

Control of reaction and thermo-acoustic instabilities of a combustion chamber

A. Debbah, A. Kerboua, R. Klaiia

Abstract— This paper deals with the improvement of the control and the vanishing of the thermo-acoustic instability in gas-turbine engine combustors via a design of an active controller. The proposed design used a multi-objective genetic algorithm (MOGA) and multi-objective particle swarm optimization (MOPSO) to stabilize the unstable system from limit cycle with uncertain model. These parameters were chosen to minimize four objective functions: the mass fraction of the fuel produced, temperature control, flame fluctuation and the variation in the flow mass rate of input fuel. In addition, an improvement in suppression of the instability was achieved by coupling the combustor and the unsteady heat perturbations and thermo-acoustic models in simulation.

Keywords— Combustion instabilities; Thermo-acoustic instability; Active control; Multi-objective optimization.

I. INTRODUCTION

Regardless of their configuration or application, all turbine engines will contain four main stages. These are the compressor, combustor, the turbine which generates mechanical work, and the exhaust generating thrust if required [1]. A large majority of research has concentrated on improving the efficiency of the compressor and exhaust stages. In recent years, recognition of global warming caused by greenhouse gases has pushed governmental organizations to restrict the allowed Carbon Monoxide and Carbon Dioxide ground emission levels in gas turbine engines, manufacturers have been able to comply with these requirements by constantly increasing the Operating Pressure Ratio (O.P.R.) of the engine. In addition to the major modifications to the compressor, this has allowed for a reduction in CO and CO₂ emissions by reducing the amount of fuel burnt for a given thrust output in counterpart flame burning temperatures have increased because of the higher pressures at the combustor inlet [1]. It is expected that the O.P.R. will rise. The issue with increasing O.P.R., is that the induced higher flame temperature is responsible for an increase in Nitrogen Oxides; these refer to nitric oxide (NO) and nitrogen dioxide (NO₂), both formed during combustion (NO_x) emissions. Combustor instability suppression presents a challenging problem for controls design due primarily to the large dead time phase

A. Debbah. is with Petrochemical and process engineering department, University of 20 Aout 1955-Skikda, Algeria. L2RCS Laboratory, Department of Electronics, University of Badji Mokhtar, Annaba, Algeria. (Phone .: + 213 0540654973 a.debbah@univ-skikda.dz.)

A. Kerboua. is with Petrochemical and process engineering department, University of 20 Aout 1955-Skikda, Algeria. (kerboua_adlen@yahoo.fr)

R. Klaiia. is with electromechanical department, University of 20 Aout 1955-Skikda, Algeria. (ridaklaia@hotmail.com)

delay (of many hundreds of degrees or more) and noise in the combustion process. Besides large phase delay and noise, there are other characteristics of combustor instabilities, such as amplitude modulations and net random phase walks, which could play an important role in the control design [1]. A number of research efforts have attempted suppression of the thermo-acoustic instability through active control. The goal of these active control efforts was to reduce the energy concentrated at the instability frequency and to reduce the overall amplitude of the combustor pressure oscillations. Some active control concepts involved speaker actuation and others involved fuel modulation [1]. The last trends in control and stabilizing dynamic combustion systems demand the research, development and applications of advance control approaches based on the principles of optimality, robustness and intelligence. Proportional integral derivative (PID) and its derived configurations such as PI [2] or PD [3] control is the conventional most commonly used controller in the engine industry. The controller's popularity can be attributed to its functional simplicity, effectiveness over a wide range of operation conditions, and easier implementation using current micro-controller technology. To compensate time-delay and gain scheduling [4], modern model-based control approaches are developed using more dynamically coupled physical models. Linear-Quadratic-Gaussian (LQG) optimal control [5,6] is a power control means. However, it lacks the accurate linear state-space models of a practical engine combustor. This situation will be changed in the future, since new generations of state-space identification methods will be commercially available. More advanced methods such as fuzzy logic [7], neural network [8,9], robust (H_∞ , H_2) [10–11], optimal (LQG), adaptive [12], predictive (hybrid) [13,14] control could also be applied to stabilize an unstable combustor. In general, there is a rich literature on robust sliding mode control [15,16,17]. The proposed robust design could be applied to design feedback controllers which are effective over a wide range of operating conditions and have great potential to apply in practice. There are some interests in the proposed optimized controller which can achieve fast response, good transient performance and robustness with respect to variations in combustor's parameters and modeled disturbances [18,19]. The paper is organized as follows; the first section is a review on control of combustion in gas turbine process and the last research and future works. The second section presented the mathematical model. In section three, we explained optimized robust control, and section four discusses the simulation and the result that we got. Finally, the last section was dedicated to conclusion.

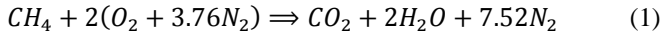
II. MATHEMATICAL MODEL

A simple combustor model is needed to incorporate into the overall fuel gas turbine system. The model must be able to simulate the temperature rise to the working fluid when it is

combusted. The model assumes the fuel input to the combustor to be methane. However, it is possible to burn the excess hydrogen straight.

A. The combustor model

The combustor model used was developed by Fannin [20]. It is called an “unsteady well stirred reactor” (WSR) model. It assumes methane (CH₄) as the fuel and air as the oxidizer. The balanced combustion reaction is as follows:



A non-linear state space model can be constructed using conservation of species and energy in the control volume of the combustor. The conservation of species equations is based on the three species present inside the combustor. They are the fuel (methane), the oxidizer (air), and the products of combustion. The amount of species created in the control volume will be driven by the Arrhenius rate term for chemical kinetics of combustion. The conservation of species equations for the fuel and oxidizer are as follows:

$$\rho V \frac{dY_{fuel}(t)}{dt} = \dot{m}_{in} Y_{fuel,in}(t) - \dot{m}_{out} Y_{fuel}(t) + W_{fuel} \dot{\omega}_{fuel}(t) V \quad (2)$$

$$\rho V \frac{dY_{oxid}(t)}{dt} = \dot{m}_{in} Y_{oxid,in}(t) - \dot{m}_{out} Y_{oxid}(t) + MW_{fuel} \frac{Y_{oxid,in}(t)}{Y_{fuel,in}(t)} \dot{\omega}_{fuel}(t) V \quad (3)$$

$$\dot{\omega}_{fuel} = (-24.100 \frac{kmol}{kg \cdot s}) \rho Y_{fuel}^{0.3} (0.233 Y_{oxid})^{1.3} e^{\frac{15.098}{RuT(t)}} \quad (4)$$

ρ : Density inside combustor.

V : Volume of combustor

\dot{m}_{in} : Total mass flow rate into combustor.

\dot{m}_{out} : total mass flow rate out of combustor.

$Y_{fuel,in}(t)$, $Y_{oxid,in}$: mass fraction of fuel & oxidizer into combustor respectively.

MW_{fuel} : molecular weight of fuel (methane).

$\dot{\omega}_{fuel}$: Arrhenius rate term for consumption of species due to combustion.

Due to the definition of the mass fraction, the equation for the mass fraction of the products is simply

$$Y_{prod} = 1 - Y_{fuel} - Y_{oxid} \quad (5)$$

As stated above, the left-hand side of equation (6) represents the change in internal energy of the species inside the combustion chamber. In order to make this term and expression of temperature only (and not Y_{fuel} , and Y_{oxid}) it is assumed that the majority of the species inside the combustor are products (CO₂, H₂O, and N₂), such that:

$$\rho V \frac{d}{dt} e(t)_{prod} \dot{m}_{in} (Y_{oxid,in} h_{oxid,in} + Y_{fuel,in} h_{fuel,in}) - \dot{m}_{out} \sum_{i=1}^3 Y_i(t) h_{i,out}(t) \quad (6)$$

B. Thermo-acoustic model

The equivalence ratio is defined as the ratio of mass of fuel to mass of air, divided by the ratio of mass of fuel to mass of air in stoichiometric conditions. This can be written as:

$$\phi_r = \frac{m_{fuel}}{m_{air}} \cdot \frac{m_{air,stoc}}{m_{fuel,stoc}} \quad (7)$$

In experimental rigs, the change in equivalence ratio is usually applied by modifying the quantity of fuel being injected m'_{fuel} , and the dispersion of imposed equivalence fluctuation and induced reduction in control authority must be taken into account. In our simulations however, it is easier to implement a change of mass of air m'_{air} and we assume a perfect actuation mechanism. According to eq. (7), both methods are analogous and therefore the simulations will only include velocity perturbations to the flame. The instabilities arise from the interactions between the combustor's oscillatory flows and heat release and the flow processes manifesting as large amplitude, organized oscillations of the combustor's flow field (comprising velocity, pressure temperature and the reactants) which generate disturbance that adds energy to the acoustic field when in phase with the pressure oscillations (Fig.1). A reduced-order behavioral model is used to represent the combustor instability. This model consists of the flame dynamics, acoustics, and saturation nonlinearity [21].

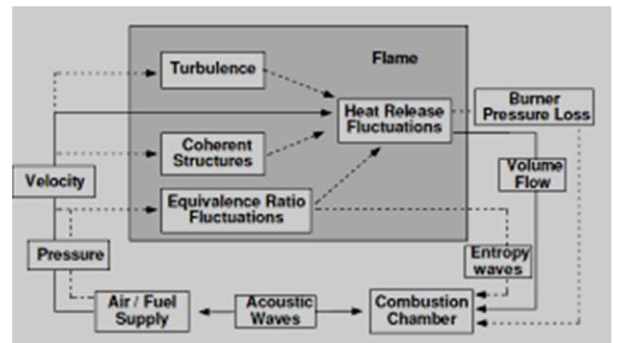


Figure. 1 Interactions between flow, acoustics and heat release in a Combustion system [21].

With the addition of noise, this instability is self-excited, that is, it requires no input via fuel injection modulation for sustained oscillations to take place. We chose the running parameters shown in Fig.2 to induce limit cycle saturation within the combustor.

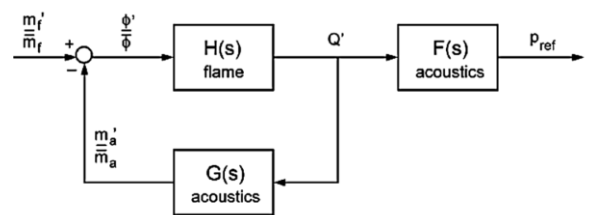


Figure. 2 Block diagram representation of a simple ducted flame combustor

We can write the block diagram representation of our combustor including only flame and acoustics modeling as shown in Fig.2, where \dot{Q} represents the fluctuating flame heat

release ratio, $\frac{\phi}{\bar{\phi}}$ is related to the air-fuel equivalence ratio, and $\frac{\dot{m}_a}{\bar{m}_a}$ is fractional air-mass flow fluctuation at the combustion zone. The open loop transfer function (OLTF) needed for control purposes is then from changes in fuel mass flow rate fluctuations \dot{m}_f , to the sensor of the acoustic pressure measured in the duct P_{ref} . The response of the pressure measurement, P_{ref} , to the heat release fluctuation, \dot{Q} , is modeled by an acoustic transfer function $F(s)$, and the response of the fractional air mass flow fluctuation at the combustion zone, $\frac{\dot{m}_a(s)}{\bar{m}_a}$, to the heat release fluctuation, \dot{Q} , is modeled by $G(s)$. Assuming a compact flame, the heat release fluctuation, \dot{Q} , is modeled by the relationship $\dot{Q}(s) = H(s) \frac{\phi(s)}{\bar{\phi}}$ where $H(s)$ is the flame model. The modified white noise is generated such that it mostly contains energy within the frequency range of interest, selected here as 30 Hz to 3000 Hz. The transfer function is then obtained by dividing the spectral transform of the output, $P_{ref}(s)$, by the spectral transform of the input $r(s) = \frac{\dot{m}_f(s)}{\bar{m}_f}$. For non-linear systems however, the system responds

at frequencies not just limited to the input frequency, and the gain and phase change applied by the system can depend on the frequency and amplitude of the input. Weakly non-linear systems, such as those modeled with flame describing function or the G Equation for example, also have an input dependent gain and phase shift, but their main output frequency will be the same as the input frequency (to first order, their response is essentially linear).

A set of finite-dimensional, low order, unstable systems is considered:

$$\frac{P_{ref}(s)}{r(s)} = \frac{(s^2 + 2c_z\omega_{z1}s + \omega_{z1}^2)(s^2 + 2c_z\omega_{z2}s + \omega_{z2}^2)}{(s^2 + 2c_{p1}\omega_{p1}s + \omega_{p1}^2)(s^2 + 2c_{p2}\omega_{p2}s + \omega_{p2}^2)} \quad (9)$$

$$\frac{V_c(s)}{r(s)} = \frac{1}{s + z_c} \quad (10)$$

With $\omega_{z1} = 150 \text{ rad/s}$, $\omega_{z2} = 250 \text{ rad/s}$, $\omega_{p1} = 200 \text{ rad/s}$, $\omega_{p2} = 500 \text{ rad/s}$, $c_{p1} = 0.02$, $c_{p2} = -0.02$, $z_c = 100 \text{ rad/s}$, $c_z = [-0.5, 0.25]$.

III. ROBUST CONTROL DESIGN

Indeed, the bifurcation behavior of the combustor [23], and the non-linear behavior of the thermo-acoustic model whilst it is in limit cycle hinders the measurement of the underlying linearly unstable system which we wish to control [1]. Because of this, model obtained from limit cycle are noisy, and do not yield the gain and phase of the thermo-acoustic system. We have shown, however, that the phase information thus obtained can be accurately corrected to provide information for the linearly unstable system. The gain of the thermo-acoustic system remains unknown, and therefore the design of a plant based controller must allow for a sufficiently large phase and gain margin [22]. An alternative to plant based control lies with robust controllers were shown to perform well on systems with a moving acoustic discontinuity. They were able to stabilize the unstable system from limit cycle with uncertain model. Let us consider the

combustor and the thermo-acoustic model (2,3,6,9,10) as a MIMO norm bounded form [24,29]:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + B(u(t) + \Delta_x(t)) \\ y(t) &= Cx(t) \end{aligned} \quad (11)$$

Where A and B are respectively the state and control matrix of the system, the state variables $x = (y_{fuel}, y_{oxid}, T, P_{ref}, V)$ belongs to R^5 and ΔA is the uncertainty in dynamic matrix. $x = (T, P_{ref})$ belongs to R^2 is a smooth measurable output vector. $u = (\bar{m}_f, m'_f)$ is the control vector.

A. Reformulating the problem to Equivalent sliding mode Control

Consider the following linear continuous sliding function [24-26]:

$$\sigma_x(t) = Sx(t) = B^T Px(t) \quad (12)$$

Where, $S \in R^{5 \times 5}$, in the design of this control the SB is non-singular. From (6) and (7), the equivalent control law may be obtained as

$$u_{eq}(t) = -(SB)^{-1} SAx(t) \quad (13)$$

Where $S = B^T P$, and SB is non-singular. It should be remarked that the obtained control law contain some uncertain terms, which can be deduced from non-linear system, this can lead to a practical controller. The non-linear part of controller will be taken as [25]

$$u_{nl}(t) = -(SB)^{-1} (|SB|\delta_f + \varepsilon_0) \text{sign}(\sigma_x) \quad (14)$$

Where $|\Delta_x| \leq \delta_f$ and ε_0 is a positive number. Select the Lyapunov function as given by [26]:

$$V_x = \frac{1}{2} \sigma_x^2 \quad \dot{V}_x = \sigma_x \dot{\sigma}_x \quad (15)$$

From equations (13), (14) and (15) the following expressions are obtained:

$$\dot{\sigma}_x(t) = -(|SB|\delta_f + \varepsilon_0) \text{sign}(\sigma_x) + SB\Delta_x \quad (16)$$

$$\dot{V}_x(t) = -(|SB|\delta_f + \varepsilon_0) |\sigma_x| + \sigma_x SB\Delta_x \leq -\varepsilon_0 |\sigma_x| \quad (17)$$

As proved in [25,26,30], is asymptotically stable if $-\varepsilon_0 |\sigma_x|$ is asymptotically stable for $\varepsilon_0 \geq 0$, with ΔA and Δ_x are bounded.

B. Auxiliary feedback and stability analysis

To solve this problem, the sliding mode controller will be designed with a feedback control as follows:

$$u(t) = -Kx(t) + v(t) = u_{eq}(t) + u_{nl}(t) \quad (18)$$

K is chosen to get a $\tilde{A} = A + \Delta A - BK = \bar{A} + \Delta A$ stable in closed loop. Selecting the Lyapunov function as:

$$\begin{aligned} \dot{V} &= 2x^T P \dot{x} = 2x^T (\tilde{A}x(t) + B(v + \Delta_x)) \\ &= 2x^T P \bar{A}x(t) + 2x^T P \Delta A x(t) + 2x^T PB(v + \Delta_x) \quad (19) \\ &= 2x^T P(\bar{A} + \Delta A)x(t) + 2x^T PB(v + \Delta_x) \end{aligned}$$

For $t \geq t_0$, the sliding variable $\sigma_x(t) = B^T Px(t) = 0$ which implies $2x^T PB(v + \Delta_x) = 0$.

Theorem 1: The uncertain sliding dynamics in (19) is asymptotically stable in closed loop with a state feedback, for Lyapunov function candidate $V = x^T Px$, if there exist a symmetric matrix $P > 0$, satisfying the following LMI:

$$\begin{aligned} & \max \alpha \\ & \text{subjected to } (A + \Delta A - BK)Q + Q(A + \Delta A - BK)^T \\ & + 2\alpha Q < 0 \quad (20) \\ & Q > 0 \\ & \alpha > 0 \end{aligned}$$

With $L = KQ$ and $Q = P^{-1}$. The closed loop system matrix has its eigen values in the strict left hand side of the line α , in complex s-plan.

C. Robust design

The equation (20) is non-linear matrix inequality, difficult to solve being non convex, can be solved by increasing α in order to shift the eigen value of $A + \Delta A - BK$ progressively toward stable plane. However, in order to make the controller more robust in face of the model uncertainties, and non-linearity, we will propose robust design [26-30].

The uncertain matrix $\Delta A = M\Delta(t)N$, where M and N are known, and $\Delta(t)$ is an unknown matrix satisfying $\Delta(t)^T \Delta(t) \leq I$ [24-26]. Note that this congruence transformation does not change the definiteness of $\Delta(t)$.

Theorem 2: The uncertain sliding dynamic in (20) can be robustly stabilized if there exists $Q^T > 0$, $K > 0$ and $\alpha > 0$ satisfying the following LMI:

$$\begin{bmatrix} AQ + QA^T - BKQ - Q^T K^T B^T & NQ \\ + 2\alpha Q + \varepsilon_1 MM^T & \\ Q^T N^T & -\varepsilon_1 I \end{bmatrix} < 0 \quad (21)$$

Proof: by replacing ΔA by $M\Delta(t)N$ in (20), it yields

$$\begin{aligned} & AQ + QA - BKQ - QBK + 2\alpha Q + M\Delta NQ \\ & + Q^T N^T \Delta M^T < 0 \quad (22) \end{aligned}$$

With the assumption $\Delta(t)^T \Delta(t) \leq I \rightarrow \|\Delta(t)\| < I$, as given in [26-27], it follows that:

$$M\Delta NQ + Q^T N^T \Delta M^T \leq \varepsilon_1^{-1} MM^T + \varepsilon_1 NQQ^T N^T \quad (23)$$

With $\varepsilon_1 > 0$, the inequality (17) is satisfied if the following equation is satisfied:

$$\begin{aligned} & AQ + QA - BKQ - QBK + 2\alpha Q + \varepsilon_1^{-1} MM^T \\ & + \varepsilon_1 NQQ^T N^T < 0 \quad (24) \end{aligned}$$

Using Schur complement, we can put (24) in the form (21) as desired. The proposed robust Sliding Mode Controller (SMC) can be constructed similar to the previous algorithm by replacing (19) by (20)[27].

D. Multi-objective optimization algorithms

A general approach to solve problems with multiple conflicting objectives is to find the set of acceptable solutions that provide tradeoffs among the conflicting objectives. The set of acceptable solutions contains non-dominated solutions, which are solutions that satisfied the condition that no other solutions exist that dominated it. attempts to directly generate multiple tradeoff solutions by considering all objectives simultaneously. It include non-Pareto-based approach, such as VEGA (vector evaluated genetic algorithm) by schaffer, and Pareto-based approach, such as MOGA (multi-objective GA) by Fonseca and Fleming, NSGA-II (non-dominated sorting GA) by Deb et al, NPGA (Niche Pareto GA) by Horn et al, SPEA (strength Pareto EA) by Zitzler and Thiele [31]. A weakness of non-Pareto-based approach is that it cannot generate compromise solutions and it is sensitive to the shape of the Pareto front. For Pareto-based method, it has advantages in that it can solve ill-posed problem, not susceptible to the shape of Pareto front, and it is highly efficient in generating multiple solutions in a single run. In addition, with the adoption of meta-heuristics (Nguyen and Kachitvichyanukul, the efficiency, the search ability, and the flexibility for handling various models have been much improved. Hence, recent research interests focus more on the search of Pareto front for MO problems via meta-heuristics [31]. Nguyen and Kachitvichyanukul developed a Pareto based multi-objective particle swarm optimization (MOPSO) algorithm which takes advantage of PSO to search for multiple solutions in the multi-objective space. An Elite

archive is adopted to store the non-dominated solutions found so far and the guidance references for the swarm movement are selected from these non-dominated solutions to ensure that the swarm tends to move toward the direction of the Pareto front. Several ways to select reference particles to guide the particle movement are proposed as different movement strategies to diversify the movement of particles. MOPSO is tested using standard test functions and such real world problems as engineering design problem and portfolio optimization problem [32]. The experimental results showed robust solutions with high quality Pareto front and MOPSO outperforms NSGA-II. MOPSO was also applied successfully to solve multi-objective job shop scheduling problems as reported in Wisittipanich and Kachitvichyanukul. The idea of movement strategies is also extended as mutation strategies for multiple objective differential evolution (MODE) as reported in Wisittipanich and Kachitvichyanukul [31,33]. The aim Our strategy is to obtained a stable system using an active controller for the thermo-acoustic model so no heat losses and zero change in kinetic or potential energy, then the maximum flame temperature can be achieved by applying a multi-objective genetic algorithm (MOGA) [34,35] or multi-objective particle swarm optimization (MOPSO) [35] to optimized the parameters of the sliding mode control algorithm (SMC). These parameters were chosen to:

1st. Get system stable with maximum power, it means the error of temperature tend to zero at maximum set point, with the least possible amount of fuel.

2nd. The output can be improved by burning the mixture under turbulent conditions (Equivalence Ratio) and increasing the fuel conversion rate and reduce the NOx emission (Output fuel tend to zero).

3rd. Achieve satisfactory actuation, by consuming less power and the actuator's frequency response and its durability must match the frequency of combustion-driven oscillations, limiting the range of applications at very high frequency (control signal variation tends to zero).

$$\begin{aligned} \min f_1(\delta_f, \varepsilon_0) &= \int_0^{t_{op}} \sigma_x (\delta_f, \varepsilon_0)^2 dt \\ f_2(\delta_f, \varepsilon_0) &= \int_0^{t_{op}} y_i (\delta_f, \varepsilon_0)^2 dt, i = fuel, oxid \\ f_3(\delta_f, \varepsilon_0) &= \int_0^{t_{op}} \bar{m}_f (\delta_f, \varepsilon_0)^2 dt \\ f_4(\delta_f, \varepsilon_0) &= \int_0^{t_{op}} m'_f (\delta_f, \varepsilon_0)^2 dt \end{aligned} \quad (26)$$

subjected to $\delta_f > 0, \varepsilon_0 > 0, -(|SB| \delta_f + \varepsilon_0) k \sigma_x^2 + \sigma_x SBA_x < 0$

We selected four objectives function: f1 Output error (temperature, acoustic pressure), f2 fuel conversion rate, f3 actuator energy, and f4 fuel fluctuation (equivalence ratio). The inequality constraint can be leads to feasible optimization

$-(|SB| \delta_f + \varepsilon_0) k \sigma_x^2 + \sigma_x SBA_x < -\varepsilon_0 k \sigma_x^2$ for $\varepsilon_0 > 0$ with $k \sigma_x^2$ is positive definite function [15],[19].

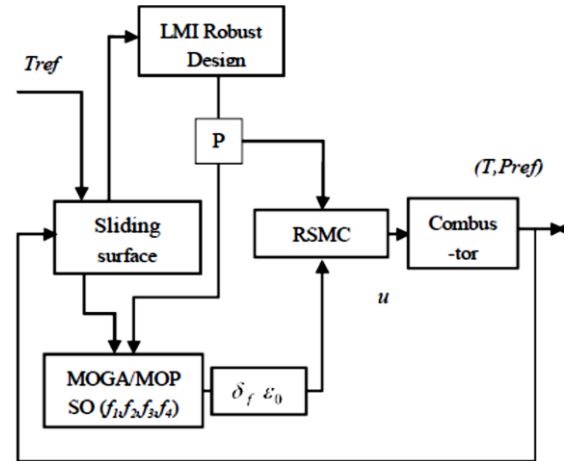


Figure 3. Block diagram of the proposed controller

IV. SIMULATION AND RESULTS

Then, with the flexibility of the GA and PSO algorithms, these numerical bounds can be used directly in an online optimization of decision variables δ_f, ε_0 .

TABLE 1. GA PARAMETERS USED IN SIMULATION

Parameter	Value
Population size	40
Generations	50
Type of selection	Roulette wheel
Type of crossover	Intermediate
Fitness function	Equation (26)
Constraints-handling methods	Penalty function
Type of mutation	Adapt feasible
Crossover Ration	0.8

TABLE 2. PSO PARAMETERS USED IN SIMULATION

Parameter	Value
Maximum iterations	50
Population size	40
Dimension	2
Value of K2	2.0
Maximum weight	0.90
Minimum weight	0.40
Fitness function	Equation (26)

By running the simulation of the thermos-acoustic and combustor models with the data provided [21] and [22], optimization algorithms provide the optimal values of decision variables as (MOGA $\delta_f = 4, \varepsilon_0 = 5.3162$, MOPSO: $\delta_f = 4,9010, \varepsilon_0 = 5.6516$). It should be noted that in this study, we focus on integrating the multi-objective meta-heuristic optimization (MOGA, MOPSO) in order guarantee the time performances and robustness

regarding model uncertainties; we aren't focused on the comparison between the performances of the both optimization strategies. The results found in the optimization are similar for the both strategies. In order to obtain realistic simulation a white noise is applied as an acoustic velocity perturbation at the upstream end of the duct.

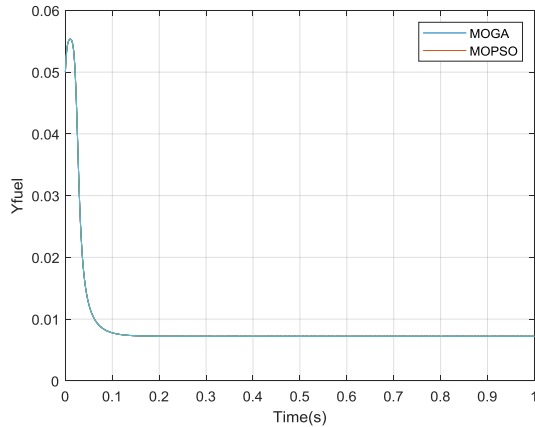


Figure 4. Fuel mass fraction inside combustor

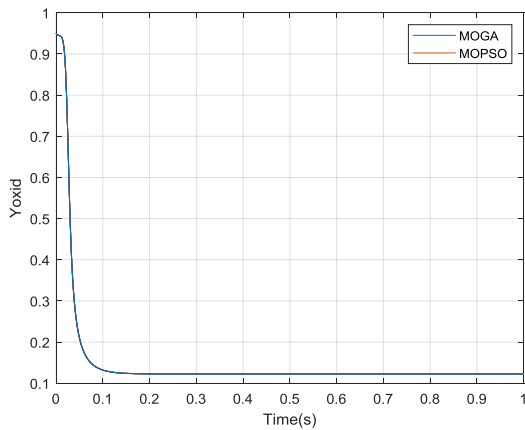


Figure 5. Oxider mass fraction inside combustor

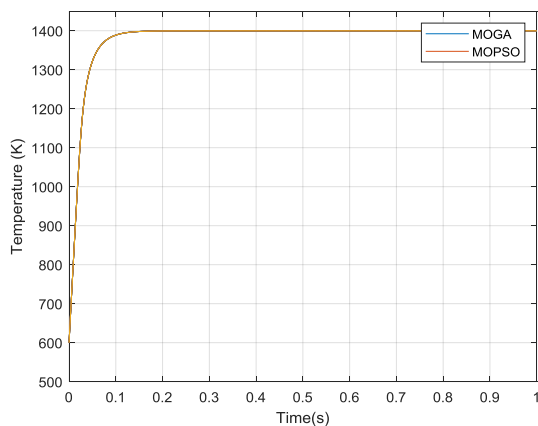


Figure 6. Output temperature of combustor

The modified white noise is generated such that it mostly contains energy within the frequency range of interest,

selected here as 30 Hz to 3000 Hz, and 3 different amplitude are applied to check the reaction of the active control.

The mass fractions of the fuel and the oxidizer fall rapidly and eventually stabilize near zero. This result implies that nearly all of the fuel and the oxidizer are being converted to products, which is consistent with the ratio of the fuel and oxidizer, which was chosen to be close to the stoichiometric value. The value of the product mass fraction inside the combustor would be close to unit. This gives credence to the assumption that the exit fluid consists of mostly products of combustion. As can be observed from Fig.6, the combustor raises the temperature of the working fluid from 600K to approximately 1400 K. These results are consistent with the test data provided in [5].

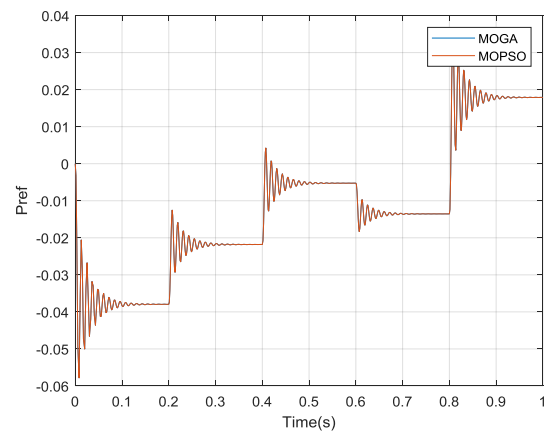


Figure 7. Output flame pressure

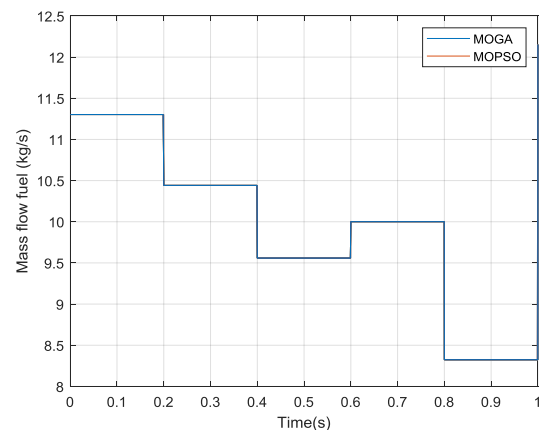


Figure 8. Fuel mass flow

As shown in Fig. 7, the pressure was stabilized due to the active controller. The actuation from the controller modifies the fuel mass flow rate as show in Fig.8 to obtain a stable flame. This result obtained due to the parameter of δ_f , ϵ_0 switching control optimized by MOSMO and MOGA under the four functions objectives that we mentioned before. Plus a stable flame due to the active controller.

V. CONCLUSION

Active control is able to stabilize combustor instabilities by interrupting the coupling between the temperature, mass

fraction and the thermo-acoustic model. Active robust controllers whose parameters optimized, offer an efficient means of maintaining control across a range of operating conditions. Robust sliding mode strategies are an attractive type of active controller as they do not require detailed prior system characterization nor on-line system identification. Instead, they utilize some general properties satisfied by a wide range of uncertain systems, and optimize according to Linear matrix inequality (LMI) based schemes. This approach was applied to a numerical model of an unstable combustion system. The modified pressure measurement resulted in the robust design being effective in cases where it was not when a single pressure measurement was used, demonstrating the success of the approach. The proposed control design method utilizes a meta-heuristic optimization added to LMI optimization algorithm, which incorporates a Pareto front to manage the compromise between objectives systems to stabilize an unstable and uncertain system

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