

DKP particle in time-dependent field

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We present the solution of Duffin–Kemmer–Petiau (DKP) equation of spin 0 and 1 in the case of the time-dependent mass in the presence of a time-dependent linear scalar field. Calculation is carried out analytically, the wave functions are then deduced for both cases and are connected respectively to the auxiliary equations. The adiabatic approximation is deduced and reveals that in this study we have an obvious absence of the geometrical amplitude factor.

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1 Introduction

The study of the systems depending explicitly on time raised a considerable interest because of its varied applications in various domains of physics such as the Berry geometric phase [1], the Aharonov–Bohm effect in molecular domain [2], the quantum Hall effect [3] and in quantum optics [4] etc..

For this purpose, several methods of analytical and approximate resolution were presented. Among them let us quote: the Lewis Riesenfeld method based on the invariant operator in non relativistic quantum mechanics [5], the Heisenberg representation [6], the standard method of separation of variables [7] and the supersymmetry method [8]. In addition, the applicability of these techniques was extended to the relativistic case [9, 10]. We note that in this domain, the spin, this fundamental physical quantity, plays a significant role in the explanation of the microscopic phenomena and its effect cannot be neglected. Consequently, it drew the attention of a

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certain number of researchers to elucidate its influence on the time-dependent systems. Indeed, this influence is concretized in the development of the exactly soluble models. The analytical solution of these models is much required since it enables us to explore relativistic and spinor nature at the same time. In other way, when the spin value increases, the problem becomes more complicated and the number of coupled equations also increases. To our knowledge in this area, the list of the exactly solved problems is restricted in spite of some attempts. For example, the case of Dirac equation in a time-dependent linear potential was recently studied [9, 10].

As it is well-known in the literature, there is another relativistic equation other than that of Dirac and Klein-Gordon namely Duffin–Kemmer–Petiau (DKP) equation [11, 12]. This latter describes the dynamics of the scalar and vectorial particles respectively (spin 0 and 1). It is time-first order, similar to that of Dirac. It replaces the algebra of the gamma matrices by another algebra whose matrices are denoted beta. The latter obey a known more complicated algebra under the name of DKP algebra which has three irreducible representations: a one-dimensional trivial representation, a five-dimensional representation associated to spin 0 and ten-dimensional representation associated to spin 1.

In this analysis, we are interested in the treatment of the one-dimensional DKP equation relative to particles of spins 0 and 1 with time-dependent masses $m(t)$ and subjected to the interaction of a time-dependent linear scalar field $zf(t)$. This problem was already treated by Maamache et al. [10] in the case of the Dirac equation. In their work, it was shown that during an adiabatic evolution, a geometrical factor appears which is related to the Hannay angle.

One then proposes to re-examine this point within the framework of the DKP equation: spin 0 and 1.

2 Resolution of the DKP equation

Before beginning the resolution of the DKP equation, it is useful to give some interesting formulas of this equation

$$[i\beta^\mu(\partial_\mu + ieA_\mu) - m]\psi(x, t) = 0 \tag{1}$$

where the β^μ matrices are singular and check the following commutation relations

$$\beta^\mu \beta^\nu \beta^\lambda + \beta^\lambda \beta^\nu \beta^\mu = g^{\mu\nu} \beta^\lambda + g^{\nu\lambda} \beta^\mu . \tag{2}$$

– For the case of spin 0, the explicit form of the β^μ is given by

$$\beta^0 = \begin{pmatrix} \theta & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } \beta^i = \begin{pmatrix} \mathbf{0} & \rho^i \\ -\rho_T^i & \mathbf{0} \end{pmatrix}, \quad i = 1, 2, 3, \tag{3}$$

where

$$\theta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{4}$$

and

$$\rho^1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^2 = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho^3 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

with ρ_T denotes the transposed matrix of ρ and $\mathbf{0}$ is the zero matrix.

– For the case of spin 1, the explicit form of the β^μ is given as

$$\beta^0 = \begin{pmatrix} 0 & \bar{0} & \bar{0} & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \bar{0}^T & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \quad \text{and} \quad \beta^i = \begin{pmatrix} 0 & \bar{0} & e_i & \bar{0} \\ \bar{0}^T & \mathbf{0} & \mathbf{0} & -is_i \\ -e_i^T & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \bar{0}^T & -is_i & \mathbf{0} & \mathbf{0} \end{pmatrix}, \quad i = 1, 2, 3, \quad (6)$$

where the matrices s_i are the standard non relativistic (3×3) spin 1 matrices and, $\mathbf{0}$ and $\mathbf{1}$ denote respectively the zero matrix and the unity matrix. The matrices $\bar{0}$ and e_i are given as

$$\bar{0} = (0 \ 0 \ 0), \quad e_1 = (1 \ 0 \ 0), \quad e_2 = (0 \ 1 \ 0), \quad \text{and} \quad e_3 = (0 \ 0 \ 1). \quad (7)$$

Now, let us return to the DKP particles with time-dependent masses $m(t)$, in the presence of a time-dependent linear potential given by

$$V(z, t) = zf(t).$$

In this case the equation (1) is written

$$\left[i\beta^0(\partial_0 + izf(t)) + i\beta^3 \frac{\partial}{\partial z} - m(t) \right] \psi(z, t) = 0. \quad (8)$$

Let us proceed in the same way as in [10]. To absorb the interaction term, we use the following time depending transformation

$$\psi(z, t) = \exp\left(-iz \int_0^t f(t') dt'\right) \tilde{\psi}(z, t), \quad (9)$$

then we substitute (9) in (8) and find

$$i\beta_0 e^{-iA(t)z} \frac{\partial \tilde{\psi}(z, t)}{\partial t} = (\beta^3 p_z + m) e^{-iA(t)z} \tilde{\psi}(z, t), \quad (10)$$

where we posed $A = \int_0^t f(t') dt'$.

Let us multiply on the left the two terms of (10) by $e^{iA(t)z}$ to have

$$i\beta_0 \frac{\partial \tilde{\psi}(z, t)}{\partial t} = e^{iA(t)z} (\beta^3 p_z + m) e^{-iA(t)z} \tilde{\psi}(z, t). \quad (11)$$

It is easy to check that

$$e^{iA(t)z} p_z e^{-iA(t)z} = p_z - A. \quad (12)$$

Let us inject the expression (12) in (11) we find

$$i\beta_0 \frac{\partial \tilde{\psi}(z, t)}{\partial t} = (\beta^3(p_z - A) + m) \tilde{\psi}(z, t) \tag{13}$$

which is exactly the DKP equation in presence of magnetic fields.

In order to separate the part in time, we introduce the following transformation

$$\tilde{\psi}_\lambda(z, t) = e^{i\lambda z} \tilde{\tilde{\psi}}(t) \tag{14}$$

where λ is a real parameter playing the role of momentum.

Inserting (14) in (13) we find

$$i\beta_0 \frac{\partial \tilde{\tilde{\psi}}_\lambda(t)}{\partial t} = (-\beta^3\eta_\lambda + m) \tilde{\tilde{\psi}}(t) \tag{15}$$

where $\eta_\lambda = A - \lambda$.

3 The case $s = 0$

In this case, the wave function $\tilde{\psi}_\lambda(t)$ has five components defined by the vector (ϕ^T, χ^T) where ϕ^T is a vector 2×1 , and χ^T a vector 3×1 , formed as follows:

$$\tilde{\tilde{\psi}}(t) = \begin{pmatrix} \phi \\ \chi \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \text{and} \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix}. \tag{16}$$

Let us substitute (3)–(5) and (16) in the equation (15); we find

$$\begin{cases} i\theta\dot{\phi} = -\eta_\lambda\rho_3\chi + m\phi, \\ \eta_\lambda\rho_3^T\phi + m\chi = 0. \end{cases} \tag{17}$$

Uncoupling this system, we obtain

$$\begin{cases} i\theta\dot{\phi} = \frac{1}{m}\eta_\lambda^2\Sigma\phi + m\phi, \\ \eta_\lambda\rho_3^T\phi + m\chi = 0 \end{cases} \tag{18}$$

where Σ is the matrix defined by

$$\Sigma = \rho_3\rho_3^T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

The preceding equation (18) in ϕ is equivalent to the following system:

$$\begin{cases} i\dot{\phi}_2 = \frac{1}{m}\eta_\lambda^2\phi_1 + m\phi_1, \\ i\dot{\phi}_1 = m\phi_2 \end{cases} \tag{19}$$

or

$$\ddot{\phi}_1 - \frac{\dot{m}}{m}\dot{\phi}_1 + \omega^2\phi_1 = 0 \tag{20}$$

with $\omega = \sqrt{\eta_\lambda^2 + m^2}$ and the initial conditions $\phi_1(t_0) = \phi_0, \dot{\phi}_1(t_0) = \dot{\phi}_0$.

The formal solution of this equation is given by [13]:

$$\phi_1(t) = \rho(t) \exp(i\gamma(t)) \tag{21}$$

where $\gamma(t)$ is a function of time which can be determined by substituting the preceding function in (20)

$$\gamma(t) = C \int_{t_0}^t \frac{m}{\rho^2} dt' + C' \tag{22}$$

with

$$C' = \gamma(t_0) = \arg(\phi_1(t_0))$$

and

$$C = \frac{|\phi_1(t_0)|}{im(t_0)} \left[\frac{|\phi_1(t_0)|}{\phi_1(t_0)} \dot{\phi}_1(t_0) - \dot{\rho}(t_0) \right].$$

According to the initial conditions $\phi_1(t_0)$ and $\dot{\phi}_1(t_0)$, a direct calculation gives

$$\dot{\rho}(t_0) = \left| \dot{\phi}_1(t_0) \right| \cos \left[\arg \dot{\phi}_1(t_0) - \arg \phi_1(t_0) \right].$$

Let us carry the latter in the expression of C , we find

$$C = \frac{|\phi_1(t_0)|}{im(t_0)} \left[\frac{|\phi_1(t_0)|}{\phi_1(t_0)} \dot{\phi}_1(t_0) - \left| \dot{\phi}_1(t_0) \right| \cos \left[\arg \dot{\phi}_1(t_0) - \arg \phi_1(t_0) \right] \right],$$

which gives

$$\phi_1(t) = \tilde{C} \rho(t) \exp \left(iC \int_{t_0}^t \frac{m}{\rho^2} dt' \right) \tag{23}$$

with $\tilde{C} = \exp(iC')$ and the function $\rho(t)$ is the solution of the auxiliary equation

$$\ddot{\rho}(t) - \frac{\dot{m}}{m} \dot{\rho}(t) - \frac{C^2 m^2}{\rho^3} + \omega^2 \rho(t) = 0. \tag{24}$$

Now, to determine $\phi_2(t)$, we substitute the derivative of (23) in (19) and obtain

$$\phi_2(t) = \tilde{C} \frac{i}{m} \left[\dot{\rho} + \frac{iCm}{\rho} \right] \exp \left(iC \int_{t_0}^t \frac{m}{\rho^2} dt' \right). \tag{25}$$

Inserting $\phi_1(t)$ and $\phi_2(t)$ by their expressions, we obtain for the vector $\phi(t)$

$$\phi(t) = \tilde{C} \left(\frac{i}{m} \left[\begin{matrix} \rho(t) \\ \dot{\rho} + \frac{iCm}{\rho} \end{matrix} \right] \right) \exp \left(iC \int_{t_0}^t \frac{m}{\rho^2} dt' \right). \tag{26}$$

To calculate the vector χ , we introduce the expression of the vector $\phi(t)$ in (17), it follows:

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \tilde{C} \frac{\eta_\lambda}{m} \begin{pmatrix} 0 \\ 0 \\ \rho(t) \end{pmatrix} \exp \left(iC \int_{t_0}^t \frac{m}{\rho^2} dt' \right). \tag{27}$$

By taking into account the relations (9) and (14), the wave function of five components, solution of the equation of DKP, will thus be written as

$$\psi(z, t) = \tilde{C} \begin{pmatrix} \rho(t) \\ \frac{i}{m} \left[\dot{\rho} + \frac{iCm}{\rho} \right] \\ 0 \\ 0 \\ \frac{\eta_\lambda}{m} \rho(t) \end{pmatrix} \exp(-i\eta_\lambda z) \exp\left(iC \int_{t_0}^t \frac{m(t')}{\rho^2(t')} dt' \right). \quad (28)$$

We note that if we fix the parameters $\dot{m}(t) = \dot{\eta}_\lambda = \dot{\rho} = 0$, the expression (28) is reduced to the solution of the stationary DKP equation in the free case.

Before ending this section of the spin 0 case, we propose to study the adiabatic approximation of our solution. Knowing that in the adiabatic mode the parameters are written $B(\varepsilon t)$, their developments will be consequently written in ε power series. By retaining the dominant quantities of order 0 and 1 in ε , we deduce from the auxiliary equation (24)

$$\rho = \sqrt{\frac{Cm}{\omega}}. \quad (29)$$

Substituting this result in the expression of the wave function, we will then have

$$\psi(x, t) \xrightarrow{\text{adiabat}} \begin{pmatrix} \sqrt{\frac{Cm}{\omega}} \\ -\sqrt{\frac{Cm}{\omega}} \\ 0 \\ 0 \\ \eta_\lambda \sqrt{\frac{C}{\omega m}} \end{pmatrix} \exp(-i\eta_\lambda z) \exp\left(i \int_{t_0}^t \omega(t') dt' \right). \quad (30)$$

Notice that, contrary to what was found for the Dirac equation, we have in our case an obvious absence of the geometrical amplitude factor.

4 The case $s = 1$

To treat this case, we follow the same steps of the preceding section. The wave function $\psi(t)$ solution of DKP equation has now ten components noted as follows $\psi^T(t) = (\varphi, \mathbf{A}, \mathbf{B}, \mathbf{C})$, where φ is a scalar and $(\mathbf{A}, \mathbf{B}, \mathbf{C})$ are vectors (3×1) definite as follows

$$\mathbf{A}^T = (A_1 A_2 A_3), \mathbf{B}^T = (B_1 B_2 B_3), \mathbf{C}^T = (C_1 C_2 C_3). \quad (31)$$

Using the relations (6), (7), (31) and by a direct calculation the equation (15) will convert to

$$\begin{cases} \varphi = \frac{\eta_\lambda e_3 \mathbf{B}}{m}, \\ \mathbf{C} = -\frac{i\eta_\lambda s_3 \mathbf{A}}{m}, \\ i\frac{\partial \mathbf{A}}{\partial t} = \eta_\lambda e_3^T \varphi + m\mathbf{B}, \\ i\frac{\partial \mathbf{B}}{\partial t} = m\mathbf{A} + i\eta_\lambda s_3 \mathbf{C} \end{cases} \quad (32)$$

or

$$\begin{cases} \varphi = \frac{\eta_\lambda e_3 \mathbf{B}}{m}, \\ \mathbf{C} = -\frac{i\eta_\lambda s_3 \mathbf{A}}{m}, \\ i\frac{\partial \mathbf{A}}{\partial t} = \Sigma_1 \mathbf{B}, \\ i\frac{\partial \mathbf{B}}{\partial t} = \Sigma_2 \mathbf{A} \end{cases} \quad (33)$$

with Σ_i are diagonal matrices given by

$$\Sigma_1 = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & \omega^2 \\ & & & m \end{pmatrix}, \quad \Sigma_2 = i \begin{pmatrix} \frac{\omega^2}{m} & 0 & 0 \\ 0 & \frac{\omega^2}{m} & 0 \\ 0 & 0 & m \end{pmatrix}. \quad (34)$$

In order to uncouple the previous system, we derive (\dot{A}_1, A_2, B_3) and then find the following three principal equations describing the dynamics of the spin one:

$$\begin{cases} \ddot{A}_1 - \frac{m}{m} \dot{A}_1 + \omega^2 A_1 = 0, \\ \ddot{A}_2 - \frac{m}{m} \dot{A}_2 + \omega^2 A_2 = 0, \\ \ddot{B}_3 - \frac{m}{m} \dot{B}_3 + \omega^2 B_3 = 0. \end{cases} \quad (35)$$

Noting that this system is exactly the same as in spin 0 case (20). Consequently, their solutions are written in the same manner (21):

$$\begin{aligned} A_1(t) &= \rho_1(t) \exp(i\gamma_1(t)), \\ A_2(t) &= \rho_2(t) \exp(i\gamma_2(t)), \\ B_3(t) &= \rho_3(t) \exp(i\gamma_3(t)) \end{aligned} \quad (36)$$

where $\gamma_i(t)$ are functions of time that can be determined by substituting the preceding functions in (35)

$$\gamma_i(t) = D_i \int_{t_0}^t \frac{m}{\rho^2} dt' + D'_i, \quad i = 1, 2, 3, \quad (37)$$

where $D'_i = \gamma_i(t_0) = \arg(A_i(t_0))$, $i = 1, 2$, $D'_3 = \gamma_3(t_0) = \arg(B_3(t_0))$ and

$$D_i = \frac{|A_i(t_0)|}{im(t_0)} \left[\frac{|A_i(t_0)|}{A_i(t_0)} \dot{A}_i(t_0) - \left| \dot{A}_i(t_0) \right| \cos(\arg \dot{A}_i(t_0) - \arg A_i(t_0)) \right], \quad i = 1, 2,$$

$$D_3 = \frac{|B_3(t_0)|}{im(t_0)} \left[\frac{|B_3(t_0)|}{B_3(t_0)} \dot{B}_3(t_0) - \left| \dot{B}_3(t_0) \right| \cos(\arg \dot{B}_3(t_0) - \arg B_3(t_0)) \right].$$

We then obtain

$$\begin{cases} A_i(t) = A_i \rho_i(t) \exp\left(iD_i \int_{t_0}^t \frac{m}{\rho_i^2} dt'\right), & i = 1, 2, \\ B_3(t) = B_3 \rho_3(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \end{cases} \quad (38)$$

where $\tilde{D}_i = \exp(iD'_i)$, $i = 1, 2$, and $\tilde{D}_3 = \exp(iD'_3)$. The functions ρ_i verify the following auxiliary equations

$$\ddot{\rho}_i(t) - \frac{\dot{m}}{m} \dot{\rho}_i(t) + \omega^2 \rho_i(t) - \frac{D_i^2 m^2}{\rho_i^3} = 0, \quad i = 1, 2, 3. \quad (39)$$

By a direct calculation, the determination of the other components is easy and the final expressions of the vectors **A** and **B** are as follows:

$$\mathbf{A}(t) = \begin{pmatrix} \tilde{D}_1 \rho_1(t) \exp\left(iD_1 \int_{t_0}^t \frac{m}{\rho_1^2} dt'\right) \\ \tilde{D}_2 \rho_2(t) \exp\left(iD_2 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ \frac{i\tilde{D}_3}{m} \left[\dot{\rho}_3 + \frac{iD_3 m}{\rho_3} \right] \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \end{pmatrix}, \quad (40)$$

and

$$\mathbf{B}(t) = \begin{pmatrix} \frac{i\tilde{D}_1}{m} \left[\dot{\rho}_1 + \frac{iD_1 m}{\rho_1} \right] \exp\left(iD_1 \int_{t_0}^t \frac{m}{\rho_1^2} dt'\right) \\ \frac{i\tilde{D}_2}{m} \left[\dot{\rho}_2 + \frac{iD_2 m}{\rho_2} \right] \exp\left(iD_2 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ \tilde{D}_3 \rho_3(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \end{pmatrix}. \quad (41)$$

To determine $\varphi(t)$, we use the equation (41) in (33); the result is

$$\varphi(t) = \frac{\eta\lambda}{m} \tilde{D}_3 \rho_3(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \quad (42)$$

and for the last vector \mathbf{C} , we substitute (40) in (33) and find

$$\mathbf{C} = -\frac{\eta\lambda}{m} \begin{pmatrix} \tilde{D}_2\rho_2(t) \exp\left(iD_2 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ -\tilde{D}_1\rho_1(t) \exp\left(iD_1 \int_0^t \frac{m}{\rho_1^2} dt'\right) \\ 0 \end{pmatrix}. \tag{43}$$

In conclusion, the explicit final form of $\psi(z, t)$ is then

$$\psi_\lambda(z, t) = e^{-i\eta\lambda z} \begin{pmatrix} \frac{\eta\lambda}{m} \tilde{D}_3\rho_3(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \\ \tilde{D}_1\rho_1(t) \exp\left(iD_1 \int_{t_0}^t \frac{m}{\rho_1^2} dt'\right) \\ \tilde{D}_2\rho_2(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ \frac{i\tilde{D}_3}{m} \left[\dot{\rho}_3 + \frac{iD_3 m}{\rho_3}\right] \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \\ \frac{i\tilde{D}_1}{m} \left[\dot{\rho}_1 + \frac{iD_1 m}{\rho_1}\right] \exp\left(iD_1 \int_{t_0}^t \frac{m}{\rho_1^2} dt'\right) \\ \frac{i\tilde{D}_2}{m} \left[\dot{\rho}_2 + \frac{iD_2 m}{\rho_2}\right] \exp\left(iD_2 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ \tilde{D}_3\rho_3(t) \exp\left(iD_3 \int_{t_0}^t \frac{m}{\rho_3^2} dt'\right) \\ -\frac{\eta\lambda}{m} \tilde{D}_2\rho_2(t) \exp\left(iD_2 \int_{t_0}^t \frac{m}{\rho_2^2} dt'\right) \\ \frac{\eta\lambda}{m} \tilde{D}_1\rho_1(t) \exp\left(iD_1 \int_{t_0}^t \frac{m}{\rho_1^2} dt'\right) \\ 0 \end{pmatrix}. \tag{44}$$

As before, let us study the adiabatic approximation of the solution. Retaining only the dominant quantities of order 0 and 1 in ε , the auxiliary equation (39) will give

$$\rho_i = \sqrt{\frac{D_i m}{\omega}}.$$

Consequently the expression of the wave function is reduced to

$$\psi(z, t) \xrightarrow{\text{adiabat}} \begin{pmatrix} \frac{\eta_\lambda}{m} \tilde{D}_3 \sqrt{\frac{D_3 m}{\omega}} \\ \tilde{D}_1 \sqrt{\frac{D_1 m}{\omega}} \\ \tilde{D}_2 \sqrt{\frac{D_2 m}{\omega}} \\ -\tilde{D}_3 \sqrt{\frac{D_3 \omega}{m}} \\ -\tilde{D}_1 \sqrt{\frac{D_1 \omega}{m}} \\ -\tilde{D}_2 \sqrt{\frac{D_2 \omega}{m}} \\ \tilde{D}_3 \sqrt{\frac{D_3 m}{\omega}} \\ -\frac{\eta_\lambda}{m} \tilde{D}_2 \sqrt{\frac{D_2 m}{\omega}} \\ \frac{\eta_\lambda}{m} \tilde{D}_1 \sqrt{\frac{D_1 m}{\omega}} \\ 0 \end{pmatrix} \exp(-i\eta_\lambda z) \exp\left(i \int_{t_0}^t \omega(t') dt'\right). \quad (45)$$

Here also, contrary to what was found for the Dirac equation, we have in our case an obvious absence of the geometrical amplitude factor.

To conclude, we have solved the Duffin–Kemmer–Petiau (DKP) equation of spin 0 and 1 in the case of the time-dependent mass and in presence of a time-dependent linear scalar field. By a direct calculation, we have solved the system and have found the explicit final form of the wave function. This solution is connected to an auxiliary equation playing the same role as the invariant system. We have deduced from this treatment the adiabatic approximation which shows the obvious absence of the geometrical amplitude factor.

In fact, it would be fundamental to connect the transformations used to gauge transformations. As we work in 1+1 dimension, we avoided to do it. With regard to this point in particular, we thank the referee for having drawn our attention by indicating us the references [14–17] necessary to develop it. This work will be the subject of another thorough study.

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