

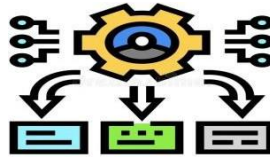


University of Larbi Ben M'hidi Oum El Bouaghi
Faculty of Exact Sciences, Natural Sciences and Life
Department of Mathematics and Computer Science



Decision Support Systems (DSS)

Course Handbook



Level: B3
Information Systems and Software
Engineering (ISSE)

Dr. Saighi Asma

Preamble

Decision Support Systems (DSS) have become essential tools in modern organizations, helping decision-makers analyze complex problems, evaluate multiple alternatives, and improve overall efficiency. This **Course Handbook** provides a structured approach to understanding the key principles, methodologies, and applications of DSS.

The handbook begins with an introduction to **decision-making processes**, highlighting the different types of decisions and the conditions under which they are made. It explores various **decision theories**, from normative to descriptive and prescriptive models, providing a theoretical foundation for effective decision-making.

A significant focus is placed on **Decision Analysis under Risk and Uncertainty**, where techniques such as **decision trees, expected value calculations, and utility theory** are discussed. The handbook also delves into **Multi-Criteria Decision-Making (MCDM)** approaches, emphasizing both **single-criteria** and **multi-criteria** models, including **outranking methods like ELECTRE and PROMETHEE**.

Furthermore, essential **mathematical techniques** such as **normalization, weight assessment, and data transformation** are introduced to help structure and quantify decision problems. Throughout the course, practical examples and real-world applications illustrate how DSS methodologies can be applied to business, engineering, healthcare, and other domains.

By the end of this handbook, students will have a **comprehensive understanding of DSS principles**, enabling them to develop, analyze, and implement decision support models effectively.

1. Course Information

- **Faculty:** Mathematics and Computer Science
- **Department:** Computer Science
- **Target Audience:** 3rd-year Bachelor's students, specializing in **Information Systems and Software Engineering (ISIL), Computer Science**
- **Course Title:** Decision Support System
- **Credits:** 04
- **Coefficient:** 02
- **Duration:** 14 weeks

2. Prerequisites

To get the most out of this course, students should have prior knowledge of:

- **Information Systems**
- **Database Fundamentals**
- **Algorithms**

3. Learning Objectives

The overall **goal** of this course is to **develop students' ability to design information systems and management tools useful for decision-makers**. The **Decision Support System (DSS) course** aims to:

- **Knowledge:** Teach students the **fundamental concepts of information systems and management tools design**.
- **Skills:** Guide students through **programming and software manipulation**, including tools like **Excel**.

4. Assessment Methods

The final assessment consists of:

- **Final Exam (graded out of 20):** A written exam covering all topics discussed throughout the semester.
- **Lab Work (graded out of 20):**

5. Course Structure & Learning Mode

The course is structured into:

- **Theoretical sessions** to introduce fundamental concepts, definitions, methodologies, and tools for designing decision support systems.
- **Practical sessions** to apply and reinforce theoretical knowledge in decision support contexts.

Table of Contents

Chapter 1: Introduction to Decision-Making and Decision Processes

1.	Course Overview	2
2.	Introduction to Decision-Making	2
2.1.	Decision Making.....	2
2.2.	Decision Support Systems (DSS).....	4
2.3.	Components of Decision Support Systems (DSS).....	5
2.3.1.	Data Management Component	5
2.3.2.	Model Management Component	6
2.3.3.	User Interface Management Component	7
2.4.	Decision Making Process	7
2.5.	Decision making conditions	9
3.	Typology of Decisions in a Company	10
3.1.1.	Strategic Decisions (High-Level Decision-Making).....	11
3.2.	Types of Decisions Based on Structure	12
4.	Modeling Decision Processes	14
5.	Introduction to Decision theories: Definition, Types, and Applications	17
5.1.	Definition of Decision Theories	17
5.3.	Decision Matrices and Decision Tables.....	20
5.3.1.	Definition.....	20
5.3.2.	Decision-Making Rules	23
2.3.	Hurwicz α -Index Rule (Weighted Optimism-Pessimism Approach)	24
2.4.	Minimax Regret Rule (Savage's Criterion)	25
3.1.	Expected Monetary Value (EMV) Criterion.....	27

Chapter 2: Decision Analysis Elements – Decision Under Risk

1.	Decision Trees.....	34
1.2	Regression Trees: Predicting Continuous Values.....	35
1.3	Structure of a Decision Tree	36
1.4	Transforming Decision Trees into Rules.....	36

2. Solving Decision Tree.....	36
2.1. Calculating Expected Monetary Value (EMV) for each option.....	39
2.2. Optimal Decision.....	40
3. Perfect vs. Imperfect Information.....	45
4. Mathematical Expectation & Utility Expectation.....	48
4.1. Utility Expectation.....	49

Chapter 3: Introduction to Multicriteria Methods

1. Quantitative and Qualitative Approaches to Decision Support.....	52
1.1. Quantitative Approach to Decision Support.....	52
1.2. Qualitative Approach to Decision Support.....	52
1.3. Combining Quantitative and Qualitative Approaches.....	52
2. Single-Criteria and Multi-Criteria Models, Dominance, and Efficiency.....	53
2.2. Dominance Principle in Decision-Making.....	55
2.3. Efficiency in Decision-Making.....	56
3. Multiple-Criteria Decision-Making (MCDM).....	56
4. MCDM Solutions.....	58
5. MCDM Methods.....	59
6. Normalization in Multi-Criteria Decision Analysis (MCDA).....	73
6.1. Purpose of Normalization.....	73
6.2. Common Normalization Techniques.....	73
6.2.1. Vector Normalization.....	73
7. Weight Assessment in Decision-Making.....	75
8. Outranking Methods.....	77
8.1. Introduction to Outranking Approaches.....	77
8.2. Common Outranking Methods.....	79

8.2.1 ELECTRE Method.....	80
8.2.2 PROMETHEE Method.....	80
8.2.3. Ordinal Priority Approach (OPA).....	80
8.2.4 Dominance-Based Rough Set Approach (DRSA)....	81
8.3. Advantages and Limitations of Outranking Methods.....	81
References...	96

Figure List

Figure 1.1: Description of Simon's model of the process of decision-making.

Figure 1.2: Types of information systems that correspond to each organizational level.

Figure 1.3: Diagram of the Decision-Making Process.

Figure 2.1: Iris Decision Tree (Géron, 2022).

Figure 2.2: A Decision Tree for regression (Géron, 2022).

Figure 2.3: Tree decision rules example.

Figure 2.4: Decision Tree representation for DSS.

Figure 3.1: Multi-Attribute Decision-Making.

Figure 3.2: Hierarchical structure of MCDM Methods (Aruldoss, 2013).

Figure 3.3: Hierarchical thinking in AHP.

Figure 3.4: Multi-Criteria Decision Analysis (MCDA) methods.

Chapter 1 Plan:

- 1. Course Overview**
- 2. Introduction to Decision-Making**
- 3. Typology of Decisions in a company**
- 4. Modeling of Decision Processes**
- 5. Introduction to Decision Theories: Definition, Types, and Applications**

1. Course Overview

This course, titled **Decision Support System**, is designed for **3rd-year Bachelor's students** specializing in **Information Systems and Software Engineering (ISSE)**. Its objective is to provide students with the **fundamental concepts** of decision-making approaches.

Decision support systems have become **ubiquitous** in modern organizations. This course aims to equip students with the **necessary knowledge** for the **development of such tools**.

The course is structured into a set of **learning units** that enable students to:

- Acquire **decision-making skills**
- Master **decision support technologies**
- Design and evaluate **decision support systems**
- Use and assess **productivity tools**

2. Introduction to Decision-Making

2.1. Decision Making:

Decision-making is the process of choosing the most appropriate option from a set of alternatives. It involves:

- **Analyzing available data** to make informed choices.
- **Understanding potential risks** associated with each option.
- **Evaluating possible outcomes** to determine the most effective course of action.

According to **Turban (1995)**, decision-making is a structured approach aimed at achieving specific goals by identifying alternatives, gathering relevant information, and systematically evaluating solutions to select the optimal one.

Examples: Decision-Making Scenarios

Example 1: Should I take my umbrella today?

The decision depends on something I do not know: whether it will rain or not.

Example 2: Looking to buy a house:

- Should I buy this one?
- It seems like a good choice, but if I keep searching, I might find a better one at the same price.
- When should I stop my search?



Example 3: A court must determine whether an accused individual is guilty or not. There are two possible errors:

- Convicting an innocent person.
- Acquitting a guilty person.
- What principles should the court apply?



Example 4: Travel Planning

Planning a trip involves choosing an itinerary based on different criteria:

- Should I select the fastest route or the most scenic one?
- Travel websites often ask users to specify their preferences before suggesting routes.



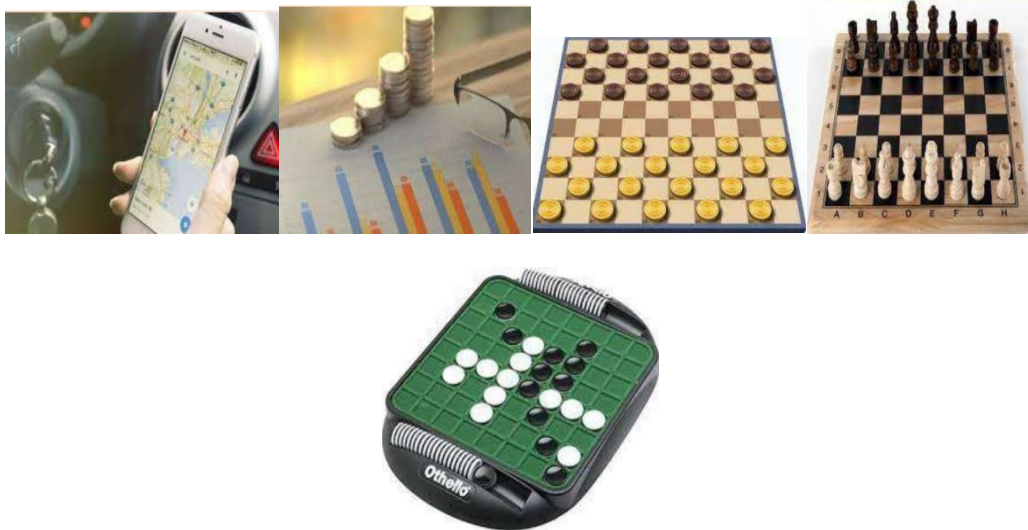
Example 5: Diet Optimization

Designing an **optimal diet plan** is a classic problem. In the 1930s, the **U.S. Army** developed a cost-effective, nutritionally balanced meal plan for soldiers. The challenge was to select the right foods based on their nutritional content while minimizing costs.

These examples highlight the **core challenge of decision-making**: selecting the best option from multiple alternatives based on a specific **optimization criterion** (cost, time, efficiency, etc.).

2.2. Decision Support Systems (DSS):

DSS are Designed to facilitate decision-making at the enterprise level. It is an interactive software tool that helps decision-makers extract useful information from raw data, documents, personal knowledge, and business models to identify and solve issues and make informed decisions. Fields of application of DSS are: **Timetable creation; Network planning and sizing, traffic routing optimization, and pricing optimization.** Industrial cutting problems, etc., and any resource allocation issue; **Strategy games** (chess, checkers, Othello, etc.).



In decision-making problems, the objective is often to minimize cost or maximize a given benefit. This involves selecting the best decision from a set of possible choices to optimize a specific criterion, making it an optimization problem.

To analyze a problem, it is essential to clearly define:

- **What needs to be done** (the type of decision),
- **What can be done** (the possible decisions),
- **How to choose** (the selection criterion).

Mathematically, a decision-making problem can be represented using:

- **Decision variable (x):** The choice to be made (e.g., a travel route or a meal plan).
- **Set of possible decisions (S):** The available alternatives (e.g., all possible routes or meal combinations).
- **Objective function (f):** The criterion used to evaluate decisions (e.g., travel time or meal cost).

The general form of an optimization problem can be expressed as follows:

$$\min_{x \in S} f(x) \quad \text{or} \quad \max_{x \in S} f(x)$$

Where:

- x represents the decision variable (e.g., a chosen route or a selected meal).
- S is the set of possible decisions, constrained by certain conditions.
- $f(x)$ is the objective function that evaluates each decision, such as cost, distance, or efficiency.

Select x from S to either minimize or maximize $f(x)$ to ensure the best possible outcome under given constraints.

2.3. Components of Decision Support Systems (DSS)

Decision Support Systems (DSS) are composed of three fundamental components: **Data Management, Model Management, and User Interface Management**. These components work together to assist decision-makers in analyzing complex problems, processing information, and generating actionable insights.

2.3.1. Data Management Component

The **Data Management Component** is responsible for **storing, organizing, and maintaining** the data used by the Decision Support System. It serves as the foundation for analytical processing and decision-making. Among its main **characteristics**:

- Stores **structured and unstructured data** from multiple sources.
- Integrates data from **internal databases, external sources, and real-time feeds**.
- Ensures **data integrity, security, and accessibility** for analysis.

Example:

A financial DSS may collect and process real-time stock market data, internal sales reports, and economic indicators to assist in investment decisions.

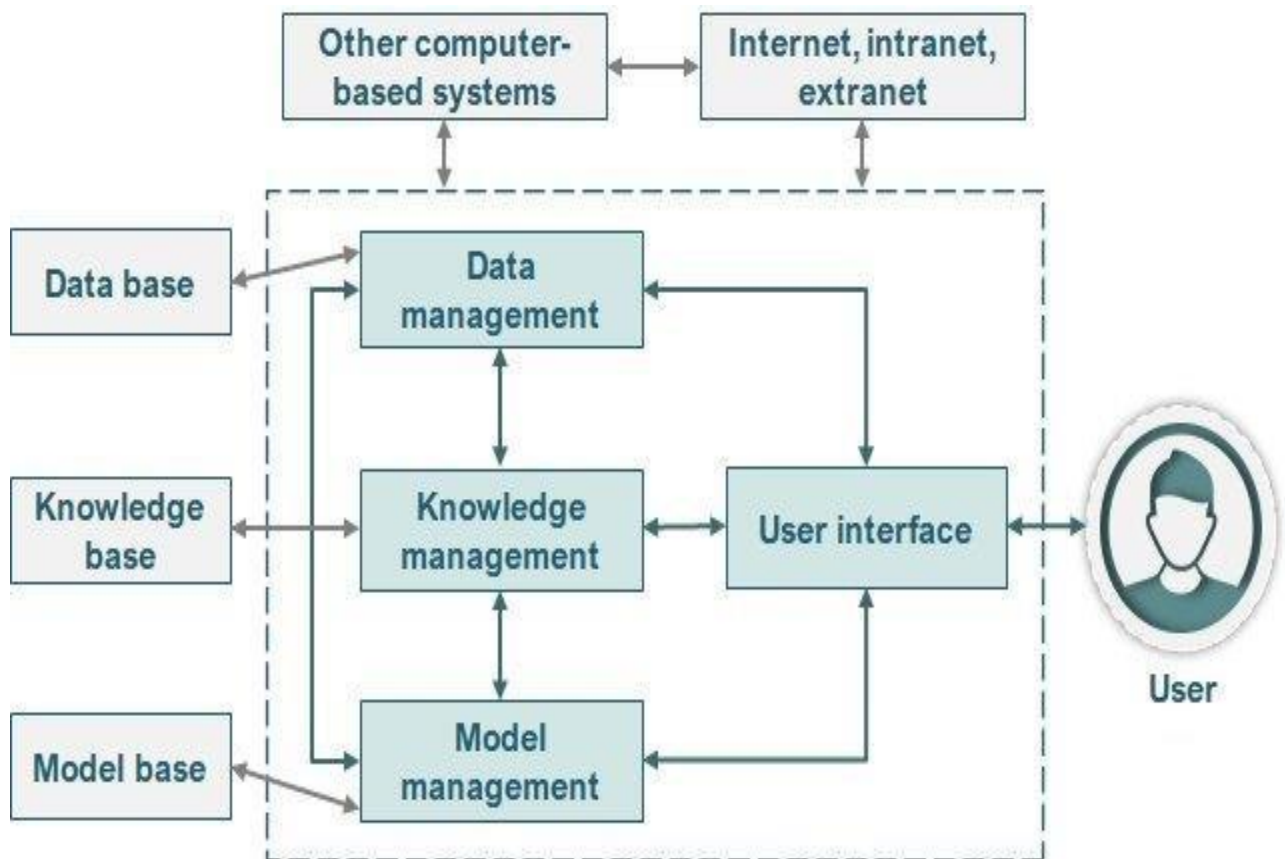


Figure 1.1. General structure of a decision support system, consisting in four components: data, model, knowledge management and user interface (Araújo, 2021).

2.3.2. Model Management Component

The **Model Management Component** is responsible for storing, managing, and executing mathematical, statistical, and analytical models that support decision-making. It acts as the **engine** of the DSS by processing data and generating insights. The key Features of such component are:

- Provides a library of **analytical, simulation, and optimization models**.
- Allows users to **build, modify, and apply models** based on their specific needs.
- Works similarly to a **database management system** but focuses on decision models instead of raw data.

Example:

A logistics DSS may use forecasting models to predict future demand and recommend optimal inventory levels.

However, the model management system does not automatically choose the best model for a given problem—**expertise and human judgment are still required** to select and interpret results.

2.3.3. User Interface Management Component

The **User Interface (UI) Management Component** is the **bridge between the user and the DSS**. It enables decision-makers to interact with the system, input data, select models, and visualize results in an intuitive way. Among its **key Features**, we can cite:

- Provides **dashboards, reports, and interactive charts** for data visualization.
- Supports **natural language processing and voice commands** in modern AI-powered DSS.
- Ensures ease of use by offering **customizable interfaces** tailored to user needs.

Example:

A retail DSS may offer a dashboard displaying **real-time sales trends, customer preferences, and AI-driven recommendations** to help managers adjust marketing strategies.

A well-structured DSS integrates **data, analytical models, and a user-friendly interface** to enhance decision-making. The **Data Management Component** provides accurate and up-to-date information, the **Model Management Component** enables data analysis and forecasting, and the **User Interface** ensures seamless interaction. Together, these components empower businesses and organizations to make informed,

2.4. Decision Making Process

Decision making process is divided into the following five steps:

1. Identification of the problem;
2. Obtaining necessary information;
3. Production of possible solutions;
4. Evaluation of such solutions;
5. Selection of a strategy for performance.

Simon (1960) proposed decision-making as a **four-phase model**:

- a. **Intelligence: Intelligence phase (problem detection and definition)** is comprised of the search for problems. This phase ends with a problem statement. The decision-maker decides what to decide or formulates the problem that needs a decision. Such a formulation is called

a decision statement or problem statement or problem definition stage. The intelligence phase may involve activities like problem classification, problem decomposition (broken down problems in simpler sub-problems that could be solved individually) and determination of problem ownership.

- b. **Design: design phase** involves extensive research to explore available alternatives and options. This stage is where a model is built, validated, and evaluation criteria are set to assess potential solutions.
- c. **Choice:** In the **choice phase**, all identified alternatives are carefully analyzed and evaluated, leading to the selection of the most suitable option. Before implementation, the chosen solution is tested conceptually or through simulations to ensure its feasibility.
- d. The **implementation phase** follows once a decision is deemed practical. If executed successfully, it addresses the initial problem. However, decision-making doesn't end with implementation. The decision-maker must continuously monitor the outcomes, assessing the new reality and making necessary adjustments. This iterative process allows for refinements at any stage, similar to Simon's review phase, ensuring the decision remains effective and adaptable over time.

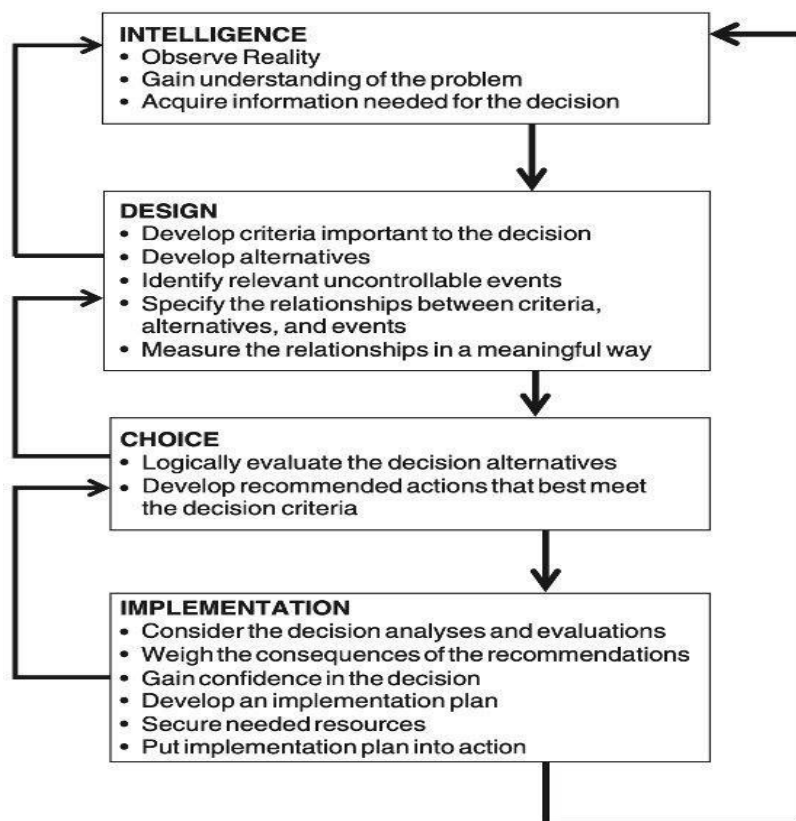


Figure 1.2. Description of Simon's model of the process of decision making, retrieved from PhillipsWren (Phillips-Wren, 2013).

2.5. Decision making conditions

When managers solve problems, they make decisions under three main conditions: **certainty, risk, and uncertainty**. Each condition affects the best action choice.

2.5.1. Decision-Making Under Certainty

In this case, **all necessary information is available**, and the outcome is **predictable**. There is only **one possible result** for each decision.

Example:

Choosing a supplier for office supplies when prices and delivery times are fixed and guaranteed.

2.5.2. Decision-Making Under Risk

Also called a **probabilistic or stochastic decision**, this occurs when there are **several possible outcomes**, but their **probabilities can be estimated**.

Example:

A company launching a new product faces uncertainty regarding market response. However, by analyzing data from similar product launches, it can estimate the likelihood of different outcomes. **The key steps in decision-making under risk are:**

1. **Identify possible outcomes** – Determine the potential results of the decision (e.g., high sales, moderate sales, low sales).
2. **Assign probabilities to each outcome** – Use historical data, market research, or expert opinions to estimate the likelihood of each scenario.
3. **Conduct risk analysis** – Evaluate the potential risks and rewards associated with each outcome.
4. **Calculate the expected value** – Compute the weighted average of all possible outcomes based on their probabilities.
5. **Select the best option** – Choose the decision with the highest expected value to maximize potential success.

2.5.3. Decision-Making Under Uncertainty

This happens when multiple outcomes exist, but their **probabilities are unknown and cannot be estimated**. Decision-making becomes more complex due to a **lack of information**.

Example:

A startup expanding into a completely new market faces **high uncertainty** since there is no historical data or prior trends to predict success. Factors such as customer demand, competition, and regulatory conditions remain unknown, making decision-making challenging. The main characteristics of Decision-Making Under Uncertainty are:

1. **Multiple possible outcomes** – The startup could experience high demand, slow adoption, or even failure, with no clear way to predict which scenario will occur.
2. **No way to estimate probabilities** – Unlike decisions under risk, there is no reliable data to calculate the likelihood of success or failure.
3. **Requires intuition and strategic thinking** – Decision-makers must rely on industry expertise, market insights, and creative problem-solving to navigate uncertainty.
4. **Flexibility and adaptability are crucial** – Companies must be prepared to pivot strategies, experiment with different approaches, and react quickly to unexpected challenges.

Conclusion

Managers must recognize which condition they are operating under to help choosing the best decision-making approach.

- **Certainty:** Simple and predictable.
- **Risk:** Manageable with data and probabilities.
- **Uncertainty:** Requires creativity and strategic thinking.

3. Typology of Decisions in a Company

In a company, decisions can be classified based on **their structure, impact, and level of management**. These classifications help organizations understand how to approach different types of decision-making.

3.1. Classification by Level of Management

Decision-making in organizations is categorized into **three hierarchical levels** (see **Figure **.**), each varying in scope, complexity, and impact. These levels are:

3.1.1. Strategic Decisions (High-Level Decision-Making)

Strategic decisions shape the long-term vision and direction of an organization. They typically involve:

- **Long-term goals** and overall corporate strategy.
- **High uncertainty and risk**, as they significantly influence the future of the organization.
- **Broad scope**, affecting multiple departments and stakeholders.
- **Creativity and foresight**, as they often involve innovation and transformation.

Example: A company deciding to enter a new international market or invest in a new technology.

3.1.2. Tactical Decisions (Mid-Level Decision-Making)

Tactical decisions serve as a bridge between strategic plans and daily operations. They are characterized by:

- **Medium-term focus**, usually spanning months to a few years.
- **Moderate significance and risk**, impacting specific business functions.
- **Resource allocation and implementation**, ensuring strategic goals are effectively executed.

Example:

A marketing department deciding on an advertising campaign strategy to promote a new product.

3.1.3. Operational Decisions (Day-to-Day Decision-Making)

Operational decisions are routine choices made daily to support tactical plans. Their features include:

- **Short-term impact**, often immediate or within a few days.
- **Low risk**, with minimal consequences for individual mistakes.
- **High structure**, as many of these decisions follow predefined rules, policies, or standard procedures.

Example:

A restaurant manager adjusting staff schedules based on daily customer demand.

Remark:

Each level of decision-making plays a crucial role in ensuring organizational success. While strategic decisions provide **long-term vision**, tactical decisions focus on **execution**, and operational decisions ensure **day-to-day efficiency**. Misalignment between these levels can lead to inefficiencies and poor organizational performance.



Figure 1.3. Levels of Decision-Making [1].

3.2. Types of Decisions Based on Structure

Decision-making in organizations can be classified based on the **degree of structure and predictability** of the problem. The three main types are **programmed (structured) decisions**, **non-programmed (unstructured) decisions**, and **semi-structured decisions** (Simon, 1960).

A. Programmed Decisions (Structured Decisions)

Programmed decisions are **routine, repetitive, and follow well-established rules or procedures**. These decisions are typically **operational** and can be automated using **decision-support systems (DSS) or software**. They: are applied to **structured problems** where the outcome is predictable, are **predefined procedures** or company policies determine the decision, can be **automated** using software and technology, are with **low risk**, as there is little uncertainty.

Examples:

- **Inventory management:** Automatically reordering stock when inventory falls below a certain threshold.
- **Payroll processing:** Employees are paid based on fixed salary structures.
- **Employee scheduling:** Work shifts are assigned based on predefined rules.

B. Non-Programmed Decisions (Unstructured Decisions)

Non-programmed decisions deal with **unique, complex, and uncertain** situations that do not have predefined solutions. These decisions require **critical thinking, creativity, and expert judgment** (Mintzberg, 1976). They: are made in **new or unexpected situations** with no established rules, require **subjective analysis, strategic thinking, and intuition**, are with **high risk**, as the outcomes are uncertain, are typically handled by **top-level management**.

Examples:

- **Launching a new product:** Determining pricing, target market, and promotional strategy.
- **Entering a new market:** Assessing economic conditions, competition, and risks.
- **Crisis management:** Responding to a major cybersecurity breach or a public relations disaster.

C. Semi-Structured Decisions

Semi-structured decisions contain elements of **both structured and unstructured** decision-making. They require **some level of analysis and judgment**, but also follow a **standard decision-making process** (Gorry & Scott Morton, 1971). They: are **partially structured** with known procedures, but require human input, often involve **data analysis, expert judgment, and experience**, with **moderate risk**, as some uncertainty exists, they typically handled by **middle management**.

Examples:

- **Developing a marketing campaign:** Using customer data to determine the best advertising strategy, but still requiring creativity.
- **Hiring a new employee:** Following recruitment policies while also assessing candidates' unique qualifications.

In fact, understanding the **structure of decisions** helps managers apply the right approach:

- **Programmed decisions** are routine and efficient, often automated.

- **Non-programmed decisions** require strategic thinking for unique challenges.
- **Semi-structured decisions** balance standard procedures with human judgment.

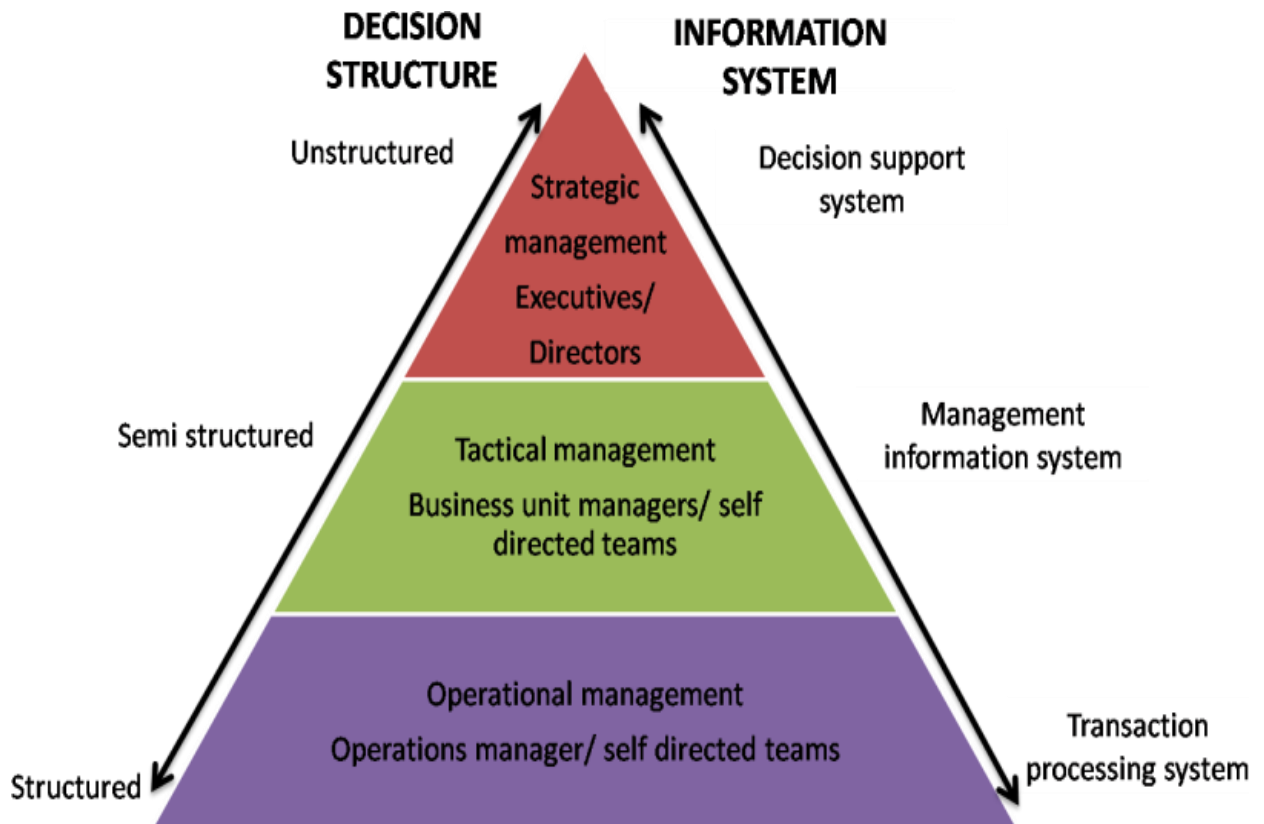


Figure 1.4. Types of information systems that correspond to each organizational level [2].

By understanding these types of decisions, managers can choose the appropriate approach to optimize outcomes and minimize risks.

4. Modeling Decision Processes

To solve a problem, a **model** is created to explore different solutions and simulate their outcomes. This model can be **mathematical** (using equations) or **graphical**. In fact, decision-making involves structuring problems, analyzing available alternatives, and selecting the best course of action. By applying a systematic approach, decision-makers can optimize their choices and achieve better outcomes.

Figure 1.5 represents the diagram of the Decision-Making general Process. The diagram represents the step-by-step process of transforming a problem into a decision. Initially, a model is chosen to address the problem, followed by selecting an appropriate resolution method.

Once a solution is obtained, it undergoes analysis to verify its validity. If the solution is deemed satisfactory, a decision is formulated and implemented. However, if the solution is inadequate,

the model may need to be revised, leading to a new iteration of the process until a viable decision is reached.

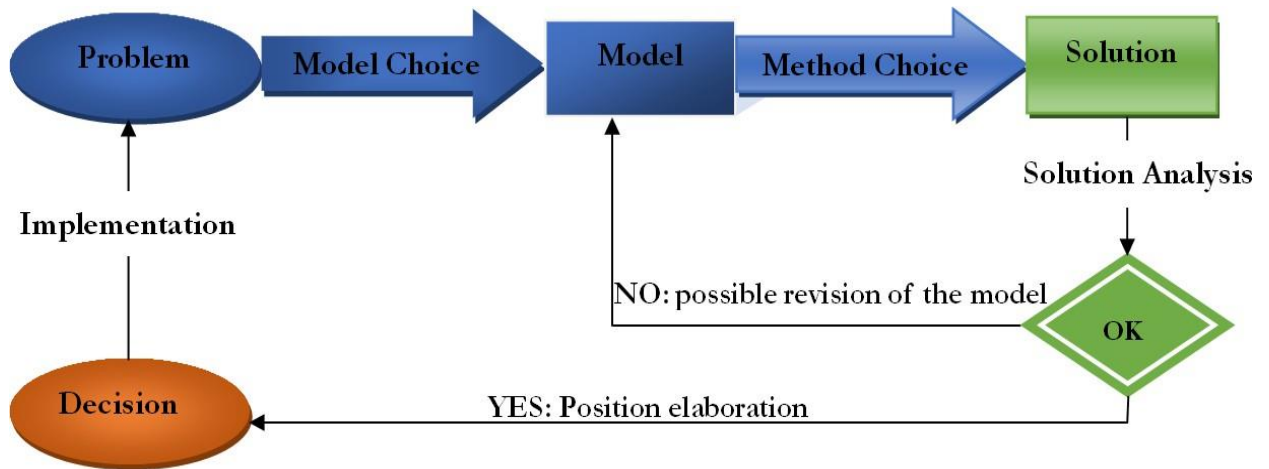


Figure 1.5. Diagram of the Decision-Making Process.

4.1. Decision-Making Approach

A. From Problem to Model

A decision-making problem can be represented using:

- **Decision variable (x):** The choice to be made (e.g., a travel route or a meal plan).
- **Set of possible decisions (S):** The available alternatives (e.g., all possible routes or meal combinations).
- **Objective function (f):** The criterion used to evaluate decisions (e.g., travel time or meal cost).

To solve a problem, a model is created to explore different solutions and simulate their outcomes. This model can be mathematical (using equations) or graphical.

Example Problem:

A truck must organize its delivery route, starting from a warehouse and sequentially visiting several customers.

For four customers (a, b, c, d), possible visit orders include:

- $a \rightarrow b \rightarrow c \rightarrow d$
- $a \rightarrow b \rightarrow d \rightarrow c$
- $a \rightarrow c \rightarrow b \rightarrow d$
- ... (24 possible orders in total)

Since there are $4! = 24$ possible routes, the number of combinations grows exponentially.

For $n=10$, we have:

$$10! = 10 \times 9 \times 8 \times \dots \times 3 \times 2 \times 1 = 3,628,800$$

Because listing all possible solutions is impractical, we build a model to represent the problem efficiently. This involves defining:

- X (decision variable)
- S (set of possible decisions)
- $f(x)$ (objective function)

N.B.: There are no strict rules for constructing a model. As Vallin-Vanderpooten (ELLIPSES) states:

"Modeling is often considered more of an art than a technique. More realistically, it is a craft that combines scientific rigor with practical know-how."

B. From Model to Solution

Once the problem is modeled, we apply appropriate methods to find solutions. Some decision problems are computationally complex, making it difficult to determine an exact optimal solution. In such cases, we use **heuristic methods**, which approximate good solutions within a reasonable time.

A model can also serve as a **simulation tool**, where variables and data are linked through relationships. This allows us to:

- Test the impact of different decisions.
- Analyze variations in input parameters.
- Evaluate outcomes in situations where real-world experimentation is impractical.

Example: Traveling Salesman Problem (TSP)

A classic example is the **Traveling Salesman Problem (TSP)**, where a delivery truck must find the shortest route to visit multiple locations.

For just **10 stops**, the number of possible routes is **3,628,800**, making an exhaustive search impractical. Instead, algorithms, are used to efficiently approximate the best route, like:

- **Greedy heuristics,**
- **Genetic algorithms,**
- **Ant colony optimization,**

C. From Solution to Decision.

Once a solution is developed, it must be analyzed to validate the model, ensuring that it accurately represents reality. A model is merely a schematic representation, and the solution may reveal that some constraints were either overlooked or incorrectly considered.

If no such issues arise, the final step is to interpret the solution in the context of the original problem, allowing the decision-maker to accept, modify, or reject it based on practical considerations.

5. Introduction to Decision theories: Definition, Types, and Applications

5.1. Definition of Decision Theories

Decision theories provide structured frameworks to **explain, predict, and guide** how individuals, businesses, and organizations make choices.

These theories help decision-makers analyze different options, assess risks, and select the most suitable course of action based on available data, constraints, and objectives.

The key questions in decision theories are:

- What criteria influence the choice between multiple alternatives?
- What factors shape decision-making (rationality, emotions, uncertainty)?
- How can decision-making be optimized using models and tools?

According to Bazerman and Moore (2022), managerial decision-making is often influenced by cognitive biases and heuristics, leading to deviations from theoretical predictions. **Hastie and Dawes (2019) emphasize** that understanding both rational models and psychological constraints is essential for making effective choices under uncertainty.

5.2. Types of Decision Theories

Decision theories are generally classified into three major categories: **Normative, Descriptive, and Prescriptive**. Each provides a distinct perspective on how decisions are made.

5.2.1. Normative Decision Theories (How decisions should be made)

Normative theories assume that decision-makers act **logically and consistently** to achieve the best possible outcome. These theories rely on **mathematical models and optimization techniques** to determine the most effective choice.

The main characteristics of such a decision theory are:

- Assumes **complete information and rational decision-making**.
- Uses **quantitative models** to identify the optimal solution.
- Commonly applied in **economics, finance, and artificial intelligence**.

Examples of Normative Theories:

1. Expected Utility Theory (EUT)

- It enables evaluating decisions by calculating the probability and expected utility of each outcome.
- **Barberis (2013) notes** that while EUT remains a cornerstone of economic theory, real-world deviations have led to alternative models such as prospect theory.

2. Game Theory

- Used to analyze **strategic interactions** where multiple decision-makers influence each other's outcomes.
- **Myerson (2013) explains** that game theory provides a mathematical framework for understanding strategic decision-making in economics and politics.

5.2.2. Descriptive Decision Theories (How decisions are actually made)

Descriptive theories focus on **real-world decision-making**, acknowledging that human choices are often influenced by **cognitive limitations, emotions, and psychological biases** rather than pure rationality. Its key characteristics are:

- Recognizes **bounded rationality** (limited time, information, and cognitive ability).
- Explains **irrational choices, cognitive biases, and heuristics**.
- Applied in **behavioral economics, psychology, and marketing**.

Examples of Descriptive Theories:

1. Prospect Theory (Kahneman & Tversky, 1979)

- Explains how individuals evaluate gains and losses differently:
 - **Risk-averse in gains:** Prefer a guaranteed reward over a higher but uncertain one.
 - **Risk-seeking in losses:** Willing to take risks to avoid definite losses.

- **Kahneman (2011) argues** that this theory challenges traditional rational decision-making by highlighting systematic biases in human choices.

2. Dual-Process Theory (Evans, 2008)

- Suggests that decision-making operates through **two cognitive systems**:
 - **System 1:** Fast, intuitive, and emotion-driven.
 - **System 2:** Slow, analytical, and logic-based.
- **Evans (2018) emphasizes** that while logical reasoning (System 2) improves decision quality, intuitive thinking (System 1) plays a crucial role in everyday choices.

5.2.3. Prescriptive Decision Theories (How decision-making can be improved)

Prescriptive theories aim to enhance decision-making by integrating elements of **both normative and descriptive approaches**. They emphasize **tools, strategies, and technology-driven solutions** to guide decision-makers toward better choices. Its key Characteristics are:

- Uses **decision aids** (AI, decision support systems, simulations).
- Bridges the gap between **theoretical rationality and human constraints**.
- Applied in **business, healthcare, public policy, and AI-driven decision-making**.

Beyond Multi-Criteria Decision Analysis (MCDA) and Decision Support Systems (DSS), structured decision-making tools such as **Decision Matrices and Decision Tables** help in evaluating multiple alternatives systematically. These tools enhance consistency in decision-making by providing a structured approach to analyse different options based on predefined criteria. Section 5.3 explores these tools in detail.

Examples of Prescriptive Theories:

1. Multi-Criteria Decision Analysis (MCDA)

- Evaluates alternatives based on **multiple, often conflicting, criteria**.
- **Greco et al. (2016) describe** MCDA as a vital tool for complex decision-making, particularly in business and policymaking.

2. Heuristics and Decision Support Systems (DSS)

- Uses **artificial intelligence and data analytics** to assist decision-makers.
- **Turban et al. (2021) highlight** that modern DSS integrate AI and big data to enhance decision processes in organizations.

We conclude that decision theories offer various perspectives on how choices are made:

- **Normative approaches** provide rational, mathematical models for optimal decision-making.
- **Descriptive models** explore the psychological and cognitive limitations that shape real-world decisions.
- **Prescriptive theories** leverage tools and technology to enhance the decision-making process.

Currently, **artificial intelligence and big data analytics** continue to evolve, prescriptive decision models are becoming increasingly valuable in improving human decision-making.

5.3. Decision Matrices and Decision Tables

In decision-making, structured tools help analyze multiple alternatives efficiently. Two widely used techniques are **Decision Matrices** and **Decision Tables**, both of which support rational, data-driven decision-making.

These methods are essential for evaluating different choices systematically based on a set of predefined criteria.

5.3.1. Definition

A **Decision Matrix** is a structured tool that helps decision-makers compare various options by scoring them against multiple criteria. It is widely used in **business, project management, and strategic planning** to facilitate objective decision-making.

Decision Matrices Facilitate a structured approach to decision-making, helps compare multiple alternatives based on weighted criteria and ensures transparency in evaluating the best choice.

Example 1: Selecting a Production Expansion Strategy

A company is considering two strategies to expand production: (1) Building a large plant, or (2) Constructing a small plant. The market demand can either be **favorable** or **unfavorable**, impacting profitability. The decision matrix below summarizes possible outcomes.

Alternatives	Favorable Market	Unfavorable Market
Construct Large Plant	\$200,000	-\$180,000
Construct Small Plant	\$100,000	-\$20,000
Take No Action	\$0	\$0

To incorporate probabilities, assume that market experts estimate a **60% chance of a favorable market** and a **40% chance of an unfavorable market**. The decision matrix with probabilities is updated as follows:

Alternatives	Favorable Market (60%)	Unfavorable Market (40%)	Expected Value (EV)
Build Large Plant	$\$200,000 \times 0.6 = 120,000$	$-\$180,000 \times 0.4 = -72,000$	\$48,000
Build Small Plant	$\$100,000 \times 0.6 = 60,000$	$-\$20,000 \times 0.4 = -8,000$	\$52,000
Take No Action	\$0	\$0	\$0

Since the small plant yields the highest expected value (**\$52,000**), it is the most favorable choice.

Example 2: Choosing an Investment Strategy

A company is considering two investment strategies for entering a new market:

1. **Launching a large-scale operation** (high investment, high risk).
2. **Launching a small-scale operation** (lower investment, lower risk).
3. **Doing nothing** (no investment, no gain or loss).

The future market demand is **uncertain**, and it could either be **favorable** (high returns) or **unfavorable** (low returns).

Investment Strategy	Favorable Market	Unfavorable Market
Launch Large-Scale Operation	\$200,000	-\$180,000
Launch Small-Scale Operation	\$100,000	-\$20,000
Do Nothing	\$0	\$0

To quantify the risk, **market analysts estimate a 60% probability of a favorable market and a 40% probability of an unfavorable market.**

The decision matrix incorporating probabilities is shown below:

Investment Strategy	Favorable Market (60%)	Unfavorable Market (40%)	Expected Value (EV)
Launch Large-Scale Operation	$\$200,000 \times 0.6 =$ \$120,000	$-\$180,000 \times 0.4 = -$ \$72,000	\$48,000
Launch Small-Scale Operation	$\$100,000 \times 0.6 =$ \$60,000	$-\$20,000 \times 0.4 = -$ \$8,000	\$52,000
Do Nothing	\$0	\$0	\$0

Solution:

1. The **expected value (EV)** is calculated by multiplying each outcome by its probability and summing them.
2. **EV(Large-Scale Operation) = \$48,000**
3. **EV(Small-Scale Operation) = \$52,000**
4. **EV(Doing Nothing) = \$0**

Best Choice: Since the **small-scale operation has the highest expected value (\$52,000)**, it is the best option based on risk-adjusted profitability.

Example 3: Multiple Criteria Decision Matrix for Selecting the Best Apartment

A person is evaluating two apartment options based on three key criteria:

1. **Price (Lower is better)**
2. **Maintenance Cost (Lower is better)**
3. **Floor Level (Lower is better for convenience)**

This simple case of **multiple attribute decision-making** involves evaluating alternatives based on several, sometimes conflicting, goals.

This situation falls under **decision-making with certainty**, as only one possible outcome is projected for each alternative. A more complex case involving uncertainty or risk can also be considered.

Criteria (Minimize)	Apartment 1	Apartment 2
Price	100	120
Maintenance Cost	20	10
Floor Level	1	2

5.3.2. Decision-Making Rules

In this section, we explore different **decision-making rules** applied under varying levels of certainty, uncertainty, and risk. The concepts are illustrated using the example below.

Example: A Grocer's Order Planning Problem

A grocer must decide how much pastry to order daily. His profit depends on **demand**, which can be **low, moderate, or high**. The profit (in dollars) for different order sizes—**small, medium, and large**—is given in the payoff table below:

Table 1: Payoff Table for Order Planning.

Order Size	Low Demand	Moderate Demand	High Demand
Small Order	50	50	50
Medium Order	42	52	52
Large Order	34	44	54

The choice of the best alternative depends on the decision conditions:

A. Decision-Making Under Complete Certainty

If demand is **known with certainty** to be **low**, the best alternative is simply the one that yields the **highest profit in that column**.

Best Alternative: The small order (profit = \$50)

B. Decision-Making Under Uncertainty

When demand is **unknown**, several decision rules can be applied:

2.1. Maximax Rule (Optimistic Approach)

- Assumes the **best possible outcome** will occur.
- Selects the alternative with the **highest maximum profit**.

Calculation:

- **Small Order:** Max = 50
- **Medium Order:** Max = 52
- **Large Order:** Max = 54

Best Alternative: Large Order.

2.2. Maximin Rule (Pessimistic/Wald's Criterion)

- Assumes the **worst-case scenario** for each option.
- Chooses the alternative with the **best worst-case outcome**.

Calculation:

- **Small Order:** Min = 50
- **Medium Order:** Min = 42
- **Large Order:** Min = 34

Best Alternative: Small Order (at least \$50 in profit).

2.3. Hurwicz α -Index Rule (Weighted Optimism-Pessimism Approach)

- Balances optimism and pessimism using a weight factor α (e.g., $\alpha = 0.7$).
- Uses the formula: $WA = \alpha \times (\text{Best Profit}) + (1 - \alpha) \times (\text{Worst Profit})$.

For $\alpha = 0.7$:

Order Size	Weighted Average (WA)
Small Order	$(0.7 \times 50) + (0.3 \times 50) = 50$
Medium Order	$(0.7 \times 52) + (0.3 \times 42) = 49$
Large Order	$(0.7 \times 54) + (0.3 \times 34) = 48$

Best Alternative: Small Order (highest weighted average).

2.4. Minimax Regret Rule (Savage's Criterion)

- Minimizes the **maximum regret** of not choosing the best option.
- The **regret** for each decision is calculated by subtracting the actual profit from the **best possible profit** in each demand scenario.

Order Size	Low Demand	Moderate Demand	High Demand	Max Regret
Small Order	0	2	4	4
Medium Order	8	0	2	8
Large Order	16	8	0	16

Table 2: Regret Table for Order Planning.

Best Alternative: Small Order (minimizes the worst regret).

2.5. Principle of Insufficient Reason (Laplace Criterion)

- Assumes **all demand levels are equally probable**.
- Computes the **average profit for each alternative** and selects the highest.

Calculation:

Order Size	Average Profit (Laplace EMV)
Small Order	$(50 + 50 + 50) \div 3 = 50$
Medium Order	$(42 + 52 + 52) \div 3 = 48.67$
Large Order	$(34 + 44 + 54) \div 3 = 44$

Best Alternative: Small Order (highest expected value).

C. Decision-Making Under Risk

If the grocer can estimate demand probabilities as follows:

- **Low demand:** 30% ($p = 0.3$)
- **Moderate demand:** 50% ($p = 0.5$)
- **High demand:** 20% ($p = 0.2$)

3.1. Expected Monetary Value (EMV) Criterion

- Calculates the **weighted average profit** for each alternative:

Calculation:

Order Size	EMV Calculation	Expected Value (EV)
Small Order	$(0.3 \times 50) + (0.5 \times 50) + (0.2 \times 50)$	50
Medium Order	$(0.3 \times 42) + (0.5 \times 52) + (0.2 \times 52)$	49
Large Order	$(0.3 \times 34) + (0.5 \times 44) + (0.2 \times 54)$	43

Best Alternative: Small Order (highest expected value).

3.2. Expected Opportunity Loss (EOL) Criterion

- Uses the **regret table** and demand probabilities to find the **expected regret**:

Calculation:

Best Alternative: Small Order (lowest opportunity loss).

Order Size	EOL Calculation	Expected Loss
Small Order	$(0.3 \times 0) + (0.5 \times 2) + (0.2 \times 4)$	1.8
Medium Order	$(0.3 \times 8) + (0.5 \times 0) + (0.2 \times 2)$	2.8
Large Order	$(0.3 \times 16) + (0.5 \times 8) + (0.2 \times 0)$	8.8

3.3. Maximum-Likelihood Decision Criterion

The **maximum-likelihood decision criterion** selects the best alternative based on the **most probable** state of nature. It assumes that the highest-probability event will occur and makes a decision accordingly.

Application in the Grocer's Order Problem:

- Given probability estimates:
 - **Low demand:** 30%
 - **Moderate demand:** 50% (most likely)
 - **High demand:** 20%
 - The decision-maker **only considers the moderate demand scenario** and selects the alternative with the highest profit under that state.

Best Alternative: Medium Order (\$52 profit under moderate demand).

4. Expected Value of Perfect Information (EVPI)

The **Expected Value of Perfect Information (EVPI)** quantifies the maximum amount a decision-maker would be willing to pay for **flawless** information regarding future outcomes. It represents the potential increase in the **best expected payoff** due to acquiring perfect foresight.

EVPI is computed using the following formula:

EVPI = EPC – max(EMV) for a maximization criterion

EVPI = min(EMV) – EPC for a minimization criterion

Where:

- **EPC (Expected Payoff under Certainty)** is the highest expected value if the decision-maker **knew the future with certainty**.
- **EMV (Expected Monetary Value)** is the best expected value **without** perfect information.

4.1. Grocer's Decision Problem: Example

The grocer's optimal decision depends on **demand levels**, which can be **low, moderate, or high**, with probabilities **0.3, 0.5, and 0.2**, respectively.

Step 1: Compute EPC (Expected Payoff under Certainty)

The highest payoff for each demand level:

- **Low demand:** 50 (from Small Order)
- **Moderate demand:** 52 (from Medium Order)
- **High demand:** 54 (from Large Order)

$$EPC=(0.3\times 50)+(0.5\times 52)+(0.2\times 54) =51.8$$

Step 2: Compute EVPI

From previous calculations, the best expected value **without** perfect information is:

$$\max(\text{EMV})=50$$

Thus:

$$\text{EVPI}=51.8-50=1.8$$

The **EVPI is \$1.8**, meaning the grocer would be willing to pay up to **\$1.8 per order** for perfect information about demand.

✓ **Alternative Interpretation:**

The **Expected Value of Perfect Information (EVPI)** can also be understood as the **minimum Expected Opportunity Loss (EOL)** across all alternatives. This highlights how much uncertainty costs the decision-maker in terms of lost potential earnings.

Mathematically, this relationship is:

$$\text{EVPI}=\min(\text{EOL})$$

Where:

- **EVPI** represents the value of knowing the future with certainty.
- **EOL** measures the regret from making a suboptimal decision under uncertainty.
- The **smallest EOL** corresponds to the best alternative under imperfect information, making **EVPI equal to the minimum EOL**.

Additionally:

$$\text{EMV}(\text{of any alternative})+\text{EOL}(\text{for the same alternative})= \text{EPC}$$

Where:

- **EMV** = Expected Monetary Value under risk.
- **EOL** = Expected Opportunity Loss due to uncertainty.
- **EPC** = Expected Payoff under Certainty (best possible decision with perfect information).

Thus, since:

$$\text{EVPI} = \text{EPC} - \max(\text{EMV}) \quad \text{and} \quad \text{Minimum EOL} = \text{EPC} - \max(\text{EMV})$$

We confirm that:

$$\text{EVPI} = \min(\text{EOL})$$

For example, in the grocer's decision problem:

- **EPC = 51.8**
- **max(EMV) = 50**
- **EVPI = 51.8 - 50 = 1.8**
- **Minimum EOL = 1.8**

EVPI and the minimum EOL both quantify the **cost of uncertainty**, showing how much the decision-maker, would benefit from perfect information.

5. Utility Theory

Economic decision-making is not solely based on **monetary values**; it also considers **personal satisfaction or utility** associated with different outcomes. **Utility** represents the relative value individuals assign to **money, goods, or experiences** based on their preferences.

Two key approaches exist in utility theory:

1. **Cardinal Utility** – Assigns a **numerical value** to the level of satisfaction (e.g., a scale from 0 to 100).
2. **Ordinal Utility** – **Ranks** alternatives in terms of preference without assigning a numerical value.

For example:

- **Cardinal Utility:** A person might assign **100 utils** to \$1 million and **150 utils** to \$3 million.
- **Ordinal Utility:** A person may prefer **\$3 million > \$1 million > \$0**, but no numerical values are given.

5.1. Utility and Diminishing Returns: Example

Consider how **utility changes** when consuming a product such as **ice cream**:

Quantity Consumed	Total Utility (Utils)	Marginal Utility
1 Unit	10	10
2 Units	18	8
3 Units	24	6
4 Units	28	4
5 Units	30	2
6 Units	30	0
7 Units	28	-2 (Discomfort)

Marks:

- **Total utility increases at a decreasing rate** (law of diminishing marginal utility).
- **Marginal utility eventually becomes negative**, meaning consuming more does not always increase satisfaction.

5.2. Expected Utility and Decision-Making Under Risk

In risky situations, **Expected Utility Theory** helps in decision-making by accounting for **both probability and individual preferences**. The decision-maker selects the option with the **highest expected utility** rather than just the highest monetary value.

Example: Choosing a Travel Option Based on Utility

A traveler must choose between:

1. A **budget airline** (cheap but unreliable in bad weather).
2. A **reliable airline** (expensive but more consistent).

Weather Condition	Cheap Airline (Utility)	Reliable Airline (Utility)
Good Weather (80%)	10	6
Bad Weather (20%)	0	5

Compute Expected Utility (EU)

$$EU(\text{Cheap Airline}) = (0.8 \times 10) + (0.2 \times 0) = 8$$

$$EU(\text{Reliable Airline}) = (0.8 \times 6) + (0.2 \times 5) = 4.8 + 1 = 5.8$$

Best Alternative: The **cheap airline** (EU = 8) is preferred because it has the highest expected utility.

5.3. Risk Attitudes and Utility Preferences

Different individuals have different **attitudes toward risk**, influencing their decisions.

Risk Classifications:

1. **Risk-Neutral:** Decisions are based purely on **expected value** (no preference for certain vs. uncertain outcomes).
2. **Risk-Averse:** Prefers **guaranteed outcomes** over uncertain ones, even if the uncertain option has a higher expected value.
3. **Risk-Seeking:** Prefers **gambling on uncertain outcomes**, even when the guaranteed outcome is better.

Example: Choosing a Lottery Ticket vs. Fixed Payout

- **Option A:** A 50% chance to win \$10, otherwise \$0.
- **Option B:** A guaranteed \$3 payout.

Expected Value Calculation:

$$EMV(A) = (0.5 \times 10) + (0.5 \times 0) = 5$$

$$EMV(B) = 3$$

A **Risk-Neutral person** chooses **Option A** (higher EMV = 5).

A **Risk-Averse person** chooses **Option B** (guaranteed payout = 3).

5.4. Application of Expected Utility in Business

Example: Grocer's Order Planning Under Expected Utility

A grocer is deciding how much pastry to order. The **profit values** and their corresponding **utility values** are:

Profit (\$)	Utility
34	0
42	0.7
44	0.8
50	0.9
52	0.95
54	1

Step 1: Compute Expected Utility (EU) for Each Alternative**Small order:**

$$EU(\text{Small})=0.9$$

Medium order:

$$EU(\text{Medium})=(0.3 \times 0.7)+(0.5 \times 0.95)+(0.2 \times 0.95)= 0.875$$

Large order:

$$EU(\text{Large})=(0.3 \times 0)+(0.5 \times 0.8)+(0.2 \times 1)=0.6$$

Best Alternative: The **small order (EU = 0.9)** is preferred because it maximizes expected utility.

- **EVPI** helps quantify the **value of perfect information**, showing how much uncertainty affects decisions.
- **Utility Theory** explains why people **do not always choose based on expected monetary value**.
- **Expected Utility Theory** incorporates **risk attitudes** into decision-making, leading to more realistic business strategies.

Chapter 2 Plan

1. Decision Trees

2. Solving Decision Tree

3. Perfect vs. Imperfect Information

4. Mathematical Expectation & Utility Expectation

1. Decision Trees

According to (Loh, 2011), *classification* and *regression* trees are machine-learning methods for constructing prediction models from data, where models are obtained by recursively partitioning the data space and fitting a simple prediction model within each partition.

1.1 Classification Trees: Categorizing Data

Classification trees are designed for scenarios where the **dependent variable** takes a **finite number of categories**. The tree organizes data into **decision nodes and leaf nodes**, ensuring a structured approach to making predictions. The **accuracy** of classification trees is often measured using **misclassification cost**, which indicates how often the model assigns an incorrect category.

Example 1: Classifying an Iris Flower

Imagine you come across an **iris flower** and want to determine its species using a **decision tree classifier**. The decision-making process begins at the **root node**, which asks:

Is the petal length less than 2.45 cm?

- **If yes**, the decision tree directs you to the **left child node**, which is a **leaf node**. No further questions are asked, and the tree **predicts** the species as **Iris-Setosa**.
- **If no**, the process continues to the **right child node**, prompting another question:

Is the petal width less than 1.75 cm?

- **If yes**, the tree predicts the flower as **Iris-Versicolor**.
- **If no**, it is classified as **Iris-Virginica**.

Outcome: With just two decision points, the model efficiently classifies the flower.

Figure 2.1 presents a **visual representation of the Iris classification tree** (Géron, 2022).

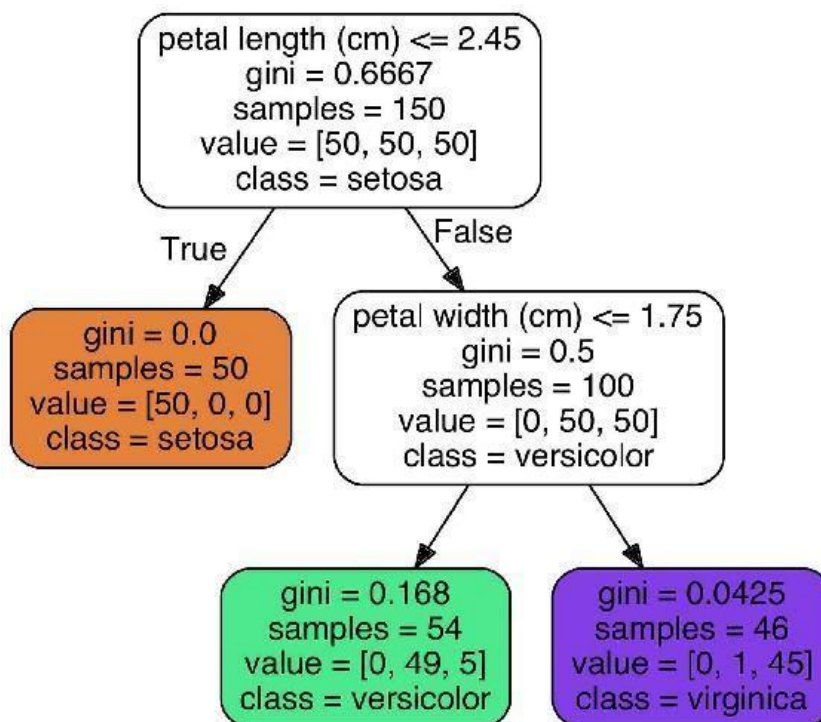


Figure 2.1. Iris Decision Tree (Géron, 2022).

1.2 Regression Trees: Predicting Continuous Values

While classification trees sort data into **distinct categories**, **regression trees** are used for **predicting numerical values**. These models are ideal when dealing with **continuous or ordinal dependent variables**, where the **Mean Squared Error (MSE)** is used to measure prediction accuracy.

Example 2: Using a Regression Tree for Prediction

Suppose you want to predict a value for a new instance where $x_1 = 0.6$. The regression tree follows these steps:

Result: The **predicted value** is **0.1106**, computed as the **average of 110 training data points**. The **MSE for this prediction** is **0.0151**, indicating a low error rate.

Figure 2.2 depicts a **regression decision tree model** (Géron, 2022).

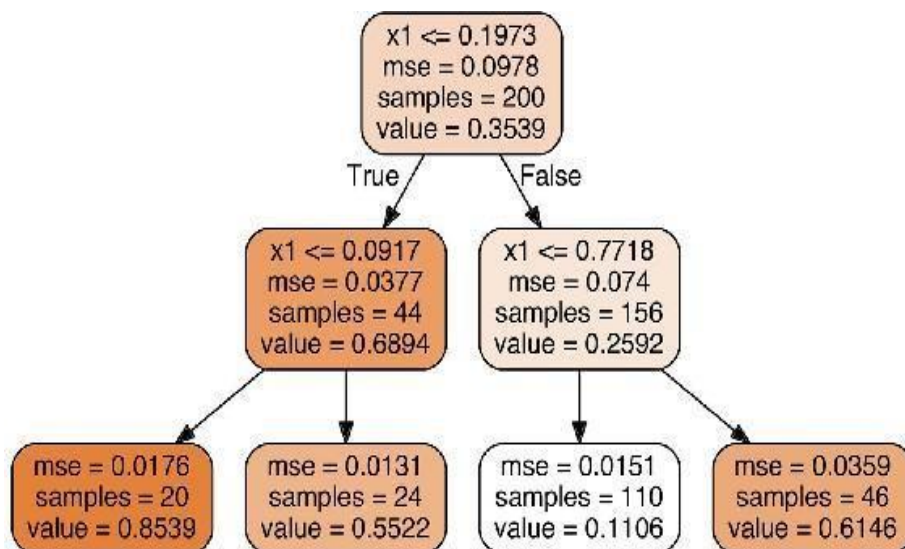


Figure 2.2. Iris Decision Tree (Géron, 2022).

1.3 Structure of a Decision Tree

Decision trees simplify **complex decision-making** by visually representing different **choices, conditions, and outcomes**. This structure makes them highly effective for **problem-solving and predictive analytics**.

Key Components of a Decision Tree:

- **Root Node:** The starting point that represents the most influential decision factor.
- **Decision Nodes:** Intermediate points where the dataset is split based on conditions.
- **Leaf Nodes:** The final classification or predicted outcome.

Example:

If a decision tree is used to predict whether to **play outside** based on weather conditions:

- The **decision node** could be "**Outlook**", with branches for "**Sunny**," "**Overcast**," and "**Rainy**".
- The **leaf nodes** would indicate whether to "**Play**" or "**Not Play**".

1.4 Transforming Decision Trees into Rules

A **decision tree** can be easily converted into a set of **if-then rules**, where each rule represents a **path from the root node to a leaf node**. These rules enhance interpretability and can be used in **rule-based decision-making systems**.

Figure 2.3 illustrates an **example of how decision trees can be mapped into decision rules**.

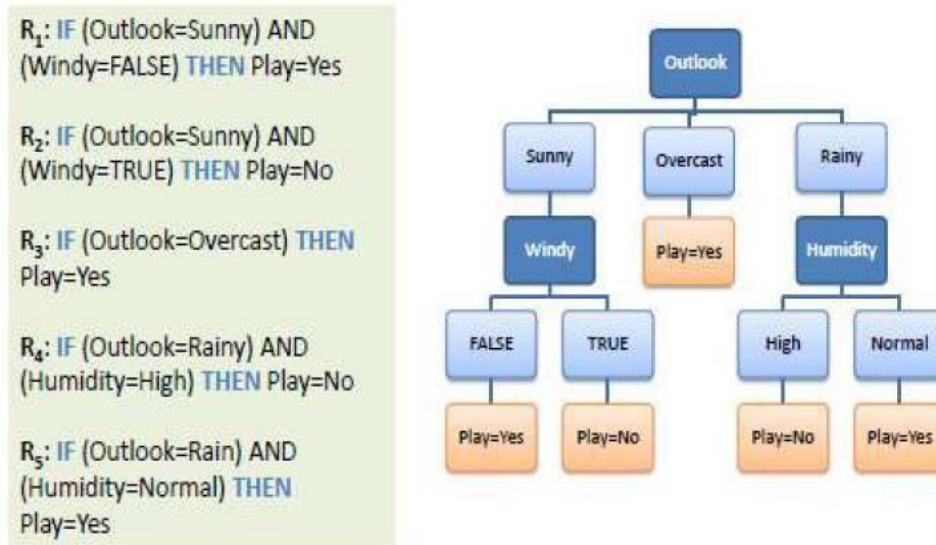


Figure 2.3. tree decision rules example.

1.5 Advantages of Decision Trees

- + **Interpretable & Easy to Understand:** No complex mathematical computations are required.
- + **Handles Both Numerical and Categorical Data:** Works well for a variety of datasets.
- + **Visual & Intuitive:** Makes it easier to analyze sequential decision-making problems.
- + **Can Be Used for Feature Selection:** Identifies key variables that impact predictions.

Decision trees structure

Decision trees can process both **categorical** and **numerical** data. They are built in a **top-down approach**, starting from the root and dividing the dataset into smaller, more uniform (homogeneous) groups based on similar values. This structured approach makes decision trees a powerful tool for classification and prediction.

Indeed, decision trees use specific symbols to represent different types of nodes:

- **Decision nodes:** Represented by squares (\square), indicating points where a decision must be made.
- **State of nature or Chance nodes:** Represented by circles (O), showing uncertain events with probabilities assigned.
- **Terminal nodes:** Represented by triangles, used optionally to indicate final outcomes.

To effectively use a decision tree, follow these steps:

1. **Define the problem** clearly, identifying all possible decisions and outcomes.
2. **Draw the decision tree**, mapping out all possible choices and their consequences.
3. **Assign probabilities** to each state of nature, representing the likelihood of different

events occurring.

4. **Estimate payoffs** for each combination of alternatives and states of nature.
5. **Solve the problem** by:
 - Computing expected values for each state-of-nature node, moving from right to left.
 - Selecting decisions that maximize the expected value to ensure the best possible outcome.

By following these steps, decision trees help in making informed and structured choices in complex decision-making scenarios.

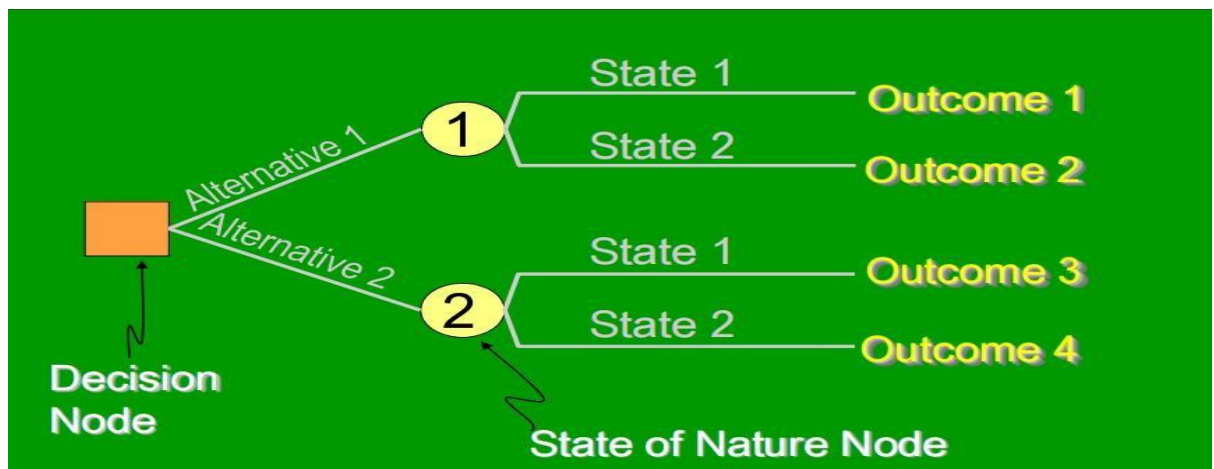


Figure 2.4. Decision Tree representation for DSS.

2. Solving Decision Tree

The process of solving a decision tree involves systematically eliminating (pruning) suboptimal choices at decision nodes while computing the expected values for all possible outcomes at chance nodes. The tree is first constructed from left to right, outlining all possible decisions and their consequences.

Once fully structured, the solution process moves from right to left, evaluating each outcome and selecting the decisions that maximize expected value. This approach ensures a logical and data-driven decision-making process, optimizing choices based on probability-weighted outcomes.

Scenario 1: A Hospital Expanding Its Emergency Unit

A hospital is considering two options to expand its emergency department to accommodate an increasing number of patients:

1. **Build a large emergency unit.**
2. **Build a small emergency unit.**

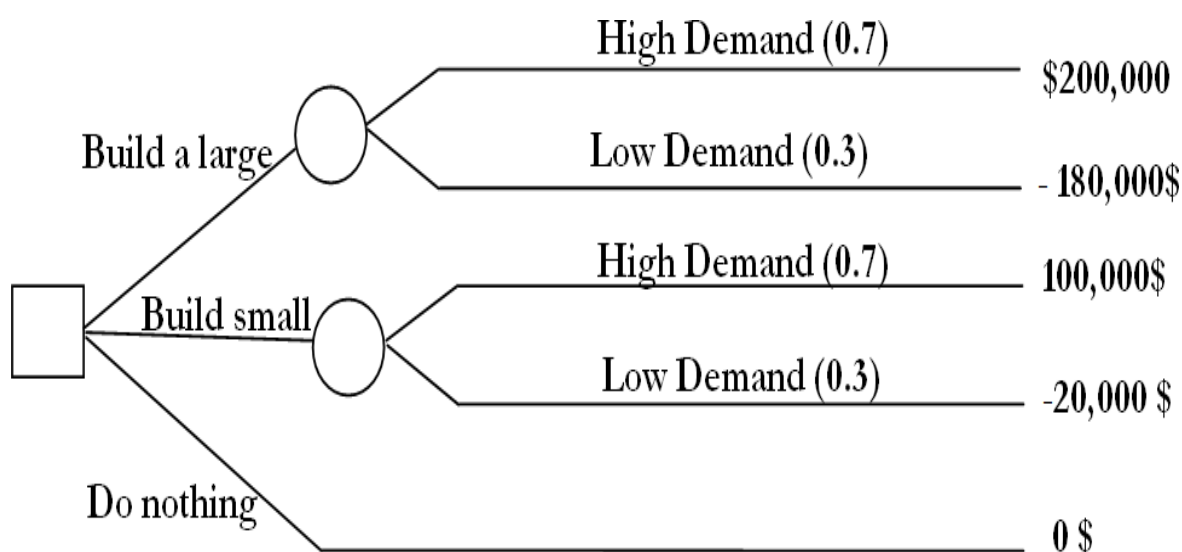
Regardless of the choice, future patient demand will be either **high** or **low**.

- If a **large emergency unit** is built and demand is high, the hospital gains **200,000\$** in additional revenue. If demand is low, it results in a **180,000\$ loss** due to underutilization.
- If a **small emergency unit** is built and demand is high, the hospital gains **100,000** in additional revenue. If demand is low, the hospital faces a **20,000 loss**.
- The hospital may also choose to "**do nothing**," resulting in neither profit nor loss.

Assume:

- **Probability of high patient demand = 0.7**
- **Probability of low patient demand = 0.3**

Using a decision tree, the hospital can compute the **expected values** for each option and determine the best course of action to maximize its financial outcomes.



2.1. Calculating Expected Monetary Value (EMV) for each option:

By using the given probabilities and the corresponding gains/losses, we can have three options.

Option 1: Build a Large Emergency Unit

$$EMV_{\text{Large Unit}} = (0.7 \times 200,000) + (0.3 \times (-180,000))$$

$$EMV_{\text{Large Unit}} = 140,000 - 54,000 = 86,000$$

Option 2: Build a Small Emergency Unit

$$EMV_{\text{Small Unit}} = (0.7 \times 100,000) + (0.3 \times (-20,000))$$

$$EMV_{\text{Small Unit}} = 70,000 - 6,000 = 64,000$$

Option 3: Do Nothing

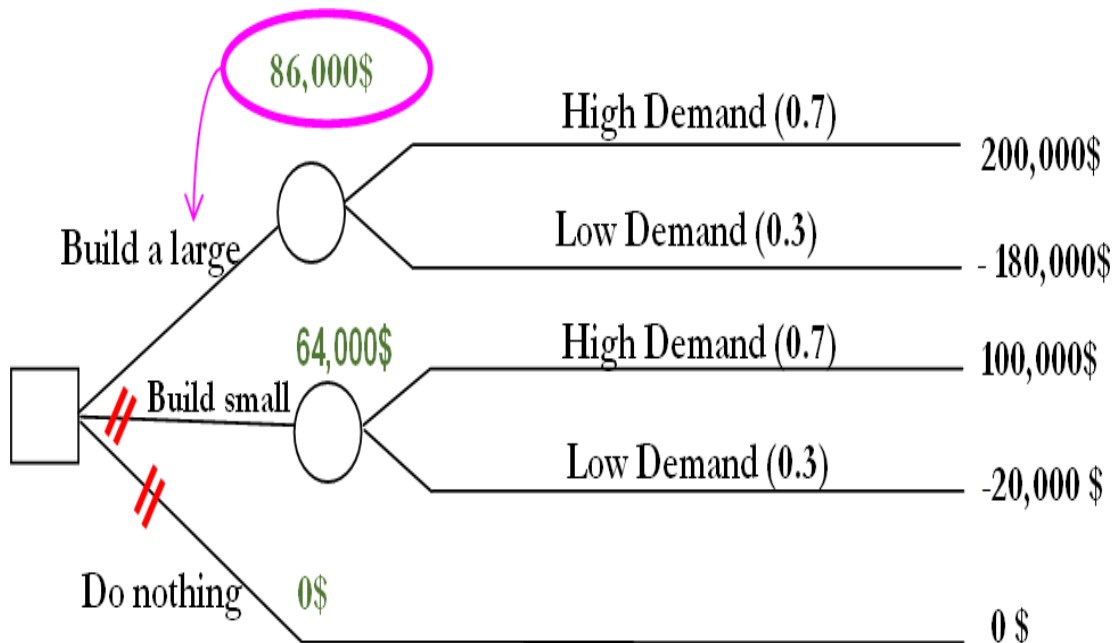
$$EMV_{\text{Do Nothing}} = 0$$

2.2. Optimal Decision

Comparing the expected monetary values:

- **Large Emergency Unit: \$86,000**
- **Small Emergency Unit: \$64,000**
- **Do Nothing: \$0**

The best choice is to **build a large emergency unit**, as it maximizes the expected financial outcome at **\$86,000**.



Scenario 2: Choosing the Right Expansion Strategy

A company preparing to launch a new product must decide how to scale production. Should it invest in a **large plant** from the start or begin with a **small plant** and expand later if needed? The demand for the product could be either **high (60% probability)** or **low (40% probability)**, making this decision critical for maximizing profitability.

Option 1: Building a Small Plant

- If demand is **low**, the company earns **\$40 million**.
- If demand is **high**, the company has two options:
 1. **Do nothing**, securing a **\$45 million** profit.
 2. **Expand production**, which introduces some risk:
 - **30% chance** demand will **drop**, leading to a **\$35 million** profit.
 - **70% chance** demand will **grow**, increasing profit to **\$48 million**.

Option 2: Building a Large Plant

- If demand is **high**, the company secures a **\$60 million** profit.
- If demand is **low**, it faces two choices:
 1. **Do nothing**, resulting in a **\$10 million loss**.
 2. **Reduce prices**, which could improve sales and lead to a **\$20 million profit**.

Main Decisions and Market Conditions

The company must navigate three critical decisions:

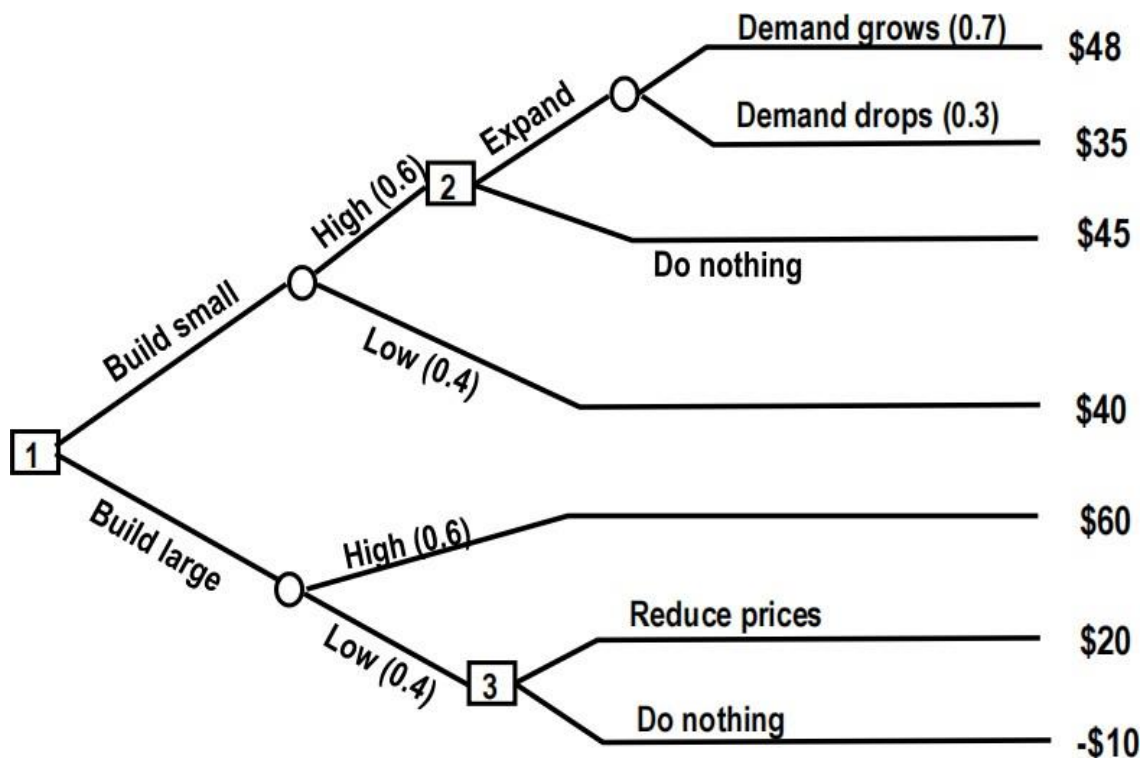
1. **Choosing the initial plant size** – Should they **build large** or **start small**?
2. **Responding to high demand with a small plant** – If they **start small** and demand is **high**, should they **expand** or **maintain current capacity**?
3. **Handling low demand with a large plant** – If they **build large** but demand is **low**, should they **reduce prices** or **accept losses**?

Market Uncertainty

The company must also consider two key uncertainties:

1. **Initial demand** – Demand could be **high (60%)** or **low (40%)**.
2. **Future demand after expansion** – If they **expand** after starting small, there is a **70% chance** demand will **grow** and a **30% chance** it will **decline**.

By utilizing a **decision tree**, the company can visualize each scenario, calculate expected values, and identify the most profitable strategy while managing potential risks.



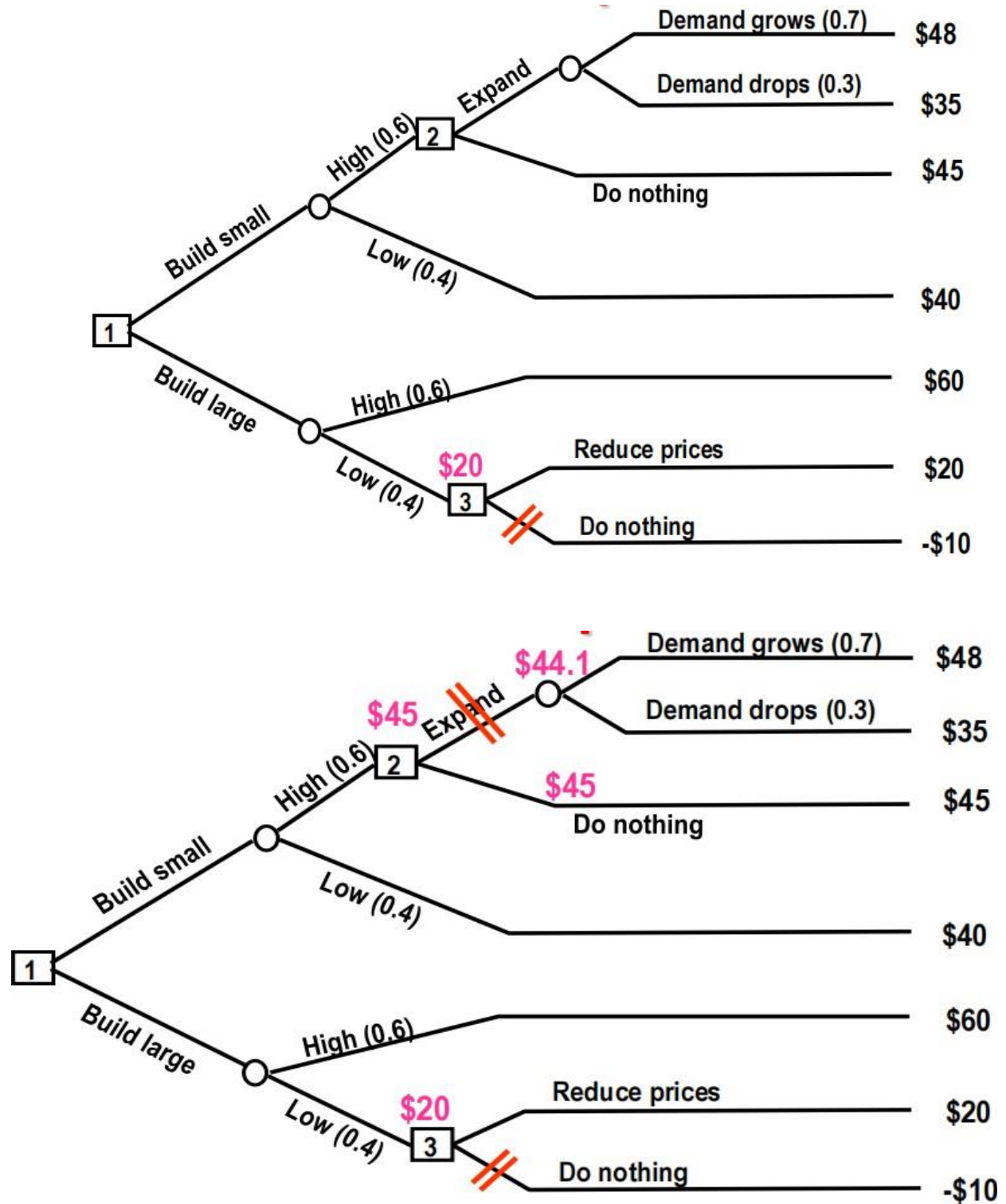
To solve the decision tree, we work **from right to left**, starting with the final decision points and evaluating the best choices based on expected outcomes.

Step 1: Evaluating Decision 3

At **Decision 3**, the company must decide whether to **reduce prices** or **do nothing** if demand is **low** after building a **large plant**. The possible outcomes are:

- **Reduce prices** → Profit of **\$20 million**
- **Do nothing** → Loss of **\$10 million**

Since **\$20 million** is greater than **-\$10 million**, the best decision at this stage is to **reduce prices**.



Step 2: Evaluating Decision 2

At **Decision 2**, the company must decide whether to **expand production** or **do nothing** if demand is **high** after building a **small plant**.

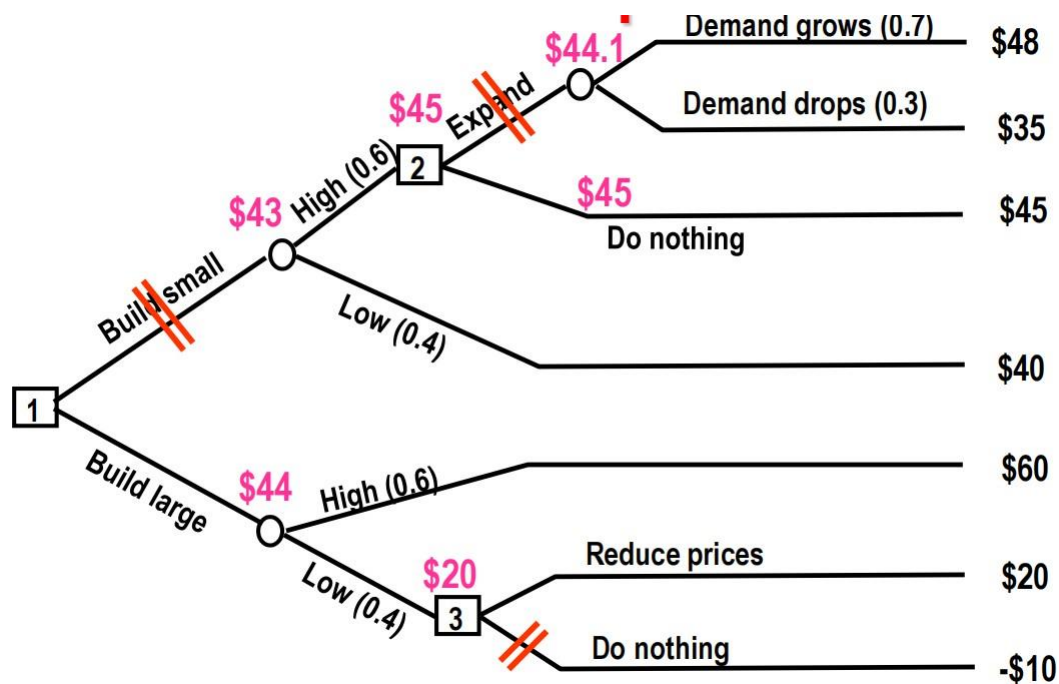
Calculating the Expected Value for Expansion:

If the company chooses to expand:

- **70% chance** demand continues to **grow**, leading to a **\$48 million** profit.
- **30% chance** demand **drops**, resulting in a **\$35 million** profit.

The expected value (EV) of expanding is calculated as:

$$EV = (48 \times 0.7) + (35 \times 0.3) = 33.6 + 10.5 = 44.1$$



Comparing the Options:

- **Expanding** has an expected value of **\$44.1 million**.
- **Doing nothing** guarantees a **\$45 million** profit.

Since **\$45 million** is greater than **\$44.1 million**, the best choice at this stage is to **do nothing**.

Final Decision: Choosing the Optimal Strategy

After evaluating all possible outcomes and expected values, the best strategy for the company is:

1. **Build a Large Plant** as the initial decision.
2. **If demand is low**, implement a **price reduction strategy** instead of doing nothing.

Final Expected Payoff:

By following this strategy, the company's expected profit is **\$44 million**, making it the most profitable and risk-adjusted decision based on the decision tree analysis.

3. Perfect vs. Imperfect Information

In decision-making, we distinguish between perfect information refers to a situation where all necessary data for making a decision is available with absolute certainty. The decision-maker knows all possible outcomes and their exact consequences in advance. This eliminates uncertainty and allows for optimal decision-making.

Example 1:

*An investor who knows with certainty how financial markets will evolve can make the best decision without risk, while **in imperfect information**, the decision-maker lacks some crucial information or faces uncertainty. Decisions must be based on estimates, probabilities, or assumptions, leading to choices under uncertainty or risk.*

Example 2:

A doctor diagnosing a patient without having all test results must make a decision with some level of uncertainty.

3.1. Perfect Information

Perfect Information provides **complete certainty** about the outcome before making a decision. It eliminates all uncertainty, allowing the decision-maker to choose the best possible option with **no risk**.

Example: An oil company knowing **exactly** whether a drilling site contains oil before investing.

The Expected Value of Perfect Information (EVPI) measures the maximum amount a decision-maker would be willing to pay for complete certainty. Mathematically, it is given by:

$$EVPI = E[\max(V1, V2)] - \max(E[V1], E[V2])$$

Where:

- $V1, V2$ are the possible values of the two alternatives.
- $E[V]$ represents the expected value before receiving any additional information.

3.2. Imperfect Information

Imperfect Information provides **partial** knowledge, reducing but not eliminating uncertainty where decisions must still be made under **some level of risk**, as the available information is not fully reliable.

Example: An oil company using **seismic surveys** that give probabilistic insights into whether a site has oil, but without absolute certainty.

Imperfect information reduces uncertainty but does not eliminate it. The **Expected Value of Imperfect Information (EVI)** measures how much a decision-maker would pay for **partial** knowledge:

$$EVI = E[\max(E[V1 | I], E[V2 | I])] - \max(E[V1], E[V2])$$

Where:

- I represents the imperfect information (e.g., a test, survey, or signal).
- $E[V|I]$ is the expected value of an alternative given the new imperfect information.

N.B. If the information signal is **strongly correlated** with the actual outcomes, the EVI approaches EVP, but, if the information is weak (i.e., provides little insight), the EVI is close to zero.

Example:

An oil company using a **seismic survey** (which provides probabilistic data about oil presence) gets partial information. The decision is still uncertain, but better than making a blind choice.

Real-World Example:

Oil Drilling Decision with Perfect vs. Imperfect Information

Let's apply the **perfect vs. imperfect information** concept using a practical case inspired by the **oil drilling example**

Scenario:

An oil company must decide whether to drill a new well. Drilling is expensive, so they must assess the probability of finding oil before investing.

Available Data:

- If oil is found, the company earns **\$100 million**.
- If oil is not found, they lose **\$50 million**.
- The probability of finding oil **without any additional information** is **0.6**.
- The probability of not finding oil is **0.4**.
- They can either drill immediately or pay for a seismic survey (which provides imperfect information about the presence of oil).

Step 1: Compute the Expected Value Without Additional Information

The **expected monetary value (EMV)** of drilling without any extra information is:

$$EMV_{No\ Info} = (0.6 \times 100) + (0.4 \times -50)$$

$$EMV_{No\ Info} = 60 - 20 = 40 \text{ million dollars}$$

If the expected value is positive, the company might consider drilling. However, they could improve their decision by gathering **more information**.

Step 2: Expected Value of Perfect Information (EVPI)

If the company had **perfect information**, they would know **with certainty** whether oil is present. This means:

- If oil is present (60% chance), they drill and earn **\$100M**.
- If oil is absent (40% chance), they avoid drilling and lose **\$0**.

The **expected value with perfect information (EVPI)** is:

$$EVPI = (0.6 \times 100) + (0.4 \times 0) = 60 \text{ million dollars}$$

The **value of perfect information** is the difference:

$$EVPI - EMV_{\text{No Info}} = 60 - 40 = 20 \text{ million dollars}$$

This means **the company would be willing to pay up to \$20M** for perfect knowledge.

Step 3: Expected Value of Imperfect Information (EVI)

Now, let's assume the company can conduct a **seismic survey**, which provides an **imperfect** signal correlated with actual oil presence.

- The test correctly detects oil **80% of the time**.
- The test incorrectly suggests oil in a dry well **20% of the time**.
- The cost of the test is **\$5 million**.

Using the **approximation**:

Step 3: Expected Value of Imperfect Information (EVI)

Now, let's assume the company can conduct a **seismic survey**, which provides an **imperfect** signal correlated with actual oil presence.

- The test correctly detects oil **80% of the time**.
- The test incorrectly suggests oil in a dry well **20% of the time**.
- The cost of the test is **\$5 million**.

$$EVI \approx \rho^2 \times EVPI$$

Using the **approximation**:

Since the test costs **\$5M** and provides information worth **\$12.8M**, it is a **good investment**.

$$EVI \approx \rho^2 \times EVPI$$

$$EVI \approx (0.8)^2 \times 20$$

$$EVI \approx 0.64 \times 20 = 12.8M$$

Since the test costs \$5M and provides information worth \$12.8M, it is a **good investment**.

Final Decision:

1. **If the test is free:** The company should definitely take it since it improves the decision.
2. **If the test costs less than \$12.8M** → The company should take it.
3. **If the test costs more than \$12.8M** → It is not worth it, and they should rely on initial probabilities.

4. Mathematical Expectation & Utility Expectation

4.1. Mathematical Expectation (Expected Value - EV)

Mathematical expectation, also known as the **expected value (EV)**, represents the **average outcome of a decision when repeated multiple times under identical conditions**. It is a key concept in decision analysis under risk, allowing decision-makers to evaluate uncertain outcomes based on their probabilities.

Formula for Expected Value (EV):

$$EV = \sum (P_i \times V_i)$$

Where:

- P_i = Probability of outcome i
- V_i = Value (payoff) associated with outcome i

Example 1: Investment Decision

A company is considering investing in a **new product launch**. The potential outcomes are:

- **High demand (50% probability, \$1M profit)**
- **Moderate demand (30% probability, \$500K profit)**
- **Low demand (20% probability, \$200K loss)**

$$EV = (0.5 \times 1,000,000) + (0.3 \times 500,000) + (0.2 \times (-200,000))$$

$$EV = 500,000 + 150,000 - 40,000 = 610,000$$

Since the expected profit is **\$610K**, the company should proceed with the investment.

4.2. Utility Expectation

While **expected value (EV)** focuses on monetary outcomes, **expected utility (EU)** considers the **decision-maker's preferences and risk attitudes**. Utility is a measure of **satisfaction or perceived value** associated with different outcomes.

$$EU = \sum (P_i \times U(V_i))$$

Where:

- P_i = Probability of outcome i .
- $U(V_i)$ = Utility function of outcome i .

Example 2: Risk Preferences in Investment

An investor has the following **risk attitudes**:

- \$1M provides **utility of 100**
- \$500K provides **utility of 80**
- Losing \$200K provides **utility of 10**

$$EU = (0.5 \times 100) + (0.3 \times 80) + (0.2 \times 10)$$

$$EU = 50 + 24 + 2 = 76$$

The best choice may be an alternative with a **higher expected utility**, even if it has a lower expected monetary value.

Differences Between Expected Value and Expected Utility

Factor	Expected Value (EV)	Expected Utility (EU)
Focus	Monetary gains/losses	Satisfaction or preference
Risk Attitudes	Assumes risk neutrality	Accounts for risk aversion/seeking
Mathematical Basis	Probability-weighted sum of payoffs	Probability-weighted sum of utilities
Decision Criteria	Choose option with highest EV	Choose option with highest EU

Conclusion :

- **EV** is useful for **objective financial decisions**.
- **EU** is better when **personal risk preferences** play a role.

Chapter 3 Plan

- 1. Quantitative and Qualitative Approaches to Decision Support**
- 2. Single-Criteria and Multi-Criteria Models, Dominance, and Efficiency**
- 3. Multiple-Criteria Decision-Making (MCDM)**
- 4. MCDM Solutions**
- 5. MCDM Methods**
- 6. Normalization in Multi-Criteria Decision Analysis (MCDA)**
- 7. Weight Assessment in Decision-Making**
- 8. Outranking Methods**

1. Quantitative and Qualitative Approaches to Decision Support

Decision support methods can be classified into **quantitative** and **qualitative** approaches. These approaches help decision-makers analyze complex problems and choose the best course of action based on available information.

1.1. Quantitative Approach to Decision Support

A **quantitative approach** relies on numerical data, mathematical models, and statistical techniques to support decision-making. This approach is objective, structured, and data-driven. Such an approach uses **mathematical models, statistical tools, and algorithms**, provides **objective and reproducible** results and commonly applied in **finance, operations research, and engineering**.

Examples: Quantitative Approach to Decision Support

- **Linear Programming:** Optimizing resource allocation in production.
- **Decision Trees:** Structuring complex decisions in business strategy.
- **Multi-Criteria Decision-Making (MCDM):** Evaluating alternatives based on multiple numerical criteria.
- **Monte Carlo Simulation:** A probabilistic technique to model uncertainty and risk in decision-making (e.g., predicting financial market trends).
- **AHP (Analytic Hierarchy Process):** A structured method for ranking alternatives based on weighted criteria (e.g., selecting the best supplier in procurement).
- **Regression Analysis:** Statistical technique for predicting future trends based on historical data (e.g., forecasting sales based on past performance).
- **Bayesian Decision Theory:** A probabilistic approach to updating decisions based on new information (e.g., medical diagnosis and treatment selection).

1.2. Qualitative Approach to Decision Support

A **qualitative approach** focuses on **expert judgment, subjective assessments, and descriptive analysis** rather than numerical data. It is commonly used when quantitative data is unavailable or insufficient.

Qualitative Approach is Based on **expert opinions, interviews, case studies, and heuristics**, it is useful for **strategic decision-making and policy analysis**, it allows **flexibility** in handling complex, unstructured problems.

Examples:

- **Expert Panels:** A group of specialists provides insights and recommendations for decision-making in uncertain situations, for instance: policy development.
- **Mind Mapping:** Visual representation of ideas and relationships in order to structure complex problems such as brainstorming strategic initiatives.
- **Case Studies:** Analyzing past decisions to understand best practices and avoid potential pitfalls (e.g., studying past market entry failures to improve future expansion strategies).
- **Focus Groups:** Gathering insights from target users or stakeholders to shape decision-making (e.g., product development based on customer feedback).

1.3. Combining Quantitative and Qualitative Approaches

Many decision support systems **integrate both approaches** to enhance decision-making. A hybrid method ensures that **hard data (quantitative)** is supported by **contextual insights (qualitative)**.

Example:

- In business decision-making, a company may use **statistical sales forecasts (quantitative)** along with **customer feedback analysis (qualitative)** to refine its market strategy.

Conclusion :

- **Quantitative approaches** provide precise, data-driven insights, ideal for structured decision-making.
- **Qualitative approaches** incorporate expert judgment and contextual analysis, useful for complex and uncertain decisions.
- **A combined approach** ensures a balanced, well-informed decision-making process.

2. Single-Criteria and Multi-Criteria Models, Dominance, and Efficiency

Multi-Criteria Decision-Making (MCDM) problems are generally classified into **continuous or discrete categories** which depend on the nature of the alternatives. **Hwang and Yoon (1981)** categorized MCDM these methods into two principal categories:

- **Multi-Objective Decision-Making (MODM)**
- **Multi-Attribute Decision-Making (MADM).**

A. Multi-Objective Decision-Making (MODM)

In case where multiple, often conflicting objectives need to be optimized simultaneously, MODM is applied. For instance, a vehicle manufacturer may aim to **maximize performance while minimizing fuel consumption**, requiring a trade-off between these objectives.

An MODM model includes:

- **Decision Variables** – Factors influencing the outcome.
- **Objective Functions** – Goals to be optimized.
- **Constraints** – Limitations defining feasible solutions.

Since MODM problems generally do not have a **single optimal solution**, decision-makers select from a **set of efficient solutions**. Several techniques are used to solve MODM problems, including **Goal Programming, Simulated Annealing, Genetic Algorithms, and Hybrid Approaches**.

B. Multi-Attribute Decision-Making (MADM)

MADM is used when decision-making involves a **finite and predefined set of alternatives**. Instead of optimizing continuous variables, MADM focuses on evaluating and ranking a **limited number of discrete choices** based on multiple attributes.

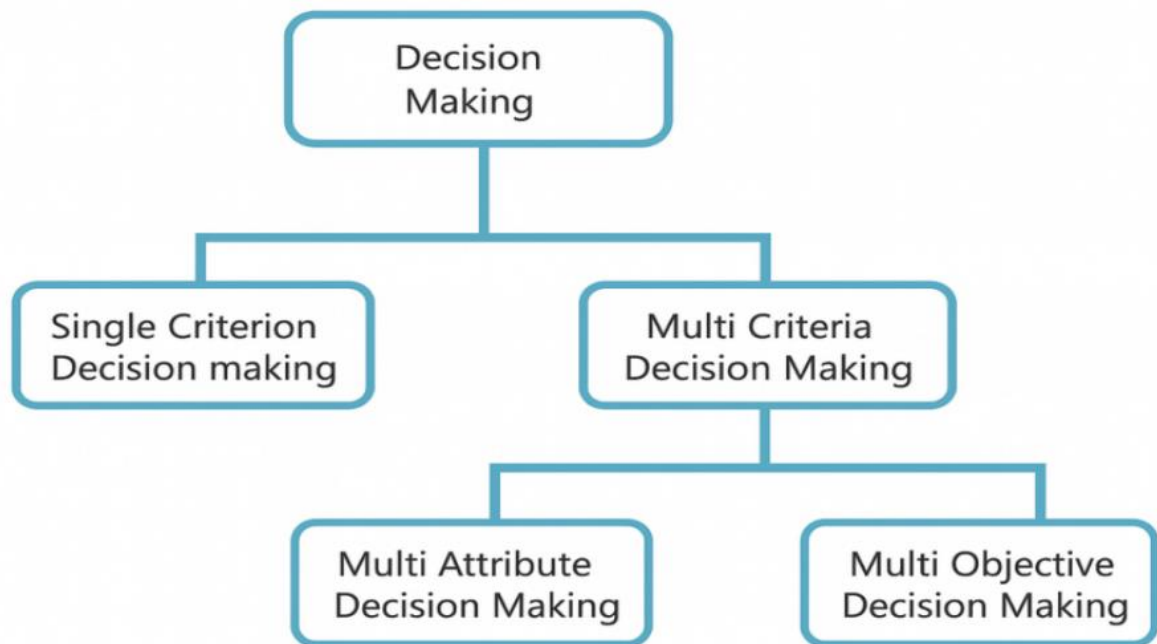


Figure 3.1. Multi-Attribute Decision-Making.

2.1. Single-Criteria Decision-Making (SCDM)

Single-Criteria Decision-Making (SCDM) is a decision-making approach where **only one criterion** is used to evaluate and select the best alternative. This method is effective when a single objective dominates the decision process, such as **cost minimization** or **profit maximization**.

For example:

- A company selecting a supplier **only based on the lowest price**, without considering quality or delivery time.
- A student choosing a university based **only on tuition fees**, without considering reputation or job placement rates.

Remark: Single-criteria decision-making often **oversimplifies complex problems**, ignoring other critical factors that may influence the outcome.

2.2. Dominance Principle in Decision-Making

The **dominance principle** states that if one alternative performs **better than another in all criteria**, then the inferior option should be eliminated. In other words, a **dominant option** is always preferable.

For instance, in **investment decision**, supposing an investor is comparing two investment options: **Investment A** with 10% return and low risk and **Investment B** with 8% return and higher risk. **Investment A dominates B** because it offers **both** a higher return **and** lower risk. Another example concerns **Product Selection** where a customer has to choose between two laptops: **Laptop X** with higher speed, better battery and lower price and **Laptop Y**: Lower speed, shorter battery, and higher price. Therefore, **Laptop X dominates Y**, so the customer should choose X. However, in real-world scenarios, **dominance does not always exist**

because some alternatives perform better in some criteria but worse in others, requiring compromise.

2.3. Efficiency in Decision-Making

Efficiency refers to the selection of an option that provides **the best possible outcome** given the available resources. In decision theory, an alternative is considered **efficient** if no other alternative provides **better results without additional costs or sacrifices**. Nevertheless, the most efficient option may not always align with **fairness, sustainability, or long-term benefits**.

Example of efficiency in Decision-Making:

- **Production Efficiency:** A manufacturing plant wants to **maximize output** while **minimizing costs**. The most **efficient** production method achieves the highest number of products with the lowest possible expenses.
- **Energy Efficiency:** A company installing **LED lighting instead of incandescent bulbs** because they provide **the same brightness with lower energy consumption**.
- **Pareto Efficiency (Economic Decision-Making):** A city allocating public transport routes in a way that **improves travel time for some citizens without making it worse for others**.

- **Single-Criteria Decision-Making** simplifies decision processes but **ignores important factors**.
- **Dominance helps eliminate inferior options**, making decisions **easier** when one alternative is clearly better.
- **Efficiency ensures the best use of resources**, but **trade-offs may still be necessary**.

3. Multiple-Criteria Decision-Making (MCDM)

Multiple-Criteria Decision-Making (MCDM) **refers to the decision-making process that involves** multiple, often conflicting criteria.

The objective of this study is not to **master every existing method** (which is impossible due to the large number of techniques and their continuous development) but rather to

understand fundamental principles, be able to **apply certain methods using software**, and **interpret the results** obtained.

According to (Aruldoss, 2013), Multi-Criteria Decision-Making (MCDM) is a powerful approach for making decisions in complex scenarios where selecting the best alternative involves multiple, often conflicting criteria. This review explores the key theoretical and practical aspects of MCDM, focusing on its various applications and methodologies.

The primary objective is to analyze different approaches, identify their strengths, and determine the most effective methods for selecting optimal alternatives. Additionally, this study examines the challenges associated with **fuzzy MCDM techniques**, which address uncertainty in decision-making.

MCDM has been widely implemented across various domains, enabling decision-makers to evaluate alternatives by considering different criteria, assigning appropriate weights, and applying suitable decision-making techniques to achieve optimal solutions. This review provides a comprehensive overview of the evolution of **fuzzy multi-criteria decision-making (FMCDM)** methods and their real-world applications.

Methods of MCDM

MCDM methods was applied to diverse applications and find the best solution for choosing the best alternative. The Figure 1 depicts the hierarchical view of MCDM methods and its types. The widely used MCDM methods have been described in following headings.

MCDM methods have been applied to different applications and find the best solution to choose the best alternative. The Figure 1 represents the hierarchical view of MCDM methods and its types.

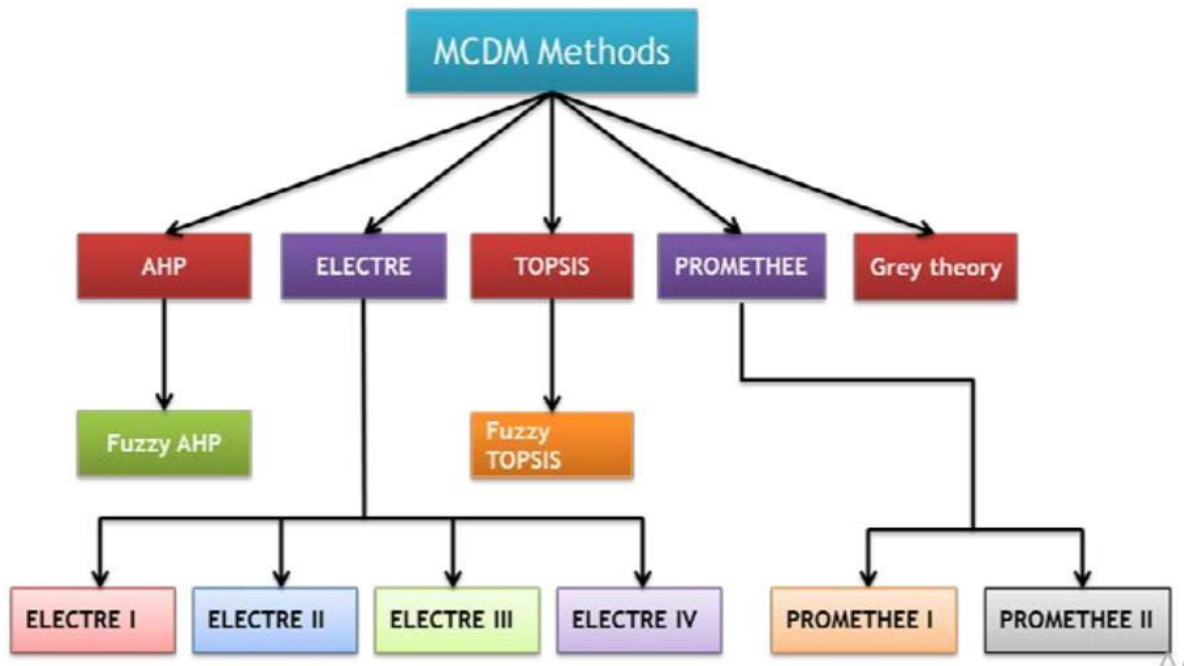


Figure 3.2. Hierarchical structure of MCDM Methods (Aruldoss, 2013).

4. MCDM Solutions

In many Multi-Criteria Decision-Making (MCDM) problems, finding a single optimal solution is often challenging. As a result, different types of solutions are defined based on their characteristics and applicability (Hwang & Yoon, 1981).

4.1. Ideal Solution

An **ideal solution** in MCDM is one that achieves the **best possible outcome** for all decision criteria. In general, criteria in decision-making can be categorized into:

- **Profit-Oriented Criteria** (to be maximized)
- **Cost-Oriented Criteria** (to be minimized)

An ideal solution would **maximize all profit criteria** and **minimize all cost criteria**. However, in most real-world scenarios, this **perfect solution does not exist**. The challenge

for decision-makers is to identify the **best possible alternative** that **closely aligns with the ideal solution**.

4.2. Non-Dominated Solutions

When an ideal solution is unattainable, decision-makers can seek **non-dominated solutions**. An alternative is considered **dominated** if there exists another alternative that is:

1. **Better in at least one criterion** and
2. **Equal or superior in all other criteria**

A **non-dominated solution** is one that **is not outperformed by any other alternative** across all decision criteria. These solutions represent **Pareto-optimal choices**, where no improvement in one criterion can be achieved without compromising another.

4.3. Satisfying Solutions

A satisfying **solution** is a feasible alternative that **meets or exceeds** the decision-maker's minimum requirements for each criterion. Instead of searching for an **absolute best** choice, this approach focuses on identifying **acceptable** solutions. **Key characteristics of this solution are:**

- A satisfying solution may **not always be non-dominated**, but it is **good enough** based on the decision-maker's expectations.
- Often used for **filtering out unacceptable alternatives** before deeper analysis.

Example:

A company selecting a supplier may not choose the absolute best supplier but **one that meets all key requirements** in price, delivery time, and quality.

4.4. Preferred Solutions

A **preferred solution** is a **non-dominated alternative** that best aligns with the **decision-maker's priorities and preferences**. This approach acknowledges that even among non-dominated solutions, **some options are more desirable** based on subjective or strategic factors.

Example:

A business considering multiple expansion locations may select one based on **strategic advantages**, even if other locations offer slightly better financial incentives.

5. MCDM Methods

Multi-Criteria Decision-Making (MCDM) methods are broadly classified into **two main categories: non-compensatory and compensatory methods (Hwang & Yoon, 1981)**. These methods differ in how they handle trade-offs between decision criteria.

5.1. Non-Compensatory Methods

Non-compensatory methods **do not allow trade-offs** between attributes. In other words, an alternative's poor performance in one criterion cannot be compensated by its strength in another. These methods evaluate alternatives by comparing each attribute **individually**, leading to a **simpler decision process**.

Common Non-Compensatory Methods:

1) Dominance Method

This method eliminates alternatives that are **outperformed in at least one criterion while being equal or worse in others**. If an alternative is dominated by another, it is discarded.

The steps of the dominance method

1. Comparison of the first two alternatives. If one is dominated, it should be eliminated.
2. Comparison of the remaining alternatives with the next one and discard the dominated options.
3. Repeat until a **set of non-dominated alternatives** is identified.

Example: In supplier selection, if Supplier A is cheaper and delivers faster than Supplier B, then **B is dominated and eliminated**.

2) Maximin Method (Wald's Criterion)

This method selects the alternative whose **worst attribute value is the best among all alternatives**. It follows the principle that "a chain is only as strong as its weakest link."

Application: Useful when **avoiding the worst possible outcome** is a priority, such as in **risk-averse financial planning**.

3) Maximax Method

Contrary to Maximin, this method selects the alternative based on its **best-performing criterion**. It is an **optimistic** approach, assuming that the most favorable condition will occur. The **Maximax Method can be used in high-risk investment decisions**, where the decision-maker aims for the highest possible return.

4) Conjunctive Constraint Method

In this approach, a **minimum acceptable threshold** is set for each attribute. Only alternatives that meet or exceed all thresholds are considered acceptable. It is used in **job recruitment**, where a candidate must meet minimum qualifications for education, experience, and skills.

5) Disjunctive Constraint Method

Unlike the conjunctive method, this approach selects alternatives **based on their strongest attribute**, ignoring weak ones. An alternative is accepted if it excels in at least **one** criterion. This method is used in **emergency response selection**, where the fastest available rescue team is chosen, even if it lacks other advantages.

6) Lexicographic Method

Based on the **most important criterion**, this method ranks alternatives. If multiple alternatives have the same ranking, the next most important criterion is considered to break the tie, and so on, until a decision is made.

The lexicographic method is used in **scholarship selection**, where the highest academic score is the primary criterion, followed by extracurricular achievements if needed.

5.2. Compensatory Methods

Unlike non-compensatory methods, compensatory models **allow trade-offs between attributes**. A lower score in one criterion can be **balanced by a higher score in another**. These methods assign numerical values to alternatives, making comparisons easier.

Common Compensatory Methods are:

5.2.1. Scoring Methods

Scoring methods evaluate alternatives based on **aggregated scores**, considering both the values of attributes and their assigned weights.

Examples:

- **Simple Additive Weighting (SAW):** Calculates the overall score of each alternative as the **weighted sum of its attribute scores**.
- **Analytic Hierarchy Process (AHP):** Uses **pairwise comparisons** to rank alternatives systematically.

Scoring methods can be **applied in supplier selection**, where price, quality, and delivery time are weighted and combined into a single score.

Analytic Hierarchy Process (AHP) in Decision-Making definition

The **AHP**, developed by **Thomas L. Saaty**, is a method for solving **complex multi-criteria decision-making problems**. It is a structured method particularly useful in situations where **subjective judgments play a crucial role** in evaluating alternatives.

AHP allows decision-makers to assess the **relative importance** of different criteria and determine the **best alternative** based on these priorities.

Steps in AHP

1. **Problem Structuring:**
 - Represent the decision problem as a **hierarchical model** consisting of three levels:
 - **Overall Goal** (the main objective).
 - **Criteria** (factors influencing the decision).
 - **Alternatives** (available choices).
2. **Pairwise Comparison of Criteria:**
 - The decision-maker evaluates the **relative importance of each criterion** using pairwise comparisons.
 - A numerical scale (typically **Saaty's 1-9 scale**) is used to quantify preferences.
3. **Ranking Decision Alternatives:**
 - Each alternative is assessed based on how well it performs for each criterion.
 - Pairwise comparisons are conducted to establish a priority ranking.
4. **Mathematical Calculation and Consistency Check:**
 - AHP **aggregates** all inputs and generates a **priority ranking** of alternatives.
 - The **Consistency Ratio (CR)** ensures the decision-maker's judgments are logically sound.

Outcome of AHP

The final output is a **ranked list of decision alternatives**, where the **highest-ranked option** represents the most preferred choice.

Hierarchy Development in AHP

The first step in applying AHP is to **visually structure** the decision problem into an hierarchy:

- **Top Level:** The **principal goal** of the decision.
- **Middle Level:** The **criteria which** influence the decision.
- **Bottom Level:** The **alternatives available** to be evaluated.

This hierarchical framework helps **break down complex decisions into manageable steps**, ensuring a clear and logical evaluation process.

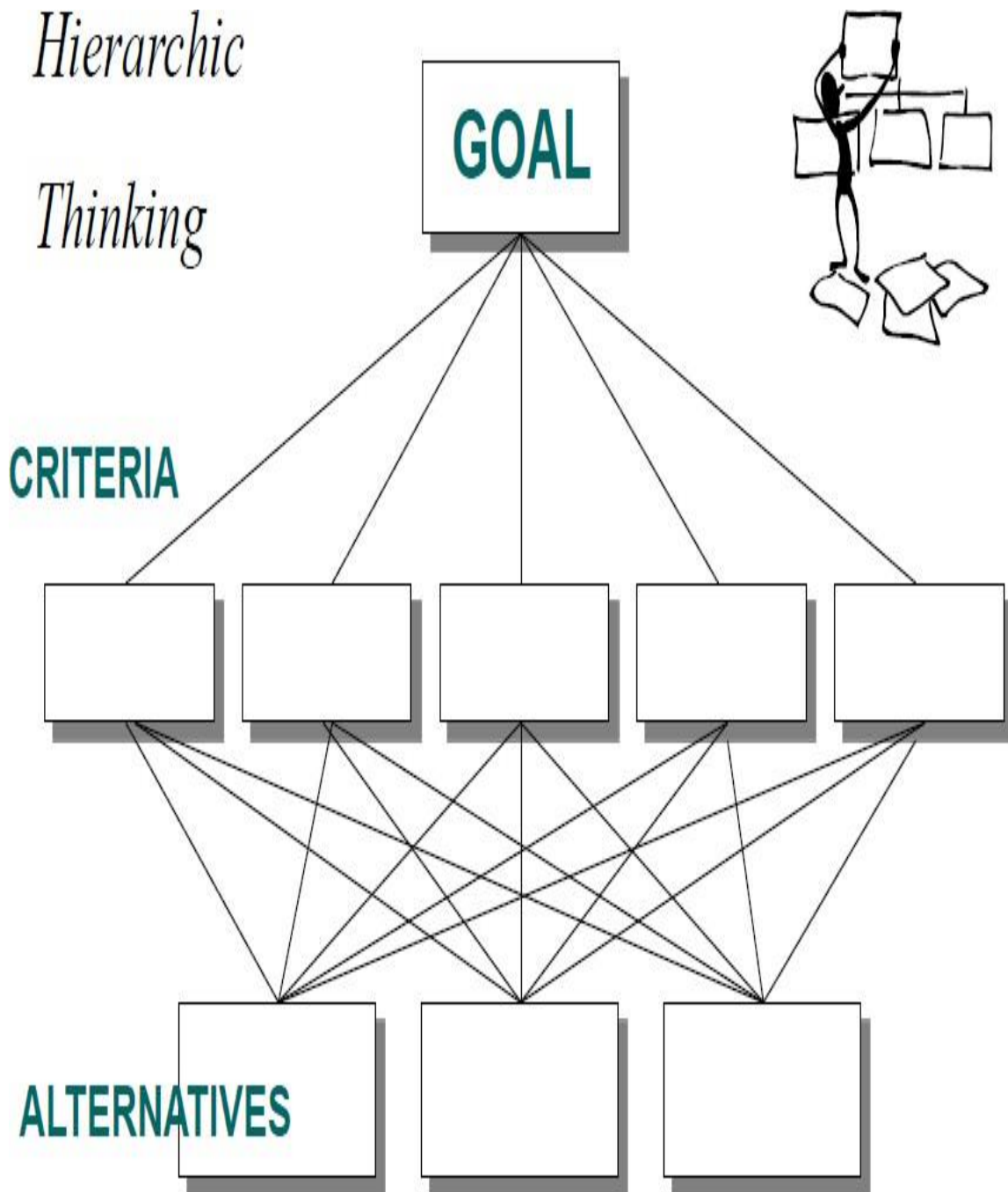


Figure 3.3. Hierarchical thinking in AHP.

Pairwise Comparisons in AHP

The **Analytic Hierarchy Process** utilizes **pairwise comparisons** for evaluating the relative importance of criteria or the preference between decision alternatives. This is done using a **numerical scale ranging from 1 to 9**, where a higher value indicates a stronger preference for one element over another.

Table 3.1. Original analytic hierarchy scale.

Numerical rating	Verbal judgments
9	Extremely more important or preferred
8	Very strongly to extremely more important or preferred
7	Very strongly preferred more important or preferred
6	Strongly to very strongly more important or preferred
5	Strongly preferred more important or preferred
4	Moderately to strongly more important or preferred
3	Moderately preferred more important or preferred
2	Equally to moderately more important or preferred
1	Equally preferred more important or preferred

Priority Calculation (Synthesis Process)

The process of the determination of the **relative importance** of each criterion or alternative in AHP is called **synthesization**. This involves converting pairwise comparison judgments into numerical priorities. The main steps in Synthesizing Priorities are:

1. **Construct the Pairwise Comparison Matrix** – A square matrix where each element represents the preference of one criterion or alternative over another.
2. **Normalize the Matrix:**
 - Sum the values of each column.
 - Divide each element by its corresponding column total.
 - The resulting matrix is known as the **normalized pairwise comparison matrix**.
3. **Calculate Priority Weights:**
 - Compute the **average of each row** in the normalized matrix.
 - These averages provide an estimate of the **relative importance of criteria** or alternatives.

Mathematical Computation:

The exact synthesis process involves **eigenvalue and eigenvector calculations**, which refine priority rankings.

Ensuring Consistency in Judgments

One of the critical aspects of AHP is **consistency** in decision-making. If judgments are inconsistent, the results may not be reliable. The key benefit is that **this ensures that decision-makers maintain logical and reliable assessments.**

Consistency Ratio (CR):

- AHP evaluates the consistency of pairwise comparisons by computing a **Consistency Ratio (CR)**.
- If **CR > 0.10**, the judgments may be inconsistent, requiring the decision-maker to revise their comparisons.

We can conclude that **Pairwise comparisons and priority synthesis in AHP** enable structured decision-making by converting subjective judgments into quantifiable rankings. The consistency check further ensures that decisions are coherent and reliable.

Estimating the Consistency Ratio in AHP

To ensure reliability in decision-making, AHP measures the **consistency** of pairwise comparisons using a **Consistency Ratio (CR)**. A CR value above **0.10** indicates inconsistencies that may require revision.

The process for calculating CR follows these steps:

A. Compute the Weighted Sum Vector

- Multiply every value in the first column of the **pairwise comparison matrix** by the priority value that corresponds.
- Repeat for all columns and sum the values across rows to obtain a **weighted sum vector**.

B. Normalize the Weighted Sum Vector

- Divide each element in the weighted sum vector by its corresponding **priority value** from the pairwise comparison matrix.

C. Compute the Largest Eigenvalue (λ_{\max})

- Now, the **average** of the normalized weighted sum values can be found. This value is referred to as λ_{\max} (largest eigenvalue).

D. Calculate the Consistency Index (CI)

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

- Where **n** is the number of criteria being compared.

E. Compute the Consistency Ratio (CR)

$$CR = \frac{CI}{RI}$$

- **RI (Random Index)** is a predefined value that depends on **n** (number of criteria).

If **CR** ≤ **0.10**, the judgments are **consistent**.

If **CR** > **0.10**, the pairwise comparisons need **revision** to improve consistency.

n	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

Example: Consistency Checking in AHP Scenario:

A company is evaluating three suppliers based on **three criteria: Cost, Quality, and Delivery Time**. The decision-maker has provided the following **pairwise comparison matrix**:

Step 1: Construct the Pairwise Comparison Matrix

Criteria	Cost	Quality	Delivery Time
Cost	1.00	3.00	2.00
Quality	1/3	1.00	1/2
Delivery Time	1/2	2.00	1.00

Step 2: Normalize the Matrix

Each element in the matrix is **divided by the sum of its respective column**, resulting in the **normalized matrix**:

Criteria	Cost	Quality	Delivery Time	Row Average (Priority Weights)
Cost	0.538	0.500	0.571	0.539
Quality	0.179	0.167	0.143	0.164
Delivery Time	0.286	0.333	0.286	0.297

Priority Weights:

- **Cost: 0.539**
- **Quality: 0.164**

- **Delivery Time: 0.297**

These weights indicate the significance of each criterion within the decision-making process.

Step 3: Calculate the Weighted Sum Vector

To obtain the weighted sum vector, each element in the original pairwise comparison matrix is multiplied by its corresponding priority weight.

$$\text{Weighted Sum Vector} = A \times \text{Priority Weights}$$

$$\begin{bmatrix} 1.00 & 3.00 & 2.00 \\ 1/3 & 1.00 & 1/2 \\ 1/2 & 2.00 & 1.00 \end{bmatrix} \times \begin{bmatrix} 0.539 \\ 0.164 \\ 0.297 \end{bmatrix} = \begin{bmatrix} 1.622 \\ 0.269 \\ 0.893 \end{bmatrix}$$

Step 4: Compute the Consistency Vector

Each element of the **Weighted Sum Vector** is **divided by** the corresponding **priority weight**:

$$\text{Consistency Vector} = \begin{bmatrix} \frac{1.622}{0.539} \\ \frac{1.269}{0.164} \\ \frac{0.893}{0.297} \end{bmatrix} = \begin{bmatrix} 3.009 \\ 3.008 \\ 3.010 \end{bmatrix}$$

Principal Eigenvalue (λ_{\max}) is the **average** of the Consistency Vector:

$$\lambda_{\max} = \frac{3.009 + 3.008 + 3.010}{3} = 3.009$$

Step 5: Computation of the Consistency Index (CI)

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

$$CI = \frac{3.009 - 3}{3 - 1} = \frac{0.009}{2} = 0.0046$$

Step 6: Computation of the Consistency Ratio (CR)

Using the **Random Index (RI)** value for $n=3$ (n is from the RI Table), **RI = 0.58**:

$$CR = \frac{CI}{RI} = \frac{0.0046}{0.58} = 0.0079$$

Step 7: Interpretation of Consistency Ratio (CR) Final

CR Value = 0.0079

- $CR \leq 0.10$ → The judgments are consistent
- $CR > 0.10$ → The comparisons need to be revised

Since **0.0079 is significantly less than 0.10**, the decision-maker's pairwise comparisons are **highly consistent** and do not require any revision.

Final results :

- **The computed priority weights:**
 - **Cost (53.9% importance)**
 - **Delivery Time (29.7% importance)**
 - **Quality (16.4% importance)**

- The consistency ratio (CR) is within the acceptable threshold ($0.0079 < 0.10$), meaning the comparisons are reliable.

Application of AHP in Leadership Selection

To demonstrate the **Analytic Hierarchy Process (AHP)** in a real-world scenario, consider the case of selecting the best leader for an organization. Leadership selection involves evaluating multiple candidates based on specific criteria that reflect key leadership qualities.

Step 1: Defining the Criteria and Alternatives

A company is looking to promote one of three candidates: **Candidate A, Candidate B, and Candidate C** for a senior leadership position. The selection process is based on four key evaluation criteria:

- **Experience (E)** – Years of managerial and industry experience.
- **Decision-Making Ability (D)** – Capacity to analyze complex situations and make effective decisions.
- **Communication Skills (C)** – Ability to articulate ideas clearly and engage with stakeholders.
- **Innovation & Vision (I)** – Creativity and strategic foresight in leadership.

Each candidate is compared based on these criteria, and **AHP is used to rank them** systematically.

Step 2: Constructing Pairwise Comparison Matrices

Pairwise comparisons are made for:

1. **The relative importance of the criteria.**
2. **The candidates' performance in each criterion.**

Pairwise Comparison of Criteria

Criteria	Experience (E)	Decision-Making (D)	Communication (C)	Innovation (I)	Priority Vector
Experience (E)	1	2	3	4	0.47
Decision-Making (D)	1/2	1	2	3	0.27
Communication (C)	1/3	1/2	1	2	0.16
Innovation (I)	1/4	1/3	1/2	1	0.10

Priority Vector (Weights for Criteria): (0.47, 0.27, 0.16, 0.10)

This means **Experience** is the most important factor (47%), followed by **Decision- Making (27%)**, **Communication (16%)**, and **Innovation (10%)**.

Step 3: Pairwise Comparison of Candidates for Each Criterion

1) Candidate Comparison Based on Experience (E)

Candidate	A	B	C	Priority Vector
A	1	3	5	0.636
B	1/3	1	2	0.246
C	1/5	1/2	1	0.118

Priority Vector for Experience (E): (0.636, 0.246, 0.118)

2) Candidate Comparison Based on Decision-Making (D)

Candidate	A	B	C	Priority Vector
A	1	2	4	0.50
B	1/2	1	2	0.30
C	1/4	1/2	1	0.20

Priority Vector for Decision-Making (D): (0.50, 0.30, 0.20)

3) Candidate Comparison Based on Communication (C)

Candidate	A	B	C	Priority Vector
A	1	1/3	1/5	0.118
B	3	1	1/2	0.246
C	5	2	1	0.636

Priority Vector for Communication (C): (0.118, 0.246, 0.636)

4) Candidate Comparison Based on Innovation & Vision (I)

Candidate	A	B	C	Priority Vector
A	1	1/2	1/4	0.118
B	2	1	1/2	0.246

Candidate	A	B	C	Priority Vector
C	4	2	1	0.636

Priority Vector for Innovation (I): (0.118, 0.246, 0.636)

Step 4: Compute Final Scores Using Weighted Sum Method

The final ranking is determined by multiplying each candidate's priority vector by the weight of the corresponding criterion:

$$\text{FinalScore} = (\text{Experience} \times 0.47) + (\text{DecisionMaking} \times 0.27) + (\text{Communication} \times 0.16) + (\text{Innovation} \times 0.10)$$

Candidate	Experience (0.47)	Decision- Making (0.27)	Communication (0.16)	Innovation (0.10)	Final Score
A	$0.636 \times 0.47 =$ 0.299	$0.50 \times 0.27 =$ 0.135	$0.118 \times 0.16 =$ 0.019	$0.118 \times 0.10 =$ 0.012	0.465
B	$0.246 \times 0.47 =$ 0.116	$0.30 \times 0.27 =$ 0.081	$0.246 \times 0.16 =$ 0.039	$0.246 \times 0.10 =$ 0.025	0.261
C	$0.118 \times 0.47 =$ 0.056	$0.20 \times 0.27 =$ 0.054	$0.636 \times 0.16 =$ 0.102	$0.636 \times 0.10 =$ 0.064	0.276

Step 5: Ranking the Candidates

- **Candidate A = 0.465** (Best choice)
- **Candidate C = 0.276** (Second best)
- **Candidate B = 0.261** (Third)

Conclusion: Based on the **AHP method**, **Candidate A is the best option** for the leadership role, as they have the highest final score. This systematic approach allows decision-makers to **prioritize key leadership criteria** and make **objective, data-driven** choices.

Why Use AHP for Leadership Selection?

- Provides a **structured framework** for evaluating candidates.
- Reduces **bias and subjectivity** in the selection process.
- Ensures **consistency** by systematically comparing **multiple factors**.
- Can be used in **business, HR, and executive recruitment**.

5.2.2. Compromise Methods

These methods identify solutions that are **closest to the ideal alternative** while being far from the worst possible option. They can be used in **investment decision-making**, where the best option should have **high returns** and **low risk**.

Example: TOPSIS Method (Technique for Order Preference by Similarity to Ideal Solution)

TOPSIS is a **multi-criteria decision-making (MCDM) technique** used to rank alternatives by evaluating their proximity to an **ideal solution** while ensuring they are farthest from the **worst-case scenario (nadir solution)**. The method follows a structured approach:

- 1) **Normalize the Decision Matrix** → Standardize the criteria to enable a fair comparison across different scales.
- 2) **Identify the Ideal and Nadir Solutions** → The **ideal solution** consists of the most favorable values for each criterion, while the **nadir solution** represents the least favorable values.
- 3) **Calculate the Distance to Both Solutions** → Measure how far each alternative is from the ideal and nadir solutions.
- 4) **Rank the Alternatives** → Select the option **closest to the ideal solution** and **farthest from the nadir solution** to determine the best choice.

TOPSIS is applied in **supplier evaluation, project selection, financial decision-making, and risk assessment**, where multiple conflicting criteria must be considered simultaneously.

5.3. Concordance Methods

These methods rank alternatives based on their overall dominance across attributes, considering how frequently an alternative **outperforms others**.

An example of concordance methods is the **Linear Assignment Method** which assigns rankings to alternatives based on multiple criteria, generating a preference order.

These methods can be **applied in performance evaluation**, where employees are ranked based on multiple competencies like productivity, teamwork, and leadership.

6. Normalization in Multi-Criteria Decision Analysis (MCDA)

In Multi-Criteria Decision Analysis (MCDA), different criteria can have varying units of measurement, such as percentages, rankings, grades, monetary values, or physical quantities. To ensure consistency in evaluation, **normalization** is applied to convert all criteria into a **dimensionless** and comparable format.

6.1. Purpose of Normalization

- **Eliminates unit discrepancies** by mapping data onto a common scale.
- **Ensures comparability** across different criteria.
- **Transforms minimizing criteria into maximizing ones** when required by certain MCDA methods.
- **Handles negative values**, as some MCDA techniques work only with positive numbers.

6.2. Common Normalization Techniques

6.2.1. Vector Normalization: The decision matrix is transformed so that each criterion's values are expressed in relation to the vector length.

$$X_{ij}^{norm} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^m X_{ij}^2}}$$

Where:

- X_{ij} = Original value of criterion j for alternative i
- m = Number of alternatives
- X_{ij}^{norm} = Normalized value

6.2.2. Linear Max-Min Normalization

It is used when data must be scaled between a defined range (e.g., [0,1]).

- For **benefit criteria** (to be maximized): The normalized value is calculated using the highest observed value.

$$X_{ij}^{norm} = \frac{X_{ij} - X_j^{min}}{X_j^{max} - X_j^{min}}$$

- For **cost criteria** (to be minimized): The normalized value is computed using the lowest observed value.

$$X_{ij}^{norm} = \frac{X_j^{max} - X_{ij}}{X_j^{max} - X_j^{min}}$$

Where:

- X_j^{max} = Maximum value for criterion j
- X_j^{min} = Minimum value for criterion j

6.2.3. Linear Sum-Based Normalization

Each value is divided by the sum of all values in its respective criterion, ensuring proportions remain consistent.

$$X_{ij}^{norm} = \frac{X_{ij}}{\sum_{i=1}^m X_{ij}}$$

Other normalization techniques include

6.2.4. Linear Max Normalization

It normalizes values by dividing every value by the **maximum value** of its criterion.

This method is useful for **proportional scaling**, where values are directly compared to the highest observed value.

$$X_{ij}^{norm} = \frac{X_{ij}}{X_j^{max}}$$

6.2.5. Gaussian Normalization

This method standardizes data by centering it around a **mean of 0** and a **standard deviation of 1**, making it useful when data follows a normal distribution.

$$X_{ij}^{norm} = \frac{X_{ij} - \mu_j}{\sigma_j}$$

Where:

- μ_j = Mean of criterion j
- σ_j = Standard deviation of criterion j

Gaussian normalization is commonly used in **machine learning, statistics, and decision-making methods sensitive to data distribution**.

Normalization is a crucial pre-processing step in MCDA, ensuring that all criteria are comparable and aligned for decision-making. Various methods are used depending on the nature of the data in addition to the requirements of the decision model.

7. Weight Assessment in Decision-Making

In many decision-making methods, it is essential to determine the **relative importance** of each criterion. This is represented by a set of **weights**, which are **normalized** so that their total sum equals **1**. Weight determination models are generally classified into **subjective** and **objective** approaches.

7.1. Subjective Weighting Methods

Subjective approaches rely on the **opinions, experience, and intuition** of decision-makers or experts. In this approach, weights are assigned based on **expert judgments**, allowing the decision-maker to directly influence the outcome.

Some of the characteristics of these methods are:

- Reflects personal preferences and expert insights.
- Often involves **pairwise comparisons** or direct ranking of criteria.
- Used in situations where **quantitative data is insufficient or unavailable**.

Common Methods:

1. **Analytic Hierarchy Process (AHP):** Compares criteria in pairs to derive relative importance.
2. **DEMATEL (Decision-Making Trial and Evaluation Laboratory):** Identifies cause-effect relationships between criteria.
3. **Best-Worst Method (BWM):** Determines the most and least important criteria to establish relative weights.

7.2. Objective Weighting Methods

Objective approaches determine weights **mathematically**, based on data from a **decision matrix**, without relying on subjective opinions. These methods ensure that weight assignment is **data-driven and independent of personal bias**.

Key Characteristics of these methods are:

- Uses mathematical models to analyze the relationships between criteria.
- Ensures consistency and minimizes human bias.
- Applied in **data-rich environments** where objective assessment is required.

Common Methods are:

1. **Entropy Method:** Measures the degree of variation in data to assign importance to criteria.
2. **CRITIC (Criteria Importance Through Inter-criteria Correlation):** Uses correlation and contrast intensity to compute weights.
3. **FANMA Method:** A statistical approach named after its authors, used for multi-criteria weight determination.

Recapitulation: Weight assessment plays a crucial role in **multi-criteria decision-making**. While **subjective methods** incorporate expert judgment, **objective methods** ensure a data-driven approach. The selection between these methods depends on **the availability of data, nature of the decision problem**, and the necessary degree of **objectivity**.

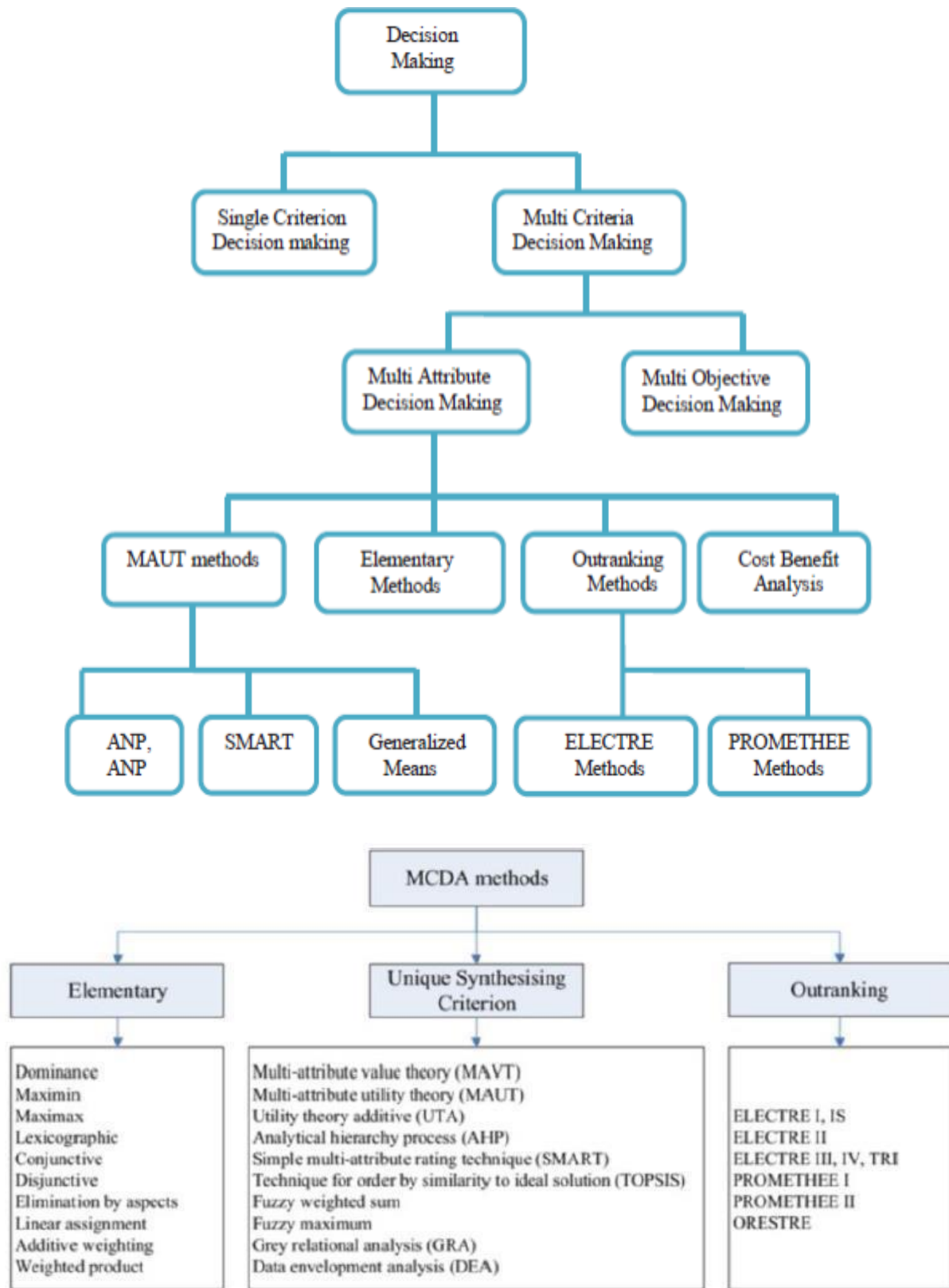


Figure 3.4. Multi-Criteria Decision Analysis (MCDA) methods.

8. Outranking Methods

8.1. Introduction to Outranking Approaches

Outranking methods are a class of **multi-criteria decision-making (MCDM) techniques** used to compare alternatives by establishing **pairwise dominance relationships** instead of aggregating scores into a single ranking. Unlike compensatory models, where trade-offs between criteria are allowed, **outranking approaches determine whether one alternative is at least as good as another across multiple criteria while considering qualitative and quantitative aspects.**

These methods are particularly effective in **complex decision-making environments** where:

- Criteria are **conflicting** (e.g., maximizing quality while minimizing cost).
- Decision-makers require a **comparative evaluation** rather than a strict ranking.
- **Uncertainty exists** in the assessment of criteria weights or performance scores.

Outranking Methods characteristics are:

Pairwise Comparisons: Alternatives are evaluated relative to each other rather than independently,

Handle incomparability: Some alternatives may remain **partially ordered** if no clear dominance exists,

Flexibility with qualitative and quantitative data (i.e., works well when some criteria are **subjective** for example customer satisfaction) and others are **numerical** for instance cost).

Adaptability to Decision-Maker Preferences: Can incorporate **weights, thresholds, and preference structures** for better alignment with strategic goals (Figueira, Greco, & Ehrgott, 2016).

8.2. Common Outranking Methods

Several outranking techniques exist, with the **ELECTRE (Elimination et Choix Traduisant la Réalité)** and **PROMETHEE (Preference Ranking Organization Method for Enrichment Evaluations)** families being the most widely applied (Roy, 1996; Brans & Vincke, 1985). Recent advancements have introduced methods such as the **Ordinal Priority Approach (OPA)** and **Dominance-Based Rough Set Approach (DRSA)**, which further refine decision-making in uncertain and large-scale environments (Mahmoudi et al., 2021).

8.2.1 ELECTRE Method

The **ELECTRE** method is a widely used **outranking approach** that compares alternatives by analyzing both **concordance (agreement) and discordance (disagreement) indices**. Rather than providing a single ranked list, **ELECTRE eliminates weak alternatives** by identifying those that **do not meet minimum preference thresholds**.

8.2.2 PROMETHEE Method

The **PROMETHEE I** (partial ranking) and **PROMETHEE II** (complete ranking) were developed by J.P. Brans and presented for the first time (Brans et al, 2016). **PROMETHEE** is another outranking technique designed to **rank alternatives based on preference flows**. Unlike **ELECTRE**, which focuses on eliminating weak alternatives, **PROMETHEE** provides a **complete ranking** by analyzing the **positive and negative preference relationships** between options.

Steps in PROMETHEE are:

1. **Define the Decision Matrix;**
2. **Normalize the Data;**
3. **Select Preference Functions;**
4. **Compute Preference Indices;**
5. **Calculate Positive and Negative Preference Flows.**

PROMETHEE is widely applied in **financial decision-making, logistics, energy planning, and supply chain management**, where a **clear ranking of alternatives** is required.

8.2.3. Ordinal Priority Approach (OPA)

The **Ordinal Priority Approach (OPA)** is a recent advancement in MCDM that **simplifies pairwise comparisons and eliminates inconsistencies**. It utilizes **linear programming to derive criteria weights** directly from **ordinal rankings provided by decision-makers**, making it especially useful when **exact numerical values are unavailable or unreliable** (Mahmoudi et al., 2021).

OPA has been applied in **project selection, supplier evaluation, and healthcare decision-making**, particularly where **decision-makers prefer to provide rankings rather than exact numerical assessments**.

8.2.4 Dominance-Based Rough Set Approach (DRSA)

The **Dominance-Based Rough Set Approach** ameliorates rough set theory to **multi- criteria decision-making** by analyzing **dominance relationships among alternatives**. Unlike traditional rough set methods, DRSA accounts for **preference information** rather than just categorical distinctions.

DRSA is a **non-compensatory approach**, it is a **Rule-Based Decision Making** (i.e., Generates decision rules based on **dominance relationships** among alternatives).

DRSA is used in **policy analysis, customer segmentation, and credit risk assessment**, where decision rules need to be **interpretable and justifiable**.

8.3. Advantages and Limitations of Outranking Methods

Feature	ELECTRE	PROMETHEE	OPA	DRSA
Decision Type	Best subset selection	Best ranking	Ranking with ordinal data	Rule-based classification
Handling of Trade-offs	No (elimination-based)	Yes (preference flows)	No (uses rankings)	No (dominance-based)
Preference Modeling	Concordance/Discordance	Preference Functions	Ordinal Rankings	Dominance Rules
Computational Complexity	High	Moderate	Low	High

Questions with some answers:

Problem Statement: A company has purchased a piece of land intended for the construction of luxury apartments. The company has preliminary architectural plans for four different project sizes

- **30 apartments**
- **60 apartments**
- **90 apartments**
- **100 apartments**

The financial success of the project depends on the **number of apartments built** and **market demand**. The company considers two possible scenarios: **high demand** and **low demand**. The objective is to determine the **optimal project size** that maximizes profit given the **uncertainty in demand**.

Profit Matrix (in millions of euros):

Demand →	High Demand	Low Demand
30 Apartments	10	8
60 Apartments	15	9
90 Apartments	20	15
100 Apartments	24	14

Question 1.1: Decision Under Total Uncertainty

Determine the best alternative (1, 2, 3, or 4) and the corresponding value using the following decision rules:

Question 1.2: Decision Under Risk

Assuming that the probability of **low demand** is **30%**, determine the best alternative and its corresponding value using the following decision methods:

Decision Method	Value	Decision
Expected Monetary Value (EMV)	21	4
Expected Opportunity Loss (EOL)	0.3	4
Maximum-Likelihood Decision Criterion	24	4

Question 1.3: Expected Value of Perfect Information (EVPI)

Calculate the **EVPI**, which measures the maximum amount the company would be willing to pay for perfect information regarding future demand.

$$\text{EVPI} = 0.3$$

Question 2: Classification of Decisions by Risk Level

- How are **decisions categorized** based on their degree of risk? Explain.

Question 3: Decision Matrices and Decision Levels

1. What **characterizes** a **decision matrix** or a **decision table**?

2. List and explain the different **decision levels**:
 - **Strategic Decisions**
 - **Tactical Decisions**
 - **Operational Decisions**

Question 4: Components of a Decision Support System (DSS)

List and explain the **main components** of a **Decision Support System (DSS)**:

- **User Interface (UI)**
- **Model Base**
- **Database**

Practical Work number 1

A grocery store owner can place an order with a wholesaler for small, medium, or large quantities of a product. The store's **daily profit** depends on **demand**, which can be **low, moderate, or high**. The profit values (in dollars) for each **order size** and **demand level** are presented in the table below:

Demand	Low	Moderate	High
Small Order	50	50	50
Medium Order	42	52	52
Large Order	34	44	54

Using **Excel**, answer the following questions:

1. Decision Under Complete Uncertainty

Determine the best alternative using the following decision rules:

- 1.1 **Maximax Rule (Optimistic Approach)**
- 1.2 **Maximin Rule (Wald's Criterion – Conservative Approach)**
- 1.3 **Hurwicz α -Index Rule**, with $\alpha = 0.75$
- 1.4 **Minimax Regret Rule (Savage's Criterion)**
- 1.5 **Principle of Insufficient Reason (Laplace Criterion)**

2. Decision Under Risk

Assuming that the grocer assigns **probabilities of 0.3, 0.5, and 0.2** to **low, moderate, and high demand**, respectively, determine the best alternative based on the following criteria:

- 2.1 **Maximum Expected Monetary Value (EMV)**
- 2.2 **Expected Opportunity Loss (EOL)**
- 2.3 **Maximum-Likelihood Decision Criterion**
- 2.4 **Maximum Expected Utility**, assuming the following utility values for each profit level:

Profit (\$)	34	42	44	50	52	54
Utility	0	0.7	0.8	0.9	0.95	1

3. Expected Value of Perfect Information (EVPI)

Calculate the **EVPI**, which measures the **maximum value of perfect information** in decision-making.

4. Decision Tree Representation

Use **TreePlan**, an Excel Add-in, to construct a **decision tree diagram**. TreePlan is an Excel-based tool designed for **decision tree analysis** and consists of a single add-in file: **TREEPLAN.XLA**.

Practical Work number 2

Decision Analysis and Decision Trees

Exercise 1: Choosing Between a Fixed Payment or Variable Returns

Olivia is a **renowned software developer** who has created a **highly anticipated mobile application**. Two companies are interested in acquiring exclusive rights to distribute her app: **a technology firm and an investment company**.

- If she **partners with the technology firm**, she will receive a **fixed payment** regardless of sales performance.
- If she **partners with the investment company**, her earnings will be based on the **market success of the app**, which could lead to **higher or lower returns**.

Projected Earnings and Probabilities

- **Investment Company (Variable Payouts):**
 - **Low Market Performance** → \$200,000
 - **Moderate Market Performance** → \$1,000,000
 - **High Market Performance** → \$3,000,000
- **Technology Firm (Fixed Payment):**
 - **Flat rate** → \$900,000
- **Market Probabilities:**
 - **P(Low Market Performance) = 0.3**
 - **P(Moderate Market Performance) = 0.6**
 - **P(High Market Performance) = 0.1**

Questions:

- Use **TreePlan**, an Excel add-in, to create and solve a **decision tree** to determine the best financial decision for Olivia.
- TreePlan is available as an **Excel add-in file (TREEPLAN.XLA)** for decision tree modeling.

Exercise 2: Evaluating the Best Alternative Using a Decision Tree

Given a **payoff table** representing different decision scenarios, construct a **decision tree** to determine the best possible alternative using TreePlan in Excel.

Investment Choice (Alternatives)	Profit in \$1,000's (States of Nature)		
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Exercise 3: Strategic Planning for a New Product Launch

A manufacturing company is considering launching a new electronic device. Initially, they must decide whether to **invest in a large-scale production facility or a smaller manufacturing unit**. The product demand could be either **high or low**, affecting potential revenues.

Market Probabilities:

- Probability of High Demand = 0.6
- Probability of Low Demand = 0.4

Payoff Scenarios:

Small Facility:

- If demand is low, the company earns \$40 million.
- If demand is high, they can:
 - Do nothing → Earn \$45 million.
 - Expand production:
 - 30% chance of demand dropping, leading to a \$35 million payoff.
 - 70% chance of demand increasing, resulting in a \$48 million payoff.

Large Facility:

- If demand is high, the company earns \$60 million.
- If demand is low, they can:
 - Do nothing, leading to a \$10 million loss (-\$10M).
 - Lower product prices, yielding a \$20 million payoff.

Questions:

- Construct a **decision tree** using **TreePlan** to analyze the best course of action for the company.
- Compare the expected values of different strategies to determine the **optimal decision**.

Exercise 4: Decision Analysis Based on Payoff Tables

Given the following **payoff table**, which represents **profits** under different states (S1, S2, S3, S4):

States (S)	S1	S2	S3	S4
Action A1	5	10	15	20
Action A2	8	12	18	25
Action A3	4	15	10	22

We need to determine the **best action (A1, A2, or A3)** using different decision criteria:

(a) Maximax Criterion (Optimistic Approach)

This rule selects the action with the **highest possible gain**, assuming the **best-case scenario** always occurs.

- **A1:** $\max(5, 10, 15, 20) = 20$
- **A2:** $\max(8, 12, 18, 25) = 25$
- **A3:** $\max(4, 15, 10, 22) = 22$

Best Decision: A2 (Highest Gain = 25)

(b) Maximin Criterion (Conservative Approach - Wald's Rule)

This rule selects the action with the **highest minimum gain**, assuming the **worst-case scenario** always occurs.

- **A1:** $\min(5, 10, 15, 20) = 5$
- **A2:** $\min(8, 12, 18, 25) = 8$
- **A3:** $\min(4, 15, 10, 22) = 4$

Best Decision: A2 (Highest Minimum Gain = 8)

(c) Minimax Regret Criterion (Savage's Rule)

This rule selects the action with the **lowest maximum regret**. The **regret table** is calculated by taking the difference between the highest value in each state and the value for each action:

Step 1: Identify the Best Payoff in Each State

- **S1:** $\max(5, 8, 4) = 8$
- **S2:** $\max(10, 12, 15) = 15$
- **S3:** $\max(15, 18, 10) = 18$
- **S4:** $\max(20, 25, 22) = 25$

Step 2: Compute the Regret Table

Each regret value is calculated as:

$$\text{Regret} = \text{Best State Value} - \text{Actual Payoff}$$

States (S)	S1	S2	S3	S4	Max Regret
A1	$8-5 = 3$	$15-10 = 5$	$18-15 = 3$	$25-20 = 5$	5
A2	$8-8 = 0$	$15-12 = 3$	$18-18 = 0$	$25-25 = 0$	3 ✓ (Best Choice)
A3	$8-4 = 4$	$15-15 = 0$	$18-10 = 8$	$25-22 = 3$	8

Best Decision: A2 (Lowest Maximum Regret = 3)

(d) Principle of Insufficient Reason (Laplace Criterion)

This rule assumes that all **states of nature are equally probable**. The decision with the **highest average payoff** is selected.

Laplace Value:

- **A1:** $(5 + 10 + 15 + 20) / 4 = 12.5$
- **A2:** $(8 + 12 + 18 + 25) / 4 = 15.75$
- **A3:** $(4 + 15 + 10 + 22) / 4 = 12.75$

Best Decision: A2 (Highest Average Payoff = 15.75)

(e) Hurwicz Criterion ($\alpha = 0.4$, Mixed Optimism-Pessimism)

This rule **weights the best and worst payoffs** for each action using the optimism index $\alpha = 0.4$.

- **A1:** $(0.4 \times 20) + (0.6 \times 5) = 8 + 3 = 11$
- **A2:** $(0.4 \times 25) + (0.6 \times 8) = 10 + 4.8 = 14.8$ ✓
- **A3:** $(0.4 \times 22) + (0.6 \times 4) = 8.8 + 2.4 = 11.2$

Best Decision: A2 (Highest Weighted Score = 14.8)

(f) Expected Monetary Value (EMV)

The **probabilities** of each state are given as:

- **P(S1) = 0.6, P(S2) = 0.1, P(S3) = 0.2, P(S4) = 0.1**

EMV

- **EMV(A1) = $(0.6 \times 5) + (0.1 \times 10) + (0.2 \times 15) + (0.1 \times 20) = 3 + 1 + 3 + 2 = 9$**
- **EMV(A2) = $(0.6 \times 8) + (0.1 \times 12) + (0.2 \times 18) + (0.1 \times 25) = 4.8 + 1.2 + 3.6 + 2.5 = 12.1$**
- **EMV(A3) = $(0.6 \times 4) + (0.1 \times 15) + (0.2 \times 10) + (0.1 \times 22) = 2.4 + 1.5 + 2 + 2.2 = 8.1$**

Best Decision: A2 (Highest EMV = 12.1)

(g) Expected Opportunity Loss (EOL)

EOL is computed using the **regret values** and probabilities:

- **EOL(A1) = $(0.6 \times 3) + (0.1 \times 5) + (0.2 \times 3) + (0.1 \times 5) = 1.8 + 0.5 + 0.6 + 0.5 = 3.4$**
- **EOL(A2) = $(0.6 \times 0) + (0.1 \times 3) + (0.2 \times 0) + (0.1 \times 0) = 0 + 0.3 + 0 + 0 = 0.3$**
- **EOL(A3) = $(0.6 \times 4) + (0.1 \times 0) + (0.2 \times 8) + (0.1 \times 3) = 2.4 + 0 + 1.6 + 0.3 = 4.3$**

Best Decision: A2 (Lowest EOL = 0.3)

(k) Expected Value of Perfect Information (EVPI)

$$EVPI = \max_{\{A_i\}}(EMV) - \min_{\{A_i\}}(EOL)$$

$$EVPI=12.1-0.3=11.8$$

The company would be willing to pay up to 11.8 for perfect market information.

Conclusion: Action A2 is the best decision under most criteria.

Exercise 5: Decision-Making Criteria Under Uncertainty and Risk

1. Decision-Making Under Uncertainty

When making decisions in **uncertain conditions**, several criteria can guide the selection of the best alternative. The following methods are commonly used:

1) Optimistic Criterion (Maximax Rule)

- Identify the **highest possible** payoff for each decision alternative.
- Choose the alternative with the **maximum** value (ideal for profit maximization).

2) Pessimistic Criterion (Maximin Rule - Wald's Criterion)

- Identify the **lowest** payoff for each alternative (worst-case scenario).
- Select the alternative with the **highest minimum** payoff, ensuring a conservative approach.

3) Equal Probability Criterion (Laplace Rule)

- Assume all **states of nature** have **equal probabilities** by using the formula:

$$\text{Probability} = \frac{1}{\text{number of states of nature}}$$

- Calculate the **expected value** for each alternative by averaging the payoffs across all states.
- Select the alternative with the **highest average payoff** (best expected outcome).

4) Hurwicz Criterion (Weighted Optimism-Pessimism Approach)

- Decide an **optimism coefficient** (α), with pessimism being ($1 - \alpha$).
- For each decision alternative:
 - Multiply the **best** payoff by α (optimism).
 - Multiply the **worst** payoff by ($1 - \alpha$) (pessimism).
 - Compute the **weighted average**:
- Choose the alternative with the **highest weighted value**.

$$D = (\alpha \times \text{Maximum Payoff}) + ((1 - \alpha) \times \text{Minimum Payoff})$$

(v) **Minimax Regret Criterion (Savage's Rule)**

- Construct an **opportunity-loss matrix** by:
 - Identifying the **best** payoff in each state.
 - Subtracting each alternative's payoff from the **best value** in that state.
- Identify the **worst regret (maximum value)** for each decision alternative.
- Select the alternative with the **lowest maximum regret**, minimizing potential losses.

2. Decision-Making Under Risk

When probabilities of different outcomes are known, the following methods are used to determine the best decision:

(i) **Expected Monetary Value (EMV)**

- Construct a **payoff matrix** listing all alternatives and their potential outcomes.
- Calculate the **EMV** for each alternative using:

$$EMV = \sum_{i=1}^m (r_{ij} \times p_i)$$

Where:

- r_{ij} = payoff for alternative j under state i .
- p_i = probability of state i .
- Choose the alternative with the **highest EMV**.

(ii) **Expected Opportunity Loss (EOL)**

- Construct a **regret table** by computing the difference between the highest payoff in each state and other alternatives.
- Compute the **EOL** for each alternative
- Select the alternative with the **lowest EOL**.

(iii) **Expected Value of Perfect Information (EVPI)**

EVPI represents the **maximum amount** a decision-maker optimal strategies when dealing with uncertainty and risk, ensuring **data-driven** and **rational choices**.

$EVPI = (\text{Expected Profit with Perfect Information}) - (\text{Expected Profit Without Perfect Information})$

$$EVPI = \sum_{i=1}^m \max(r_{ij})p_i - EMV$$

N.B. The purpose of these exercises is to help understanding **decision-making under uncertainty and risk**, using **decision trees** to evaluate and compare **alternative choices**.

Practical Work number 3

Applying the Analytic Hierarchy Process (AHP) to Decision-Making

Example: Selecting the Best Car

Imagine you need to choose between three car models: **Car A, Car B, and Car C**. The decision is based on four key criteria: **Comfort, Price, Fuel Efficiency (MPG), and Style**. The **Analytic Hierarchy Process (AHP)** can help determine which car is the best option by ranking them based on **pairwise comparisons**.

Step 1: Compute the Priority Vectors

Each car is compared against the others **for each criterion** using pairwise comparison matrices.

- **Determine the priority vector for each criterion:**
 - Compute the priority values for Comfort.
 - Compute the priority values for Price, MPG, and Style.
- **Determine the priority vector for the criteria themselves:**
 - Assign weights to each criterion to reflect their relative importance in the decision.

Step 2: Construct the Decision Matrix

The **decision matrix** is built by organizing the priority values of each alternative (Car A, Car B, Car C) under each criterion.

Step 3: Compute the Overall Priority Score

To evaluate the final ranking:

- Multiply the **priority vector of each criterion** by the **priority values of the alternatives** (sum product method).
- This results in a final **weighted score** for each car.

Step 4: Rank the Alternatives

The cars are ranked based on their computed priority scores, helping to **identify the best option**.

Conclusion: AHP provides a **structured decision-making approach**, ensuring that **subjective preferences are quantified** systematically. This method is widely used in complex decision scenarios, such as selecting suppliers, evaluating projects, or making strategic business choices.

Applying the Analytic Hierarchy Process (AHP) to Car Selection – Numerical Example

Let's assume you are choosing between three cars: **Car A, Car B, and Car C**, based on four key criteria: **Comfort, Price, Fuel Efficiency (MPG), and Style**. You will use **AHP** to rank the alternatives.

Comfort	Car A	Car B	Car C
Car A	1	2	8
Car B	1/2	1	6
Car C	1/8	1/6	1

Price	Car A	Car B	Car C
Car A	1	1/3	1/4
Car B	3	1	1/2
Car C	4	2	1

MPG	Car A	Car B	Car C
Car A	1	1/4	1/6
Car B	4	1	1/3
Car C	6	3	1

Style	Car A	Car B	Car C
Car A	1	1/3	4
Car B	3	1	7
Car C	1/4	1/7	1

Criterion	Price	MPG	Comfort	Style
Price	1	3	2	2
MPG	1/3	1	1/4	1/4
Comfort	1/2	4	1	1/2
Style	1/2	4	2	1

References

- Simon, H. A. (1960). *The New Science of Management Decision*. Harper & Row.
- Mintzberg, H., Raisinghani, D., & Theoret, A. (1976). *The structure of "unstructured" decision processes*. *Administrative Science Quarterly*, 21(2), 246-275.
- Gorry, G. A., & Scott Morton, M. S. (1971). *A framework for management information systems*. *Sloan Management Review*, 13(1), 55-70.
- Barberis, N. C. (2013). *Thirty Years of Prospect Theory in Economics: A Review and Assessment*. *Journal of Economic Perspectives*, 27(1), 173-196.
- Myerson, R. B. (2013). *Game Theory: Analysis of Conflict*. Harvard University Press.
- Kahneman, D. (2011). *Thinking, Fast and Slow*. Farrar, Straus and Giroux.
- Evans, J. St. B. T. (2018). *Thinking and Reasoning: A Very Short Introduction*. Oxford University Press.
- Greco, S., Ehrgott, M., & Figueira, J. R. (2016). *Multiple Criteria Decision Analysis*. Springer.
- Turban, E., Pollard, C., & Wood, G. (2021). *Information Technology for Management: On-Demand Strategies for Performance, Growth, and Sustainability*. Wiley.
- Greco, S., Ehrgott, M., & Figueira, J. R. (2016). *Multiple Criteria Decision Analysis*. Springer
- Aruldoss, M., Lakshmi, T. M., & Venkatesan, V. P. (2013). A survey on multi criteria decision making methods and its applications. *American Journal of Information Systems*, 1(1), 31-43.
- Hwang, C. L., & Yoon, K. (1981). *Multiple attribute decision making methods and applications*. Berlin: Springer-Verlag.
- Brans, J. P., & De Smet, Y. (2016). PROMETHEE methods. *Multiple criteria decision analysis: state of the art surveys*, 187-219.
- Mahmoudi, A., Deng, X., Javed, S. A., & Yuan, J. (2021). Large-scale multiple criteria decision-making with missing values: project selection through TOPSIS-OPA. *Journal of Ambient Intelligence and Humanized Computing*, 12(10), 9341–9362.
- Géron, A. (2022). *Hands-on machine learning with Scikit-Learn, Keras, and TensorFlow: Concepts, tools, and techniques to build intelligent systems*. " O'Reilly Media, Inc."

Loh, W. Y. (2011). Classification and regression trees. *Wiley interdisciplinary reviews: data mining and knowledge discovery*, 1(1), 14-23.

Bickel, J. E. (2008). The relationship between perfect and imperfect information in a two-action risk-sensitive problem. *Decision Analysis*, 5(3), 116-128.

(Araújo, 2021) Araújo, S. O., Peres, R. S., Barata, J., Lidon, F., & Ramalho, J. C. (2021). Characterising the agriculture 4.0 landscape—emerging trends, challenges and opportunities. *Agronomy*, 11(4), 667.

Phillips-Wren, G.: Intelligent decision support systems. In: Doumpos, M. and Grigoroudis, E. (eds.) *Multicriteria Decision Aid and Artificial Intelligence*. pp. 25–41. Wiley (2013).

Cerejo, J., & Carvalhais, M. (2020). The lens of Anticipatory Design under AI-driven Services. In *International Conference on Digital Design & Communication (DIGICOM 2020) ATAS: PT & ES, Barcelos, Portugal* (pp. 345-357).

Mintzberg, H., Raisinghani, D., & Theoret, A. (1976). The structure of "unstructured" decision processes. *Administrative science quarterly*, 246-275.

Simon, H. A. (1960). *The New Science of Management Decision*. Harper & Row.

Anthony, R. N. (1965). *Planning and Control Systems: A Framework for Analysis*. Harvard Business School.

Web sites:

[1] <https://www.futurecioclub.com/blog/organizational-planning-and-execution-in-three-levels-strategic-tactical-operational>

[2] <https://eternalsunshineoftheismind.wordpress.com/2013/02/24/different-types-of-information-systems-and-why-they-fail/>