Fuzzy Approximation-Based Model Reference Adaptive Control of Nonlinear Systems

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Abstract— **Based on the universal approximation capability of the Takagi-Seguno (TS) fuzzy systems, we present a solution to the model reference control of continuous-time nonlinear systems problem. Using the assumption that a fuzzy model exists for the considered nonlinear systems class, a direct TS fuzzy adaptive controller is designed to achieve the tracking objective. It is not required to estimate the fuzzy model, only its existence assumption is required. It is proved, using Lyapunov stability tools, that this adaptive scheme is asymptotically stable and the tracking error converges to zero. Simulation results illustrate the proposed algorithm performance.**

Keywords— **Fuzzy systems, Universal approximation, Reference model, Adaptive control, Nonlinear systems, Stability.**

I. Introduction

Model reference control is one of the well established techniques in the linear system field, however, few results exist for the nonlinear systems case [1-3]. On the other hand, fuzzy systems have been proved to be universal approximators for nonlinear systems [4-7], and various algorithms were proposed to train fuzzy systems as nonlinear identifiers, and were useful in the modelling and control of nonlinear plants [4, 8-11]. Recently, fuzzy model-based control succeeded to exploit the particular structure of TS fuzzy systems [12]. This quasi-linear structure, i.e., the TS fuzzy system being nonlinear superposition of linear local models, has prompted efficient analysis techniques established mainly on the concept of quadratic stability [13-14]. In recent years, fuzzy systems have been applied to model reference adaptive control [15-20]. In [15] and [16] a good performance is shown through Lyapunov stability approach. In [17] and [18] an indirect model-following control, using fuzzy linguistic model and neural networks, is proposed. In [19] a model-reference control is developed based on fuzzy basis functions of Wang [4], and in [20] a direct approach, based on Takagi-Seguno system, is proposed and it's stability is analyzed in the hyperstability framework [1].

This work proposes a direct adaptive model reference approach to control the nonlinear systems represented by Takagi-Sugeno (TS) models. The TS fuzzy model is only assumed to exist, and it is not necessary to be estimated. Based on this assumption, a TS fuzzy adaptive controller is implemented to achieve the tracking task. The stability of the proposed scheme is established using Lyapunov tools. It is shown that the tracking error converge to zero. Also, it is shown that the this approach robust against external disturbance and approximation error.

The rest of the paper is organized as follows. Section II poses the control problem. Section III develops the fuzzy adaptive approach, section IV develops the stability analysis, section V presents the simulation results and section VI concludes the paper.

II. Problem Formulation

We consider the continuous-time nonlinear systems given by

$$\dot{x} = Ax + B\left[f(x) + g(x)u + \eta\right] \qquad (1)$$

where $x \in R^n$ is the state vector, $f(x)$ and $g(x)$ are smooth unknown functions, u is the control input, η is a bounded external disturbance, and

$$A = \begin{bmatrix} 0 & I_{n-1} \\ & 0 \end{bmatrix}, \; B^T = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix}_{1\times n}$$

The above system is required to follow the stable LTI reference model given by

$$\dot{x}_m = A_m x_m + B b_m r \qquad (2)$$

where $x_m \in R^n$ is the state vector, r is a bounded reference input, $b_m > 0$ is a scalar and A_m is given by

$$A_m = \begin{bmatrix} 0 & I_{n-1} \\ & -a_m \end{bmatrix}$$

and $a_m \in R^n$.

Since the nonlinearities $f(x)$ and $g(x)$ are unknown, we investigate, in what follows, the fuzzy systems approach to solve the control problem.

III. Fuzzy Adaptive Control

A. Fuzzy Approximation

Consider the TS fuzzy model constituted by a set of if-then rules of the form

$$\text{R}_i\text{: If } z \text{ is } Z_i \text{ Then } y = a_i x + b_i u, \;\; i = 1 \dots m \qquad (3)$$

where $a_i \in R^n$, $b_i \in R$ are the ith rule consequence parameters, m is the number of rules, $z \in R^q$ is the fuzzy model input vector, and the fuzzy sets Z_i operate a fuzzy partition of the fuzzy model input space (i.e., the fuzzification operators).

The output of the fuzzy model (3) is inferred as follows

$$y = \frac{\sum_{i=1}^{m} \mu_i(z)(a_i x + b_i u)}{\sum_{i=1}^{m} \mu_i(z)} \tag{4}$$

where $\mu_i(z)$ is the grade of membership of z in Z_i (i.e., the firing strength of the rule i).

Further, (4) can be rewritten as

$$y = \sum_{i=1}^{m} \varphi_i (a_i x + b_i u) \tag{5}$$

where φ_i is the normalized firing strength, given by

$$\varphi_i = \frac{\mu_i}{\sum_{l=1}^{m} \mu_l} \tag{6}$$

Based on the universal approximation results [3-7], there exists an optimal set of parameters a_i^* and b_i^*, such that the nonlinear system (1) can be modelled by the fuzzy model (5) such as

$$\dot{x} = Ax + B\left[\sum_{i=1}^{m} \varphi_i (a_i^* x + b_i^* u) + d\right] \tag{7}$$

where d is the combination of the external disturbance and the approximation error introduced by the optimal fuzzy model.

B. Fuzzy Controller

The TS fuzzy controller to be designed is a multi-input single-output TS fuzzy system of the form

$$\text{R}_j\text{: If } v \text{ is } V_j \text{ Then } u_f = k_{1j} x + k_{2j} r, \quad j = 1 \ldots m \tag{8}$$

where $k_{1j} \in R^n$, $k_{2j} \in R$ are the jth rule consequence parameters, m is the number of rules, $v \in R^m$ is the fuzzy controller input vector, and the fuzzy sets V_j operate a fuzzy partition of the fuzzy controller input space.

The final output of the fuzzy controller (8) is inferred as follows

$$u_f = \frac{\sum_{j=1}^{m} \rho_j(v)(k_{1j} x + k_{2j} r)}{\sum_{j=1}^{m} \rho_j(v)} \tag{9}$$

where $\rho_j(v)$ is the grade of membership of v in V_j. We assume that, the fuzzy sets V_j are singleton, chosen such as

$$\rho_j(v) = \begin{cases} 1 & if \quad j = i \\ 0 & if \quad j \neq i \end{cases} \tag{10}$$

The fuzzy controller output (9) can also be rewritten in following compact form

$$u_f = \sum_{j=1}^{m} \xi_j (k_{1j} x + k_{2j} r) \tag{11}$$

where

$$\xi_j = \frac{\rho_j}{\sum_{l=1}^{m} \rho_l} \tag{12}$$

C. Closed Loop Dynamic

Using (2) and (7), the tracking error can be rewritten as

$$\dot{e} = A_m e - B\left[\sum_{i=1}^{m} \varphi_i (a_i^* x + b_i^* u) + a_m x - b_m r + d\right] \tag{13}$$

To ensure tracking objective and to overcome the uncertainties effect, we use the following control input

$$u = u_f + u_s \tag{14}$$

where u_s are additional control term to be defined later, and u_f is the fuzzy adaptive controller defined by (11).

Using (14) and (11) in (13) yields

$$\begin{aligned} \dot{e} \;=\; & A_m e - B\left[\sum_{i=1}^{m} \varphi_i a_i^* x + a_m x + \sum_{i=1}^{m} \varphi_i b_i^* \sum_{j=1}^{m} \xi_j k_{1j} x \right. \\ & \left. + \sum_{i=1}^{m} \varphi_i b_i^* \sum_{j=1}^{m} \xi_j k_{2j} r - b_m r + \sum_{i=1}^{m} \varphi_i b_i^* u_s + d\right] \end{aligned} \tag{15}$$

Exploiting the fact that $\sum_{i=1}^{m} \varphi_i = \sum_{j=1}^{m} \xi_j = 1$, (15) can be arranged as

$$\begin{aligned} \dot{e} \;=\; & A_m e - B\left[\sum_{i=1}^{m} \varphi_i \sum_{j=1}^{m} \xi_j (a_m + a_i^* + b_i^* k_{1j}) x \right. \\ & + \sum_{i=1}^{m} \varphi_i \sum_{j=1}^{m} \xi_j (b_i^* k_{2j} - b_m) r \\ & \left. + \sum_{i=1}^{m} \varphi_i b_i^* u_s + d\right] \end{aligned} \tag{16}$$

Then, using the definition (10) for $\rho_j(v)$, (16) can be further, simplified as

$$\begin{aligned} \dot{e} \;=\; & A_m e - B\left[\sum_{i=1}^{m} \varphi_i (a_m + a_i^* + b_i^* k_{1i}) x \right. \\ & \left. + \sum_{i=1}^{m} \varphi_i (b_i^* k_{2i} - b_m) r + \sum_{i=1}^{m} \varphi_i b_i^* u_s + d\right] \end{aligned} \tag{17}$$

At this point, we recall the following result from the linear systems theory.

Lemma [3]: there exists a set of parameters k_{1i}^* and k_{2i}^* such as

$$a_i^* + b_i^* k_{1i}^* + a_m = 0 \tag{18}$$

$$b_i^* k_{2i}^* - b_m = 0 \tag{19}$$

Also, (18)-(19) can be rewritten as

$$a_i^* + b_i^* k_{1i} + a_m = b_i^* \widetilde{k}_{1i} \tag{20}$$

$$b_i^* k_{2i} - b_m = b_i^* \widetilde{k}_{2i} \tag{21}$$

where $\widetilde{k}_{1i} = k_{1i} - k_{1i}^*$ and $\widetilde{k}_{2i} = k_{2i} - k_{2i}^*$ are the parameters estimation errors.

Then, substituting with (20)-(21) in (17) yields

$$\dot{e} = A_m e - B\left[\sum_{i=1}^{m} \varphi_i b_i^* \widetilde{k}_{1i} x + \sum_{i=1}^{m} \varphi_i b_i^* \widetilde{k}_{2i} r + \sum_{i=1}^{m} \varphi_i b_i^* u_s + d\right] \quad (22)$$

IV. Stability Analysis

To establish the stability the following assumptions are used.

Assumption 1: The uncertainty term is bounded by $|d| < \overline{d}$ where $\overline{d}$ is a known upper bound.

Assumption 2: $k_{2j}^* > 0$ for $j = 1..m$.

Consider the following Lyapunov function

$$V = \frac{k_2^*}{2} e^T P e + \frac{1}{2\gamma_1} \sum_{i=1}^{m} \widetilde{k}_{1i} \widetilde{k}_{1i}^T + \frac{1}{2\gamma_2} \sum_{i=1}^{m} \widetilde{k}_{2i}^2 \quad (23)$$

where

$$k_2^* = \sum_{j=1}^{m} \xi_j k_{2j}^* \quad (24)$$

and, $\gamma_1, \gamma_2 > 0$ are design parameters. The matrix $P = P^T > 0$ is the solution, for a given $Q = Q^T > 0$, of the following Lyapunov equation

$$A_m^T P + P A_m = -Q \quad (25)$$

The differentiation of (23) along the trajectory of (22) yields

$$\begin{aligned} \dot{V} &= -\frac{k_2^*}{2} e^T Q e - k_2^* e^T P B \left[\sum_{i=1}^{m} \varphi_i b_i^* u_s + d\right] \\ &\quad - k_2^* e^T P B \left[\sum_{i=1}^{m} \varphi_i b_i^* \widetilde{k}_{1i} x + \sum_{i=1}^{m} \varphi_i b_i^* \widetilde{k}_{2i} r\right] \\ &\quad + \frac{1}{\gamma_1} \sum_{i=1}^{m} \widetilde{k}_{1i} \dot{\widetilde{k}}_{1i}^T + \frac{1}{\gamma_2} \sum_{i=1}^{m} \widetilde{k}_{2i} \dot{\widetilde{k}}_{2i} \end{aligned} \quad (26)$$

which can be arranged as

$$\begin{aligned} \dot{V} &= -\frac{k_2^*}{2} e^T Q e - e^T P B \left[\sum_{i=1}^{m} \varphi_i k_2^* b_i^* u_s + d\right] \\ &\quad + \frac{1}{\gamma_1} \sum_{i=1}^{m} \widetilde{k}_{1i} \left(\dot{\widetilde{k}}_{1i}^T - \gamma_1 k_2^* e^T P B \varphi_i b_i^* x\right) \\ &\quad + \frac{1}{\gamma_2} \sum_{i=1}^{m} \widetilde{k}_{2i} \left(\dot{\widetilde{k}}_{2i} - \gamma_2 k_2^* e^T P B \varphi_i b_i^* r\right) \end{aligned} \quad (27)$$

Using (24) and the definition (10), we have that $k_2^* = k_{i2}^*$. Then, since $k_{2i}^* b_i^* = b_m$, (27) can be rewritten as

$$\begin{aligned} \dot{V} &= -\frac{k_2^*}{2} e^T Q e - e^T P B \left[b_m u_s + d\right] \\ &\quad + \frac{1}{\gamma_1} \sum_{i=1}^{m} \widetilde{k}_{1i} \left(\dot{\widetilde{k}}_{1i}^T - \gamma_1 e^T P B \varphi_i b_m x\right) \\ &\quad + \frac{1}{\gamma_2} \sum_{i=1}^{m} \widetilde{k}_{2i} \left(\dot{\widetilde{k}}_{2i} - \gamma_2 e^T P B \varphi_i b_m r\right) \end{aligned} \quad (28)$$

Let's define the following update laws

$$\dot{k}_{1i} = \gamma_1 \varphi_i b_m x^T B^T P e \quad (29)$$

$$\dot{k}_{2i} = \gamma_2 \varphi_i b_m r B^T P e \quad (30)$$

Hence, replacing with (29)-(30) in (28) gives

$$\dot{V} = -\frac{k_2^*}{2} e^T Q e - e^T P B \left[b_m u_s + d\right] \quad (31)$$

Now, choosing the additional control term as

$$u_s = \frac{\overline{d}}{b_m} sgn\left(e^T P B\right) \quad (32)$$

and substituting in (31) gives

$$\dot{V} \leq -\frac{k_2^*}{2} e^T Q e \quad (33)$$

The stability results for this approach are summarized by the following theorem.

Theorem : The control system composed by the nonlinear system (1) the reference model (2) the fuzzy model (7), the fuzzy controller (11), and the update laws (29)-(30), is asymptotically stable and the tracking error converges to zero.

Proof : from (33) $\dot{V}$ is always negative in the e space if $e \neq 0$, then $e, \widetilde{k}_{1i}$ and $\widetilde{k}_{2i} \in L_\infty$, therefore $V \in L_\infty$. Since all variables in the right-hand side of (22) are bounded, $\dot{e}$ is bounded, i.e., $\dot{e} \in L_\infty$. Integrating both sides of (33) yields

$$\int_0^\infty |e|^2 \, dt \leq \frac{2}{k_2^* \lambda_{\min}(Q)} V(0) \quad (34)$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q. Since the right side of (34) is bounded, $e \in L_2$. Hence, using Barbalat's lemma [2], we have that the error converges asymptotically to zero, i.e., $\lim_{t \to \infty} e = 0$.□

Remark 1: Because of the discontinuous nature of the switching term (32), chattering may occur. To eliminate this problem, the control term (33) can be smoothed as

$$u_s = \frac{\overline{d}}{b_m} \frac{e^T P B}{|e^T P B| + \sigma} \quad (35)$$

where $\sigma > 0$ is design parameter. It can be also shown that the error converge to a bounded region. However, the parameters boundedness is no longer ensured. To avoid parameters drift, various techniques from robust identification, such as dead zone or projection can be used [21].

Remark 2: The proposed approach can be easily extended, without further modifications, to the decoupled or decentralized control of multivariable nonlinear systems.

V. Simulation

To illustrate the performance of the proposed adaptive scheme, we consider the manipulator of fig. 1. The end effector of the manipulator can be extended or extracted from, and rotated around the vertical axis O. The dynamic of the manipulator motion is given by

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{\left((m_c+m)\,x_1+ma\right)x_4^2}{m_c+m}+\frac{u_1}{m_c+m} \qquad (36)\\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{-\left((m_c+m)\,x_1+ma\right)x_2x_4}{J_1+J_2+m_cx_1^2+m\,(x_1+a)^2} \\
&\quad +\frac{u_2}{J_1+J_2+m_cx_1^2+m\,(x_1+a)^2} \qquad (37)
\end{aligned}
$$

where x_1 is the distance from the center of mass and the axis O, and x_3 is the angular position of the link. m is the payload mass, m_c is the link mass, J_1 and J_2 are the moments of inertia of the link with respect to the vertical axis through C and O respectively, and a is the distance from C to the center of the payload.

The reference models, for the both motions, are given by $a_{1m} = a_{2m} = \left[\begin{array}{cc} 1 & 2 \end{array}\right]$ and $b_{1m} = b_{2m} = 1$.

The proposed fuzzy adaptive controllers are given by

$$
\begin{aligned}
&\mathrm{R}_i^1 : \text{If } \; x_1 \text{ is } V_i^1 \text{ and } x_3 \text{ is } V_i^2 \\
&\qquad\qquad \text{Then } u_{1f} = k_{1i}^1 x + k_{2i}^1 r_1\,, \quad i = 1..9
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathrm{R}_i^2 : \text{If } \; x_1 \text{ is } V_i^1 \text{ and } x_3 \text{ is } V_i^2 \\
&\qquad\qquad \text{Then } u_{2f} = k_{1i}^2 x + k_{2i}^2 r_2\,, \quad i = 1..9
\end{aligned}
$$

where $x^T = \left[\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\right]$, and the fuzzy sets Z_i^1, Z_i^2 are designed as shown in fig .2.

The manipulator's parameters, used in this simulation, are $m_c = 12$, $m = 10$, $J_1 = 10$, $J_2 = 8$ and $a = 1$. No a priori knowledge is assumed in this simulation, and the controllers parameters are initialized to zero.

The matrices $Q_{1,2} = I_2$ are chosen, and the $P_{1,2}$ are found to be

$$
P_{1,2} = \left[\begin{array}{cc} 1.5 & 0.5 \\ 0.5 & 0.5 \end{array}\right]
$$

. The switching control terms are chosen as in (35) with $\sigma_1 = \sigma_2 = 0.01$. The controllers parameters are updated using (29)-(30) with $\gamma_{1x} = \gamma_{2x} = \gamma_{1r} = \gamma_{2r} = 5$ The upper bounds on the uncertainties terms are estimated to be $\overline{d}_1 = 2$ and $\overline{d}_2 = 5$.

In the simulation the reference inputs are chosen as $r_1 = r_2 = sign(\sin(0.3t)) + 1$. The fuzzy adaptive control results are depicted on Fig. 3(a-c) for the translational motion and in Fig. 4(a-c) for the rotational motion. It is clear that the positions and velocities converge rapidly to their respective references, the fuzzy controllers achieve full rejection of the uncertainties effect, and the control inputs are seen to be smooth. As can be seen, the tracking errors transient dynamics are very acceptable and that the convergence to a small values around zero is fast.

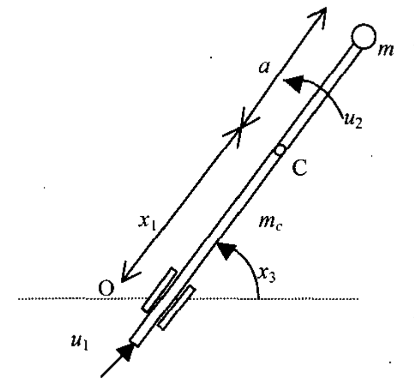

Fig .1 : The manipulator schematic structure.

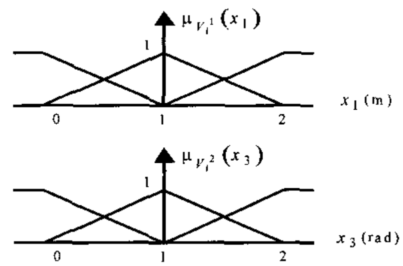

Fig. 2 : The membership functions.

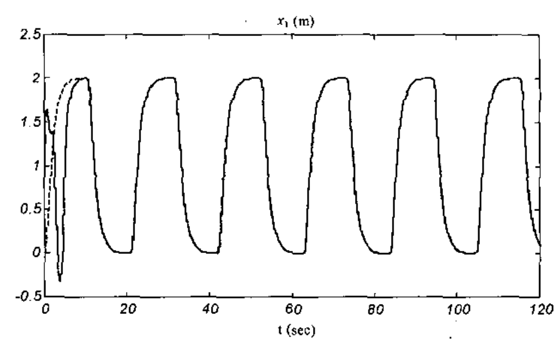

Fig. 3(a) : Position (— robot, ... ref).

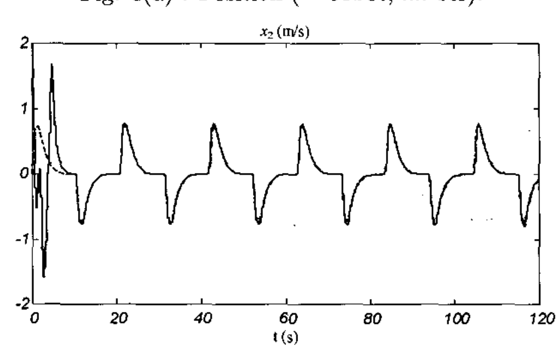

Fig. 3(b) : Velocity (— robot, ... ref).

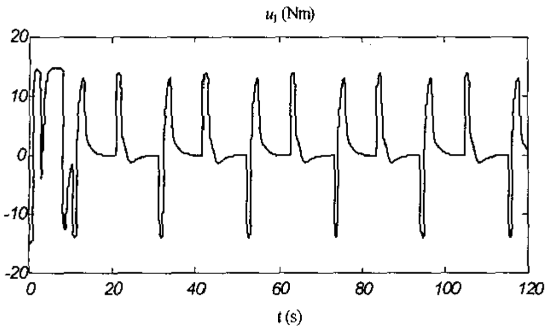

Fig. 3 (c) : Force.

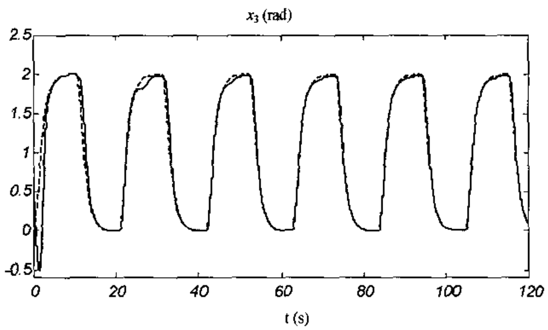

Fig. 4(a) : Angular position (— robot, ... ref).

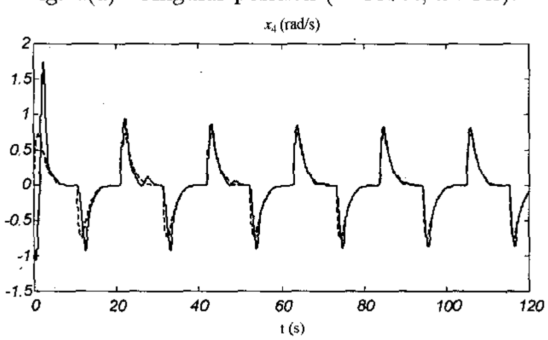

Fig. 4(b) : Angular velocity (— robot, ... ref).

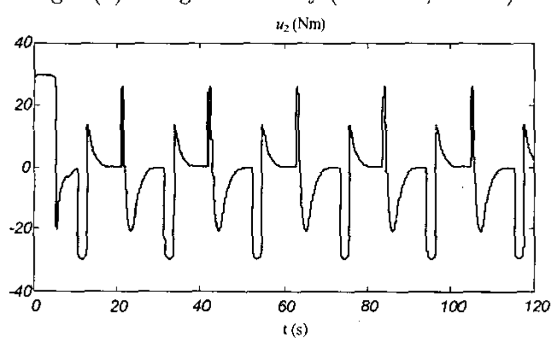

Fig. 4(c) : Torque.

VI. Conclusion

Based on the fuzzy systems approximation results, a direct fuzzy adaptive controller is developed. This controller doesn't need the identification of the fuzzy model, only it's existence is used. The stability analysis has shown that, this adaptive scheme tracks asymptotically a stable reference model, and the tracking error converges to zero. The simulation study demonstrates the performance of the proposed approach, and it's possible use for the control of multivariable systems.

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