Contribution to Inverse Kinematic Modeling of a Planar Continuum Robot Using a Particle Swarm Optimization

Ammar Amouri[1], Chawki Mahfoudi[2], and Abdelouahab Zaatri[1]

[1] Department of Mechanical Engineering, University1, Constantine, Algeria
{ammar_amouri,azaatri}@yahoo.fr
[2] Department of Mechanical Engineering,
University Labri Ben M' Hidi, Oum el Bouaghi, Algeria
mahfoudi.chawki@gmail.com

Abstract. According to the literature, research on modeling continuum robots is focused on ways to develop the kinematic models, because of the lack of analytical models for these robots and the complexity of the problem which reside in the coupling of operational variables and infinite of possible solutions for a desired configuration. This paper presents a numerical approach for solving the inverse kinematic model of a planar continuum robot (PCR), assuming that each section of the manipulator is curved as a circular arc, with an inextensible central axis of the structure. At first, this paper presents an inverse kinematic model solution for one bending section, whereas the extreme points, of each section, used in calculating the inverse kinematic model for multi-sections is calculated numerically using a particle swarm optimization (PSO) technique. Finally, Simulation examples of this method are carried to validate the proposed approach.

Keywords: Planar continuum robot, Modeling, Inverse kinematic model, Optimization, Particle swarm optimization.

1 Introduction

An important research theme for study the continuum robots is the calculation of kinematic models, where the traditional approaches to modeling, in which frames are associated with each joint, are inappropriate for this case, because of the absence of discrete and rigid links in their architecture and the complexity of the problem which reside in the coupling of operational variables. However, the hyper-redundant robotic systems, discrete links or continuum form cannot be control by considering a finite number of degrees of freedom, thus admitting a reduced set of physical solutions (Robinson and Davies 1999). Among contributions on the inverse kinematic model of these structures, we can cite the model for one bending section with a special configuration proposed by (Hannan and Walker 2003), to

M. Haddar et al. (eds.), *Multiphysics Modelling and Simulation for Systems Design and Monitoring*, Applied Condition Monitoring 2, DOI: 10.1007/978-3-319-14532-7_15

validate the model; the authors used a specific software symbolic computation and have simulated the kinematic behavior of the robot at two sections of Air-Octor and Oct-Arm. The same authors studied the workspace and configuration space describing the constraints due to limitations with respect to the lengths of robots. (Escande et all. 2011), synthesized and validated an inverse kinematic model for one bending section. The experimental results of this model show an accurate follow-up identification of all existing uncertainties on the bench test and measurement systems for both models and also in (Lakhal et all. 2014), the authors used an optimization algorithm, based on a Sequential Quadratic Program (SQP). The case of a multi-section is studied in (Neppalli et all. 2009), where the end-point of each section is assumed to be known. The overall points can be chosen by solving a system of inequalities. The authors did not validate experimentally the model but they simulated the overall behavior using appropriate software. (Iqbal et all. 2009), presenting a complete inverse kinematic model for one bending section based on the development realized by (Escande et all. 2011).This inverse kinematic model is formulated as an optimization problem with a cost function and constraints using the principle of interval analysis. The validation of this model has done on a micro-robot and showed that the model was not sufficiently robust against uncertainties of environment and inaccuracies of materials. The solution of inverse kinematic model is posed also in terms of optimizing an objective function under inequality constraints by (Bailly and Amirat 2005). The validation of this model was made on a micro-robot dedicated to surgery. Finally, a state of the art report in modeling these classes of robots is presented by (Robert et all. 2010). The lack of analytical models for the kinematic of continuum robots, guide us to use a powerful numerical method such as Particle Swarm Optimization (PSO). The first idea based on swarm intelligence algorithm was introduced in 1995 (Kennedy and Eberhart 1995), who studied social relationships in a group to develop new methods of calculation. This method has showed its capacity to solve different optimization problems compared to other heuristics (Elbeltagi et all. 2005). The main contribution of this paper is the calculation of operational coordinates of each section, by using an optimization method, under the constraints of conserving the length of each section. The rest of the paper is organized as follows: Section 2 describes the bending section modeling. Section 3 shows the optimization technique for inverse kinematic modeling of a PCR. Finally, simulation examples are performed to validate the proposed approach.

2 Inverse Kinematic Model for One Bending Section

One bending section of a PCR is modeled as an inextensible arc of circle, with one end point o_{k-1} fixed to the origin of the reference frame R_{k-1}, the other end point P_k located at anywhere in the workspace, where the first tangent of this bending section is collinear with the y_{k-1} axis (physical constraints) and individually controlled . This section of a PCR is parameterized by its arc length s_k, its curvature κ_k, and its orientation θ_k as shown in Fig.1.

From the Fig.1 and using trigonometrically relations we have:

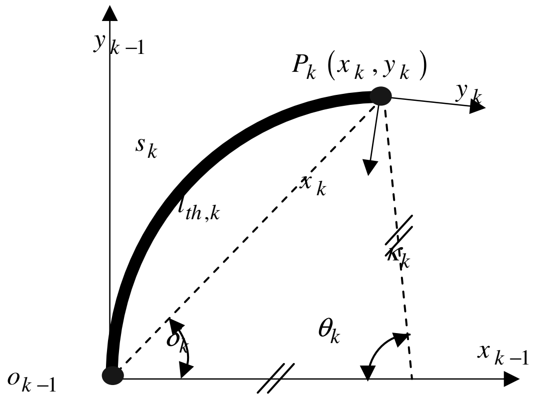


Fig. 1 Circle arc parameters

$$\begin{cases} \delta_k = a\tan 2(y_k, x_k) \\ \theta_k = \pi - 2\delta_k \end{cases} \tag{1}$$

$$\kappa_k = \frac{2\sin\left(\frac{\theta_k}{2}\right)}{\sqrt{x_k^2 + y_k^2}} \tag{2}$$

$$s_k = \frac{\theta_k}{\kappa_k} \tag{3}$$

3 PSO and Inverse Kinematic Model of Multi-sections

3.1 PSO

Particle Swarm Optimization (PSO) is a biologically inspired computational search and optimization method based on the social behaviors of some biological organisms, especially the group's ability of some animal species to locate a desirable position in the given area. It was proposed first by (Kennedy and Eberhart 1995) and its fast convergence compared to other Evolutionary Algorithms (Elbeltagi et all. 2005). Thus, using the simple rules of displacement (in the space of the solutions), the particles can progressively converge to a global optimum. During each iteration *t*, the velocity changes by applying equation (4) to each particle:

$$v_{p_i}(t+1) = \omega v_{p_i}(t) + c_1\rho_1\left(P_{i,best}(t) - P_i(t)\right) + c_2\rho_2\left(P_{g,best}(t) - P_i(t)\right) \tag{4}$$

The position update is applied by equation (5) based on the new velocity and the current position.

$$P_i(t+1) = P_i(t) + v_{P_i}(t+1) \tag{5}$$

Where $P_i(t)$, $v_{p_i}(t)$ represent the current position and the velocity of the particle i at iteration t, respectively; ω is called the inertia weight, c_1 and c_2 are weighting factors, also called the cognitive and social parameters, respectively; and ρ_1, ρ_2 are random variables uniformly distributed within interval [0, 1]. The basic algorithm is as follows, from which we can obtain the good solution of our problem:

1) Initialize each particle of the swarm, with random values for position in the search space.
2) For each particle, evaluate of the cost function.
3) Compare the value obtained by the cost function from particle i, with the value of $P_{i,best}$. If the current value is better than $P_{i,best}$, than set $P_{i,best}$ equal to the current value.
4) If the value $P_{i,best}$ is better in the swarm, then $P_{g,best} = P_{i,best}$.
5) Change the velocity and position of the particle using equations (4) and (5), respectively.
6) If the maximum number of iterations or the ending condition is not achieved, return to step 2.

3.2 *Problem Formulation*

In 2D research space, the particle i can be represented by a two-dimensional position vector $P_i(t) = \{P_{x,i} \quad P_{y,i}\}^T$ and the velocity vector $v_{P_i}(t) = \{v_{P_{x,i}} \quad v_{P_{y,i}}\}^T$. The best position visited by the particle is denoted $P_{i,best}(t) = \{P_{x,i,best} \quad P_{y,i,best}\}^T$ and the best position found by the swarm is $P_{g,best}(t) = \{P_{x,g,best} \quad P_{y,g,best}\}^T$.

Given the desired position $P_n(x_n^*, y_n^*)$ and following the above assumptions, the operational Cartesian coordinates $P_k(x_k^*, y_k^*)$ of each section, with respect to the reference frame R_0, located on the virtual central axis, are calculated numerically by the PSO method, where the cost function is given by:

$$F\left(l_{th,k}, l_{r,k}\right) = \sum_{k=1}^{n} f_k\left(l_{th,k}, l_{r,k}\right) \tag{6}$$

With:

$$f\left(l_{th,k}, l_{r,k}\right) = \frac{1}{2}\left(l_{th,k} - l_{r,k}\right)^2 \tag{7}$$

Such that:

$$\begin{cases} l_{th,k} = \sqrt{\left(x_k - x_{k-1}\right)^2 + \left(y_k - y_{k-1}\right)^2}\,\dfrac{\theta_k}{\sin\left(\theta_k\right)} \\ \theta_k = a\tan 2\left(\left(y_k - y_{k-1}\right), \left(x_k - x_{k-1}\right)\right) \end{cases} \tag{8}$$

Where $l_{th,k}$, $l_{r,k}$: the real length and the theoretical length calculated using equation (8) of a bending section k respectively and where n is the number of bending section.

The intermediate operational coordinates $\left(x_k^*, y_k^*\right)$ of each section, are calculated by solving the following optimization problem:

$$\begin{cases} \min\limits_{x_k^*, y_k^*; k=1,\ldots,n} F\left(x_k^*, y_k^*\right) \\ \text{under the constraints } g_j \\ g_j \leq 0; j = 0,1,\ldots,m \end{cases} \tag{9}$$

Where g_j the constraints are could be used for example: obstacle avoidance, taking objects, energy minimization, etc. And m is the number of added constraints.

Initially, the population of particles is located with random positions in the problem space. During each iteration *t*, each particle changes their velocity according to equation (4) and move to a new position using (5). Once the displacement of the particles carried out, the new positions are evaluated using equation (6) and the vector $P_{i,best}$ and $P_{g,best}$ are determined. The algorithm stops if maximum number of iterations is achieved or any other stopping criteria are satisfied. The solution of this algorithm represents the intermediate Cartesian coordinate researched with respect to the reference frame R_0.

3.3 Inverse Kinematic Model of Multi-sections

Knowing the intermediate Cartesian coordinates of each bending section, the inverse kinematic model presented in the previous section can be iteratively applied to several sections connecting in series to model a PCR of *n* sections. Where each found point (calculated by optimization method PSO) will be transformed to a local reference frame of each section, by the following equation:

$$\left[x_k, y_k, 0\right]^T = T_0^{k-1} P_k^0 \quad (10)$$

Such that:

$$T_k^0 = T_1^0 T_2^1 \cdots T_k^{k-1} \quad (11)$$

Where $T_k^{k-1}\left(k = 1, 2\ldots, n\right)$, is a homogeneous transformation matrix, contains both rotational and translational terms, defines the origin of the reference frame R_k in the reference frame R_{k-1} and n is the total number of the bending sections.

4 Simulation Results

The effectiveness of proposed approach is verified by some simulations through a personal computer with i3 3.30 GHz processor. The first example was performed for a planar robot consists of two bending sections for the following settings of the trajectory in operational space: $\left(x_2^* = \frac{1}{2}t^2, y_2^* = 200 - 4t\right)$ The trajectory has been accomplished in 10 sec of time duration, corresponding to 51 positions captures. Fig. 2 represents the configurations (solutions) for a desired point of the given trajectory, while respecting the imposed assumptions. The execution time of the optimization process for each point found is shown in Fig. 3. Fig. 4 shows the convergence of cost function. In this simulation, the tolerance error is fixed to 10^{-6} while the time execution is equal to 23.91 msec.

For the second case, knowing the general movement of a superior mobile platform generated by the trajectory in operational space $\left(x_2^* = 0, y_2^* = 200 - 10t\right)$ where t is time variable varying from 0 to 10 sec, with a step equal to 2 sec. Fig.5.a represents the possible configurations for a desired point of the given trajectory. The problem of infinite solutions is solved by adding some other constraints. To illustrate this, two cases be imposed. The first case uses the constraint of $g_1 = \theta_1 \langle 0$ and the second that uses $g_2 = \theta_1 \rangle 0$, as shown in Fig.5.b and 5.c, respectively.

In the third example, the trajectory of equation: $\left(x_2^* = 0, y_2^* = 200 - 10t\right)$ is used as input in the proposed model and lengths variation is determined. This trajectory has been accomplished in 10 sec of time duration, corresponding to 121 position captures. Fig. 6 shows the length profiles obtained by unconstrained model $\left(g_j = 0\right)$. Fig. 7 show the length profiles, where the constraint is given by $g_1 = \theta_1 \langle 0$.

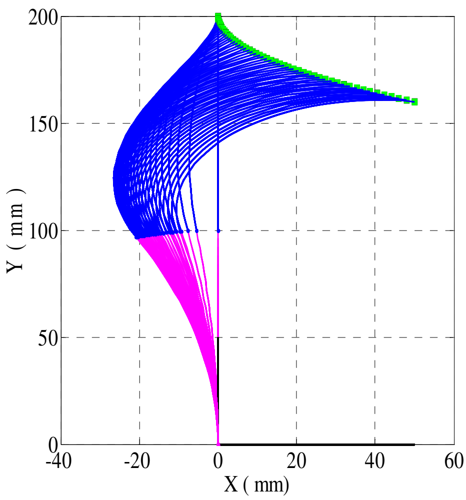


Fig. 2 Virtual axis of the planar robot

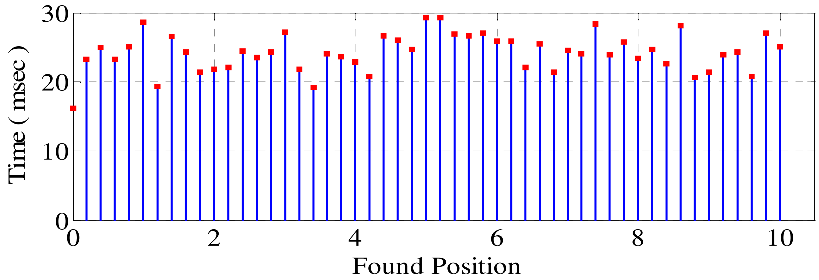


Fig. 3 Execution time of the optimization algorithm

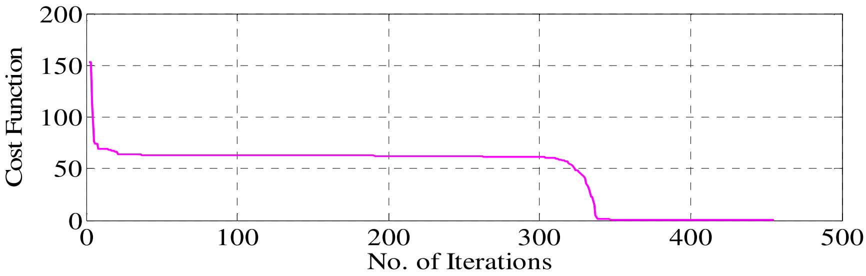
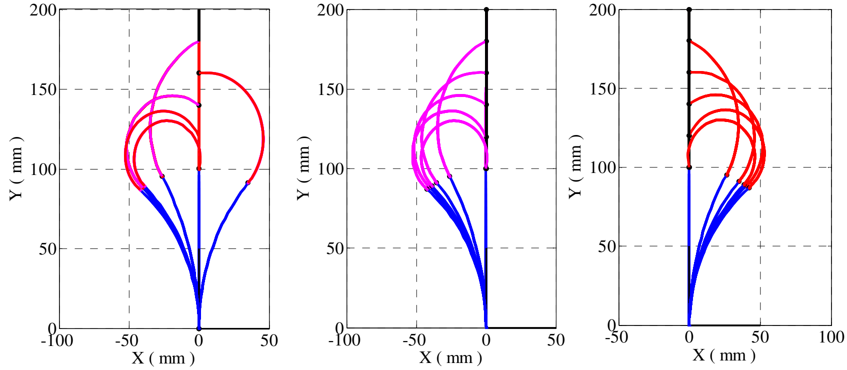


Fig. 4 Convergence of the cost function



(a) Without constraint (b) With: $g1 = \theta_1 \langle 0$ (c) With: $g_2 = \theta_1 \rangle 0$

Fig. 5 Virtual axis of the planar robot

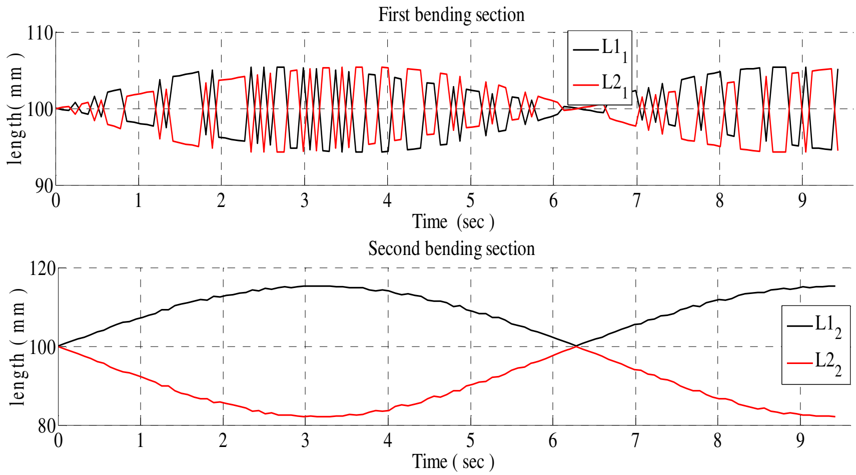


Fig. 6 Lengths variation without constraint

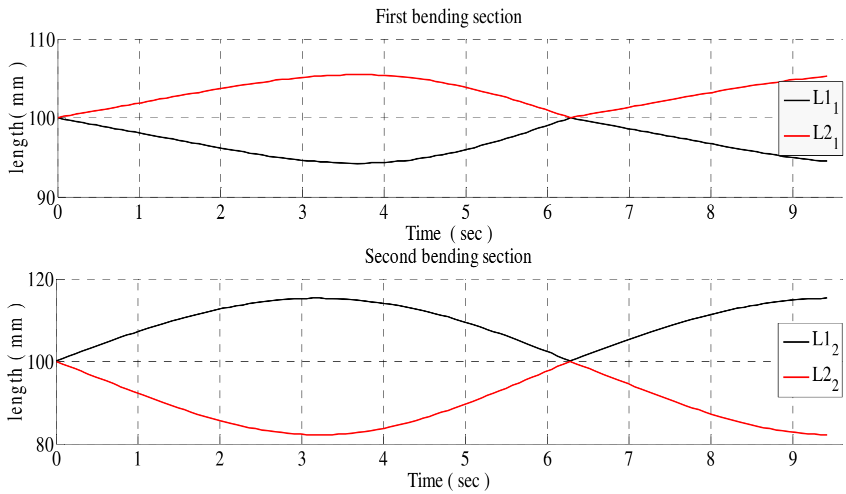


Fig. 7 Lengths variation with constraint

5 Conclusion

This article presents a new approach for the synthesis of the inverse kinematic model for a planar continuum robot. The mathematical formulations defining the kinematic of a bending section are given. This approach allows us to calculate the Cartesian coordinates of the origin of intermediate platforms using the PSO optimization method, where the search space that is chosen arbitrarily. Future work is to improve the execution time, one taking the space of research in the workspace of each section, in order to find a solution close to real time and the generalization of the approach in the three-dimensional.

References

Robinson, G., Davies, J.B.C.: Continuum Robots: A State of the Art. In: IEEE International Conference on Robotics and Automation, Detroit, Michigan, pp. 2849–2854 (1999)

Hannan, M., Walker, I.D.: Kinematics and the Implementation of an Elephant's Trunk Manipulator and Other Continuum Style Robots. Journal of Robotic Systems 20(2), 45–63 (2003)

Escande, C., Pathak, P.M., Merzouki, R., Coelen, V.: Modeling of Multi-section Bionic Manipulator: application to Robotino XT. In: 2011 IEEE International Conference on Robotics and Biomimetics (ROBIO), Phuket, Thailand, pp. 92–97 (2011)

Lakhal, O., Chibani, A., Escande, C., Merzouki, R.: Contribution to Inverse Kinematic Modeling of a class of Continuum Bionic Handling Arm. In: 2014 IEEE/ASME, Interntional Conference on Advanced Intelligent Mechatronics (AIM), Besançon, France (2014)

Neppalli, S., Csencsits, M.A., Jones, B.A., Walker, I.D.: Closed-Form Inverse Kinematics for Continuum Manipulators. Advanced Robotics 23, 2077–2091 (2009)

Iqbal, S., Mohammed, S., Amirat, Y.: A guaranteed approach for kinematic analysis of continuum robot based catheter. In: IEEE International Conference on Robotics and Biomimetics (ROBIO), Guilin, China, December 19-23, pp. 1573–1578 (2009)

Bailly, Y., Amirat, Y.: Modeling and control of a hybrid continuum active catheter for aortic aneurysm treatment. In: Proc. of the International Conference on Robotic and Automation, Barcelona, Spain, pp. 936–941 (2005)

Webster III, R.J., Jones, B.A.: Design and Kinematic Modeling of Constant Cur-vature Continuum Robots: A Review. The International Journal of Robotics Research 29(13), 1661–1683 (2010)

Kennedy, J., Eberhart, R.C.: Particle Swarm Optimization. In: Proceeding of the IEEE International Conference on Neural Networks IV, pp. 1942–1948 (1995)

Elbeltagi, E., Hegazy, T., Grierson, D.: Comparison among five evolutionary-based optimization algorithms. Advanced Engineering Informatics 19(1), 43–53 (2005)