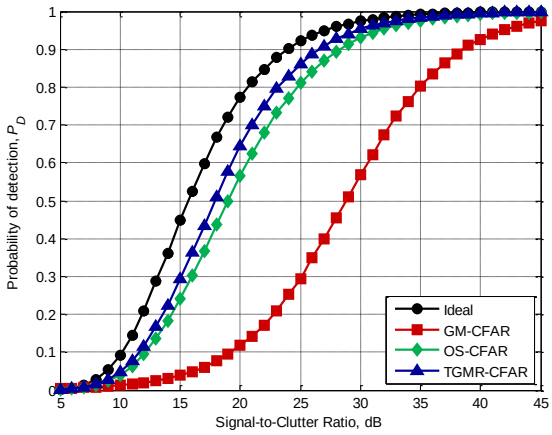

Fig. 5. Detection performance of Ideal, GM, OS and TGMR processors in the presence of two interferring targets with *ICR* = 20dB and for *M* = 24.

V. Conclusion

In this paper, we have considered the problem of Constant False Alarm Rate (CFAR) detection in homogeneous and heterogeneous Pareto clutter. Assuming Pareto distributed clutter, we have proposed the TGMR-CFAR detection scheme, which is based upon a trimmed geometric mean of ratios of consecutive OS. The corresponding expression of the probability of false alarm has been derived and given in closed-form. However, due to the lack of mathematical background, we have revealed that, at the present time, closed-form of its respective probability of detection could not be obtained. To alleviate such a problem and for comparison purposes, we have conducted Monte-Carlo simulations. Performance analysis carried out here showed that the GM-CFAR process worked very well in homogeneous clutter, especially for small size of the reference window *M*. However for increasing *M*, the performance of the proposed TGMR improved clearly and approach that of the GM. In the presence of interfering targets, the new CFAR processes worked very well in managing interference. This provided a significant improvement on the Pareto detection processes discussed in the literature.

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