Indirect Fuzzy Adaptive Control: Hyperstability Approach

Noureddine Golea
E.E. Institute
Oum El-Bouaghi University
04000 Oum El-Bouaghi, Algeria.
n.golea@eudoramail.com

Amar Golea
E.E. Institute
Biskra University
07000 Biskra, Algeria.
agolea@lycos.com

Mohamed Kadjoudj
E.E. Institute
Batna University
05000 Batna, Algeria.
m.kadjoudj@lycos.com

Abstract— **A new fuzzy indirect adaptive controller for continuous-time nonlinear systems, with a poorly understood dynamics, is developed. The proposed adaptive scheme uses a single Takagi-Seguno (TS) fuzzy model with few parameters to learn, which results in low implementation complexity and a fast learning rate. In addition, the use of TS fuzzy model permits the inclusion of a priori knowledge about the plant dynamics in terms of exact mathematical models or qualitative information. Using the hyperstability approach, it is proved that this adaptive controller is globally asymptotically stable, and achieves asymptotic tracking of a stable reference model. The performance of the developed approach is illustrated with simulation results.**

Keywords— **Fuzzy systems, adaptive control, hyperstability, robustness**

I. Introduction

Despite their learning capabilities and practical implementations, the earlier fuzzy adaptive systems suffer from the lack of stability analysis, i.e. the stability of closed loop system is not guaranteed and the learning process do not lead to a well defined dynamic. Recently, and important class of a fuzzy adaptive systems, which use Mamdani or TS fuzzy models, have been developed, and their stability is guaranteed using the Lyapunov theory (see [1]-[6] and references therein).

This investigation develops a new stable indirect fuzzy adaptive controller for nonlinear continuous systems, which requires only a single TS fuzzy model, with few rules, to approximate the nonlinear plant dynamics. This adaptive scheme presents the following advantages: i) the qualitative information about the plant operating points can be used to design the fuzzy model antecedents, ii) if for some operating points, identified linear models of the plant are available, they can be directly incorporated into the fuzzy model rule consequences, and iii) it allows fast control update, which is limit factor for some applications. The stability of the proposed adaptive scheme, in presence of approximation error and external disturbance, is established in the hyperstability framework [7, 8]. The potentials of the Lyapunov approach and the hyperstability approach are theoretically the same. However, there is no general effective way for finding the Lyapunov function. Moreover, as mentioned in [9], in many adaptive situations, the hyperstability approach may be easier to apply than the Lyapunov method. This is because the search for suitable Lyapunov function is replaced by both positivity condition and popov's inequality to be satisfied independently. The simulation results for the inverted pendulum benchmark show that the proposed adaptive scheme maintains a consistent performance under approximation error and external disturbance.

II. Problem statement

Consider for the continuous-time nonlinear system

$$\dot{x}_n = f(x,u) + \eta \tag{1}$$

where $f(.)$ is unknown continuous function, η is unknown bounded external disturbance, $u \in R$ is the input of the system, and $x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T \in R^n$ is the state vector of the system, which is assumed to be available.

The stable, linear time invariant and controllable reference model is defined by the following state equation

$$\dot{x}_m = A_m x_m + b_m r \tag{2}$$

where $x_m = \begin{bmatrix} x_{1m} & x_{2m} & \dots & x_{nm} \end{bmatrix}^T \in R^n$ is the state vector of the reference model, r is a bounded reference input, and A_m, b_m are given by

$$A_m = \begin{bmatrix} 0 & I_{n-1} \\ & -a_m \end{bmatrix}, \; b_m = \begin{bmatrix} 0 & \dots & 0 & b_{nm} \end{bmatrix}^T$$

The control problem can be stated as that of designing the input control u such that the states of the plant (1) follow those of the reference model (2), under the condition that all involved signals in the closed loop remain bounded. Since the nonlinear function is not known, and the input do not appear explicitly in (1), the TS fuzzy model will be used to estimate the unknown function $f(.)$.

III. Fuzzy modeling

The TS fuzzy model is characterized by a set of If-Then fuzzy rules expressed as

R$_i$: If z is Z_i Then $\hat{\dot{x}}_n = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n + b_i u$

where $z = \begin{bmatrix} z_1 & z_2 & \dots & z_p \end{bmatrix}^T$ is the fuzzy model input vector, and the fuzzy sets Z_i operate a fuzzy partition of the fuzzy model input space.

The final output of the fuzzy model is inferred as follows

$$\hat{\dot{x}}_n = \frac{\sum_{i=1}^{M} \mu_i(z) \left(\sum_{j=1}^{n} a_{ij}x_j + b_i u \right)}{\sum_{i=1}^{M} \mu_i(z)} \tag{3}$$

where $\mu_i(z)$ is the grade of membership of z in Z_i (i.e., the firing strength of the rule i). In this paper, it assumed that there exist always at least one active rule, i.e. $\sum_{i=1}^{M}\mu_i(z) > 0$.

The fuzzy model output (3) can also be written in following matrix form

$$\dot{\hat{x}}_n = \phi \sum_{j=1}^{n} \theta_{a_j} x_j + \phi\theta_b u \tag{4}$$

where

$$\theta_{a_j} = \left[\begin{array}{cccc} a_{1j} & a_{2j} & \dots & a_{Mj} \end{array}\right]^T$$
$$\theta_b = \left[\begin{array}{cccc} b_1 & b_2 & \dots & b_M \end{array}\right]^T$$

and ϕ is the vector of the normalized firing strengths of the rules, given by

$$\phi = \frac{1}{\sum_{i=1}^{M}\mu_i}\left[\begin{array}{cccc} \mu_1 & \mu_2 & \dots & \mu_M \end{array}\right] \tag{5}$$

Following the universal approximation results [1-2, 12], the fuzzy model (4) is rich and able to approximate the nonlinear function $f(.)$ on a compact operating space to any degree of accuracy. Next, we define the optimal fuzzy model parameters $\theta_{a_j}^*$ and θ_b^* be such that

$$\dot{x}_n = \phi \sum_{j=1}^{n} \theta_{a_j}^* x_j + \phi\theta_b^* u + \eta + \omega \tag{6}$$

where ω is the minimum approximation error achieved by the fuzzy model with the optimal parameters.

Since the optimal fuzzy model parameters are not known, we make use of their estimates, then (5) can be rewritten as

$$\dot{x}_n = \phi \sum_{j=1}^{n} \theta_{a_j} x_j + \phi\theta_b u + \phi \sum_{j=1}^{n} \widetilde{\theta}_{a_j} x_j + \phi\widetilde{\theta}_b u + \eta + \omega \tag{7}$$

where $\widetilde{\theta}_{a_j} = \theta_{a_j}^* - \theta_{a_j}$ and $\widetilde{\theta}_b = \theta_b^* - \theta_b$ are the parameter estimation errors.

A. *Closed loop dynamic*

The Tracking error between the nonlinear system (1), represented by the fuzzy model (7), and the reference model (2) is given by

$$\begin{aligned} \dot{e} \;=\; & A_m e - b_c \left[\phi \sum_{j=1}^{n} \theta_{a_j} x_j + \phi\theta_b u + a_m x - b_{nm} r \right. \\ & \left. + \phi \sum_{j=1}^{n} \widetilde{\theta}_{a_j} x_j + \phi\widetilde{\theta}_b u + u_s + \eta + \omega \right] \end{aligned} \tag{8}$$

where $b_c = \left[\begin{array}{cccc} 0 & \dots & 0 & 1 \end{array}\right]^T$.

The control input is chosen as (see fig. 1)

$$u = \frac{1}{\phi\theta_b}\left[b_{nm} r - \phi \sum_{j=1}^{n} \theta_{a_j} x_j - a_m x + u_s \right] \tag{9}$$

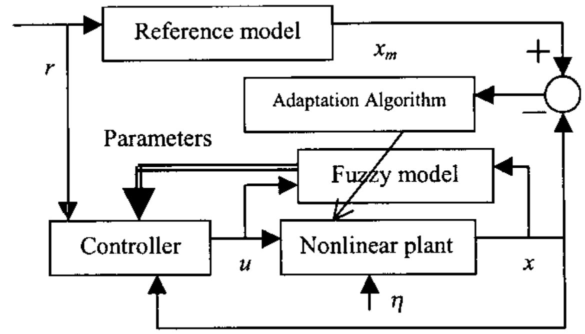

Fig. 1. Fuzzy adaptive control scheme.

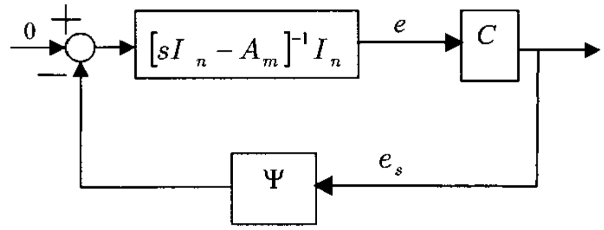

Fig. 2. Feedback structure.

where u_s is an additional control input used to attenuate the external disturbance and the approximation error effects.

The substitution of (9) in (8) yields the following closed loop dynamic

$$\dot{e} = A_m e - b_c \left[\phi \sum_{j=1}^{n} \widetilde{\theta}_{a_j} x_j + \phi\widetilde{\theta}_b u + u_s + \eta + \omega \right] \tag{10}$$

Which can be arranged as

$$\dot{e} \;=\; A_m e - I_n \Psi \tag{11}$$
$$\Psi \;=\; b_c \left[\phi \sum_{j=1}^{n} \widetilde{\theta}_{a_j} x_j + \phi\widetilde{\theta}_b u + u_s + \eta + \omega \right] \tag{12}$$

IV. Stability analysis

Following the hyperstability approach [7, 8], the feedback system (11)-(12) is split into two blocks (fig. 2): A linear time invariant feedforward block and a nonlinear time varying block. If the following conditions are satisfied:

1. The feedforward transfer function matrix $C\left[sI - A_m\right]^{-1} I_n$ is strictly positive real (SPR).
2. The nonlinear time varying block satisfies the following inequality

$$\int_0^T \Psi^T e_s dt \geq -\delta, \qquad \forall T \geq 0 \tag{13}$$

where $e_s = Ce$, and δ is a positive constant independent of T. Then the asymptotic hyperstability (or global asymptotic stability) of the feedback system is ensured for all bounded initial conditions on x, r, e and u.

Following the Kalman-Yacubovicth lemma [10], the transfer function matrix $C[sI - A_m]^{-1} I_n$ function is strictly positive real (SPR) if there exists a symmetric positive definite matrices P and Q so that

$$A_m^T P + P A_m = -Q \tag{14}$$
$$C = I_n P \tag{15}$$

Since A_m is a Hurwitz matrix, a symmetric positive definite matrix P always exists for any symmetric positive definite matrix Q [10]. Thus, SPR condition is always verified independently of the plant dynamics.

Note that, as required by the hyperstability theory, the system described by (11)-(12) is controllable and observable (it is easy to show that the pairs $\{A_m, I_n\}$ and $\{P, A_m\}$ are controllable and observable, respectively).

In order to prove the second condition, inequality (13) is expanded to the following two terms

$$\int_0^T \Psi^T e_s dt = s_1 + s_2 \tag{16}$$

where

$$s_1 = \int_0^T \left(\sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T \phi^T p_n e x_j + \widetilde{\theta}_b^T \phi^T p_n e u \right) dt \tag{17}$$

and

$$s_2 = \int_0^T (u_s + \eta + \omega) p_n e dt \tag{18}$$

with p_n is nth row of the matrix P. Hence, the sum of the two terms must verify the condition (13).

By choosing the following parameters update laws

$$\dot{\theta}_{a_j} = -\gamma_1 \phi^T p_n e x_j \tag{19}$$
$$\dot{\theta}_b = -\gamma_2 \phi^T p_n e u \tag{20}$$

and using the fact that $\dot{\widetilde{\theta}}_{a_j} = -\dot{\theta}_{a_j}$ and $\dot{\widetilde{\theta}}_b = -\dot{\theta}_b$, then (16) can arranged as

$$s_1 = \int_0^T \left(\frac{1}{\gamma_1} \sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T \dot{\widetilde{\theta}}_{a_j} + \frac{1}{\gamma_2} \widetilde{\theta}_b^T \dot{\widetilde{\theta}}_b \right) dt \tag{21}$$

which yields the following result

$$s_1 \geq - \left[\frac{1}{2\gamma_1} \sum_{i=1}^{n} \widetilde{\theta}_{a_i}^T (0) \widetilde{\theta}_{a_j} (0) + \frac{1}{2\gamma_2} \widetilde{\theta}_b^T (0) \widetilde{\theta}_b (0) \right] \tag{22}$$

To analyze (18), the additional control term u_s is defined as

$$u_s = k_s . \mathrm{sgn}(p_n e) \tag{23}$$

If the constant k_s is chosen

$$|k_s| = \overline{\eta} + \overline{\omega} \tag{24}$$

where $\overline{\omega}$ and $\overline{\eta}$ are the upper bounds on the approximation error and the external disturbance, respectively, required to design the switching control signal u_s. Then s_2 is always ≥ 0. Hence, the inequality (12) is verified, with

$$s_1 + s_2 \geq -\delta \tag{25}$$

where

$$\delta = \left[\frac{1}{2\gamma_1} \sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T (0) \widetilde{\theta}_{a_j} (0) + \frac{1}{2\gamma_2} \widetilde{\theta}_b^T (0) \widetilde{\theta}_b (0) \right] \tag{26}$$

Thus, the feedback system given by (11)-(12) is globally asymptotically stable, and the error e is guaranteed to converge to zero.

Remark 1:

The above stability result is achieved under the assumption that the fuzzy model is well designed such that: $\phi\theta_b > 0$ for all the time. If the above condition is not fulfilled, the control input may be large at certain time, and the internal dynamic of (10) will be instable. To prevent this case, assume (without loss of generality) that a lower bound $b_0 > 0$ of the control gain is known, then using the following update law

$$\dot{b}_i = \begin{cases} 0, \text{ if } b_i = b_0 \text{ and } \phi_i p_n e u > 0 \\ -\gamma_2 \phi_i p_n e u, \text{ else} \end{cases} \tag{27}$$

assures that $\phi\theta_b \geq b_0$ for all the time, if the vector $\theta_b(0)$ is initialized properly. The stability of fuzzy adaptive system, with the modified update law, can verified using the same analysis. Let the expression (17) be rewritten as

$$s_1 = \int_0^T \left(\sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T \phi^T p_n e x_j + \sum_{i=1}^{M} \widetilde{b}_i \phi_i p_n e u \right) dt \tag{28}$$

then, if all the elements of θ_b are above the limit b_0, we get the same result. If any element b_k reaches the limit, then $\phi_k p_n e u > 0$ and $\widetilde{b}_k = (b_k^* - b_0) > 0$ (because b_0 is a lower bound). Then (28) becomes

$$s_1 = \int_0^T \left(\frac{1}{\gamma_1} \sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T \dot{\widetilde{\theta}}_{a_j} + \frac{1}{\gamma_2} \sum_{i=1, i \neq k}^{M} \widetilde{b}_i \dot{\widetilde{b}}_i + \widetilde{b}_k \phi_k p_n e u \right) dt \tag{29}$$

since $\int_0^T \widetilde{b}_k \phi_k p_n e u dt \geq 0$ for all $T \geq 0$, it follows that

$$s_1 \geq - \left(\frac{1}{2\gamma_1} \sum_{j=1}^{n} \widetilde{\theta}_{a_j}^T (0) \widetilde{\theta}_{a_j} (0) + \frac{1}{2\gamma_2} \sum_{i=1, i \neq k}^{M} \widetilde{b}_i^2 (0) \right) \tag{30}$$

thus, the global stability of fuzzy adaptive scheme is also guaranteed with the modified update law. Note that, the stability analysis for the case of $b_0 < 0$ can be handled in a similar way.

Remark 2:

Since the additional control input u_s is discontinuous, it may lead to control chattering, which is undesirable in practice because it involves high control activity and may

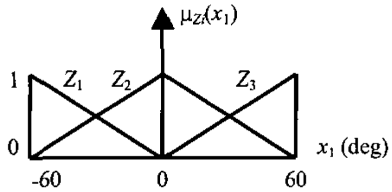

Fig. 3. Membership functions.

excite high frequency plant dynamics. To overcome this problem, the control input can be smoothed as

$$u_s = k_s \frac{p_n e}{|p_n e| + \sigma} \tag{31}$$

with

$$|k_s| = \left(1 + \frac{\sigma}{\beta}\right)(\overline{\eta} + \overline{\omega}) \tag{32}$$

where $\sigma > 0$ and $\beta > 0$ are arbitrary constants. The constant σ is selected based on engineering consideration to achieve the admissible tracking error amplitude. The ratio σ/β determines the gain amplitude. Using (31) and (32), (18) becomes

$$s_2 = \int_0^T \left((\overline{\eta} + \overline{\omega}) \left(1 + \frac{\sigma}{\beta}\right) \frac{(p_n e)^2}{|p_n e| + \sigma} + (\eta + \omega) . p_n e \right) dt \tag{33}$$

if $p_n e$ is $\geq \beta$ then $p_n e \geq 0 \; \forall T \geq 0$ and the inequality is verified. If $p_n e$ is $\leq \beta$ no indication is given. But, this result indicates that the tracking error is guaranteed to converge to a bounded region defined by the design parameters, and due to the bounded input-bounded output property of hyperstable systems the stability is preserved [7].

V. Design example

The performance of the proposed fuzzy adaptive control is studied using an inverted pendulum system. The dynamics of the pendulum system is given by

$$\begin{aligned} \dot{x}_2 &= \frac{(m_c + m)\, g \sin(x_1) - \cos(x_1) \left(m l x_2^2 \sin(x_1) - u(t)\right)}{\frac{4}{3}(m_c + m)\, l - m l \cos(x_1)^2} \\ &\quad + \eta(t) \end{aligned} \tag{34}$$

where x_1 and x_2 are the angular position and velocity of the pole. $u(t)$ is the control input torque applied to the cart, $g = 9.8$ (m/s^2) is the gravitational constant, $m_c = 1$ kg is the mass of the cart, and $m = 0.5$ kg, $l = 0.5$ m are the pole mass and half-length, respectively. $\eta(t) = 0.1 \sin(2\pi t)$ is the bounded external disturbance.

The control objective is to maintain $x_1(t) = x_{1m}(t)$ and $x_2(t) = x_{2m}(t)$, where $x_{1m}(t)$ and $x_{2m}(t)$ are the states of the reference model given by

$$A_m = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \; b_m = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The fuzzy adaptive control design procedure consists of the following steps:

1. Select the TS fuzzy model structure and initial parameters, which will be used to approximate the nonlinear system (34). For this purpose, the relevant region of the state space is partitioned using three fuzzy sets (see fig. 3). The fuzzy model is constituted by three rules of the form

$$\text{R}_i\text{: If } x_1 \text{ is } Z_i \text{ Then } \dot{x}_2 = a_{i1}x_1 + a_{i2}x_2 + b_i u, \;\; i = 1..3$$

If the expert knowledge about the plant is available, it can be used to initialize the fuzzy model parameters. In this simulation no prior knowledge is assumed, and the parameters are initialized as: $a_{ij} = 0$ and $b_i = 1$.

2. select

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

then solving the Lyapunov equation (14) yields

$$P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}$$

3. Obtain the adaptive control law (9), with the switching control term as defined in (23) or (31).

4. Update the fuzzy model parameters using (19) and (27). For a high penalty on the initial parameter errors, the learning rates $\gamma_1 = 0.9$ and $\gamma_2 = 0.2$ are selected. A lower bound on the control gain is fixed to $b_0 = 0.6$.

In this simulation, the fourth-order Runge-Kutta method, with a step size of 0.01 s, is applied to integrate the nonlinear system (34). To achieve effective rejection of the approximation error and external disturbance effects, the smoothed switching control (31) is used, with $k_s = 0.3$ and $\sigma = 0.04$. Simulation results (fig. 4 a, b and c), for the stabilization problem (i.e. $r(t) = 0$), show satisfactory performance, and the errors converge to bounded small value. The remaining errors are essentially due to the external disturbance. The tracking problem results, for the reference input $r(t) = \pi/3 \sin(0.2\pi t)$, depicted in (fig.5a, b and c), demonstrate also, that the fuzzy control has a consistent tracking performance, specially for the pole position (fig. 5a). The control input depicted in fig .4c is seen to be smooth.

From fig. 6 it is seen that the fuzzy model parameters converge to (almost) stationary values after about few seconds. The remaining oscillations are due to the fast learning rate and approximation error. Decreasing the learning rate may reduce those oscillations, but in turn, the convergence of the tracking error will be slower. This result means that the most of the fuzzy model regressors are persistently exciting. As pointed out in [6] for Mamdani fuzzy model type, the persistent excitation condition is fulfilled if the regressors (i.e. ϕ) are linearly independent. The problem here is more complicated, since the regressors depend on ϕ, $\mathbf{x}$ and u. In summary, the persistent excitation mechanism depends on the fuzzy model construction (i.e. fuzzy sets definition and rules selection), the nonlinear plant and the reference signal. The persistent excitation in fuzzy control systems has received little interest in the literature, and further work is needed to clarify this problem.

Compared to the adaptive schemes in [2],[4] and [6], this approach is computationally simpler, since only three rules are used, only nine parameters are to be updated and stored. This allows for fast tracking and adaptation rates. Moreover, the perturbations rejection is better, with less control effort.

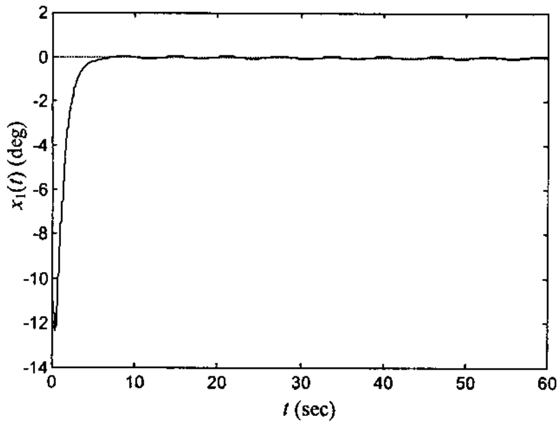

Fig. 4.a: Position.

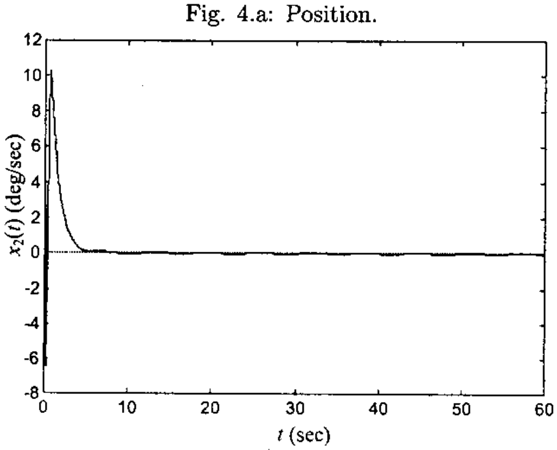

Fig. 4.b: Velocity.

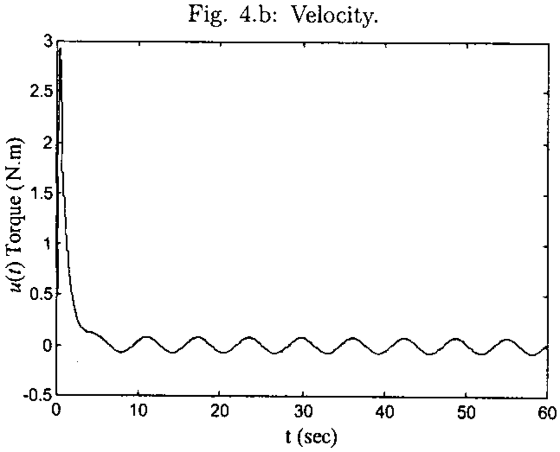

Fig. 4.c: Torque.

Fig. 3: Stabilization (– system, ... ref).

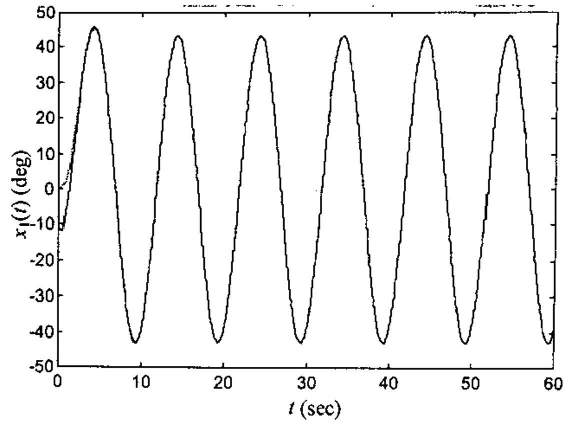

Fig. 5.a: Position.

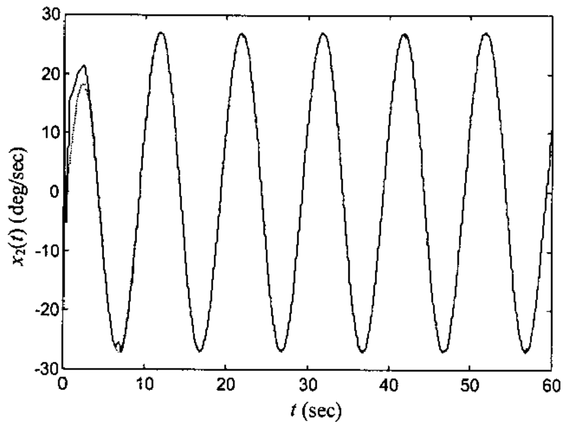

Fig. 5.b: Velocity.

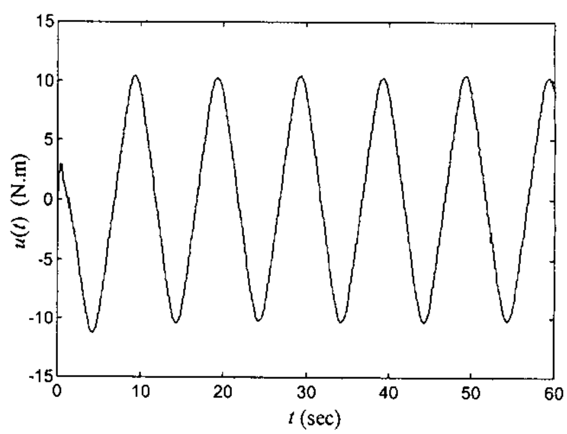

Fig. 5.c: Torque.

Fig. 4: Tracking (– system, ... ref).

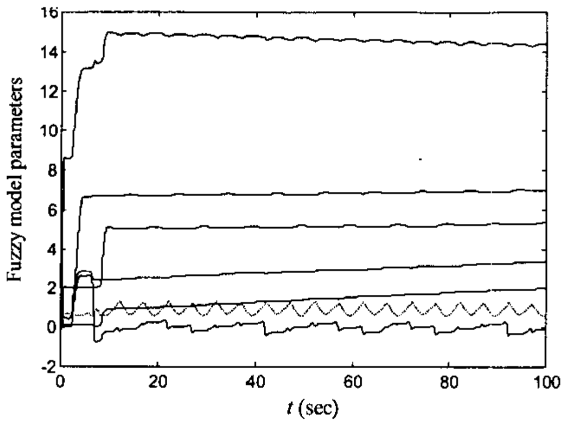

Fig. 6: The parameters evolution.

VI. Conclusion

A new fuzzy indirect adaptive control for SISO nonlinear continuous systems, is developed. The stability of this adaptive scheme, under few requirements on the nonlinear system and the uncertainties (i.e. the plant order and the lower bound on the control gain must be known, and the perturbations are bounded), is established, in easier manner, using the hyperstability approach. Simulation results show that good performance is achieved even when the switching control is not used. Furthermore, the TS fuzzy model parameters are shown to converge to (almost) constant values. Major features of this approach are low computation cost, application to a broader of nonlinear systems, and fast tracking performance. Further research are directed to extend this approach to multivariable nonlinear systems.

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