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Calculation of the contact angle of a spherical surface with a bubble in flotation

M. Bouhenguel*, S. Kouachi, A. Bouchemma
Applied Chemistry and Materials Technology Laboratory, Larbi Ben M'hidi University Center, Oum El Bouaghi 04000, Algeria
email : mbouhenguel@yahoo.fr

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Abstract

We have developed a method for the calculation of contact angle of spherical particle in interaction with a bubble in the process of flotation. The balance of forces acting on the instantaneous TPC perimeter in the radial direction could connect the central angle α and the contact angle θ in only one equation depending on the modified Bond number Bo, which made it possible to couple this equation with the one of the contact angle in function of the linear tension. Some theoretical results are presented.

Keywords: Contact angle; Flotation; Models

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1. Introduction

The measurement of contact angles, as described by Thomas Young in 1805, remains the simplest and the most precise method to characterize solid surface properties and to determine the energy of interaction between a liquid and a solid. To be able to understand and calculate contact angles, we need to know the phenomena happening during the contact between a solid and a gas, a solid and a liquid or between these three phases and the balances of the phases present during the process of separation. In order to calculate the latter, one calculates the contact angle of the plane surfaces, especially of a surface supposed to be spherical and linked to an air bubble, during the ores separation by flotation.

The study of an interaction particle–bubble lays out a very important conventional and practical significance at the time of the process of flotation [1–4]. Since the majority of the ores of the world are recovered by flotation, much attention was given to these interactions so as to understand them.

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*Corresponding author.

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In the aqueous medium, at least three suitable forces of surface act between a bubble of air and a particle, namely; the electrostatic double layer force; the Van der Waals force; and if the contact angle is larger than zero, it is the hydrophobic attraction force. In the case of the hydrophobic solids, a strong attraction is observed at the time of the contact with the bubble of air leading, thus, to the realization of the three subprocesses controlling the process of flotation which are: the collision, the attachment and the stability of the aggregate particle bubble. The subprocess of attachment is the decisive point in which the best flotation would be possible. Because of this, much of the studies were aimed at understanding the stages thus produced. The attachment of air bubble on the mineral surface is possible after thinning down the liquid layer separating the air bubble from the ore until achieving a critical thickness. Then, the breaking of the thin film, the formation of a three-phase core of contact (a hole of a critical ray), and the expansion of the contact area between the air bubble and the ore surface to form a stable perimeter for damping all these stages will be possible if the formed contact angle is more than zero.

In this paper, the measurement of the contact angle of a spherical surface during flotation is achieved by calculating the plane surface using two models: Owens and Wendt model and Good Van Oss model. Determining the contact angle of a spherical surface is possible after coupling the equation depending on the linear tension and the equilibrium equation of three contact phases connecting the angle which describes the position of the particle in the interface and the contact angle θ [5–7].

2. Methods

2.1. Owens and Wendt model

In the model of Owens and Wendt, one considers that the energy of surface is expressed in the form [8]

$$\gamma_S = \gamma_S^d + \gamma_S^{nd}$$

in which γ_S^d is a dispersive component and γ_S^P is a nondispersive component.

The equation connecting the components to the contact angle is then written:

$$\theta = \arccos\left(\frac{2\sqrt{\gamma_S^d}\sqrt{\gamma_L^d} + 2\sqrt{\gamma_S^P}\sqrt{\gamma_L^d}}{\gamma_L} - 1 \right) \quad (1)$$

In this model, one needs two different liquids to obtain the energy of surface. However, an approximation is made on the nondispersive term by considering that it is the geometric mean of γ_S^P and of γ_L^P. This approximation does not account for the possible polar polymer reaction in an aqueous medium.

2.2. Good Van Oss model

In the Good Van Oss model, the energy of surface is written (Fig. 1):

$$\gamma_S = \gamma_S^d + 2\sqrt{\gamma_S^+ \gamma_S^-}$$

where γ_S^d is the dispersive component (dipole–dipole interaction) and γ_S^+ and γ_S^- are the polar components.

The relation between solid surface energy and liquid components and the contact angle of the drop is written:

$$\theta = \arccos\left(\frac{2\left(\sqrt{\gamma_S^d \gamma_L^d} + \sqrt{\gamma_S^+ \gamma_L^-} + \sqrt{\gamma_L^+ \gamma_S^-}\right)}{\gamma_L} - 1 \right) \quad (2)$$

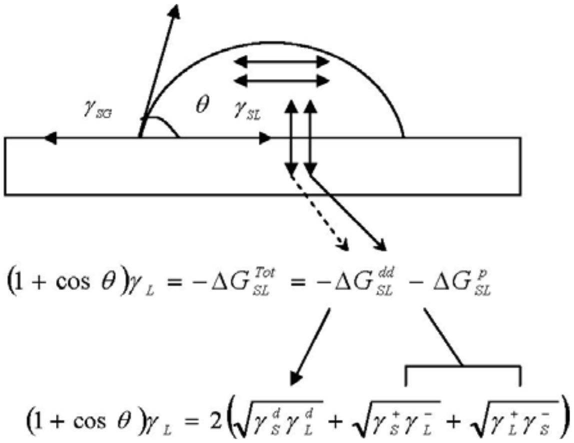

Fig. 1. Surface free energy according to the Good Van Oss model.

2.3. *Calculation of the contact angle of a spherical surface related to the linear tension*

When the spherical particle is attached to the air bubble, a three-phase contact system is formed. The equivalent force of the jump is the capillary force due to meniscus formation. This force depends on the contact angle given by the relation [9,10]:

$$F_{Cap} = 2\pi R_P \gamma \cdot \sin\alpha \cdot \sin(\theta - \alpha)$$

where γ is the surface tension of the liquid and R_P is the radius of particle. The parameter α describes the position of the particle in the interface (Fig. 2). The capillary force is equal to zero for $\alpha = \theta$, which is the case when no meniscus is formed, and vice versa; i.e, a meniscus inevitably leads to a capillary force. This assumption is used to determine the contact angle of a spherical particle.

The capillary force depends on the contact angle which in turn is related to the linear tension κ^0 via the formula [11]

$$\cos\theta = \cos\theta^0 + \frac{\kappa^0}{R_P\gamma}\cot\alpha \tag{4}$$

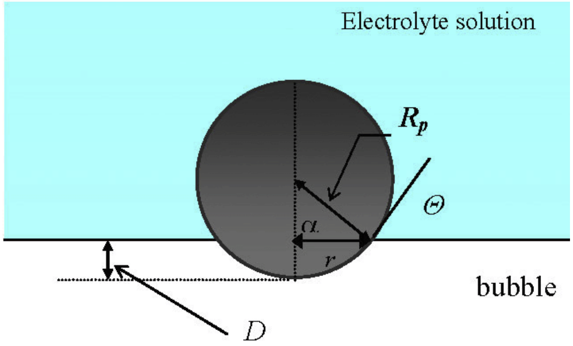

Fig. 2. Spherical particle in three contact phases on the surface of a bubble in an electrolyte solution.

where θ^0 is the contact angle measured on a plane surface.

At the three contact phase's equilibrium, the equation relating angle α to the contact angle θ is given by the following expression [12]:

$$\sin\alpha\sin(\alpha - \theta) = Bo \tag{5}$$

with Bo as the modified Bond number, which is equal to:

$$Bo = \{2/3\}\{R_P/l_c\}^2\left[\left(\frac{\rho_P}{\rho_L}\right) - f(\alpha)\right] \tag{6}$$

where l_c is the capillary length, and f is a function of α equal to $f(\alpha) = (2 + 3\cos\alpha - \cos^3\alpha)/4$.

Let us replace these parameters in Eq. (6), and with Eq. (4) one obtains the system of equations according to:

$$\begin{cases} \{2/3\}\{R_P/l_c\}^2\left[\left(\dfrac{\rho_P}{\rho_L}\right) - \left((2 + 3\cos\alpha - \cos^3\alpha)/4\right)\right] \\ \cos\theta = \cos\theta^0 + \dfrac{k^0}{R_P\gamma}\cot\alpha \end{cases}$$

This system of two equations with two variables θ and α needs a numerical method to be solved.

The Newton method of resolution for a system of two equations is given. Supposing θ_n, α_n as an approximate solution of the system of the equations:

$$F(\theta,\alpha) = \sin\alpha \sin(\alpha-\theta) - \{2/3\}\{R_P/l_c\}^2 \left[\left(\frac{\rho_P}{\rho_L}\right) - \left(\left(2+3\cos\alpha-\cos^3\alpha\right)/4\right)\right] = 0 \quad (7)$$

$$G(\theta,\alpha) = \cos\theta - \cos\theta^0 - \frac{k^0}{R_P\gamma}\cot\alpha = 0 \quad (8)$$

where F and G are continuously derivable functions. While posing

$$\theta = \theta_n + h_n\ ; \quad \alpha = \alpha_n + k_n$$

Then one has

$$F(\theta_n + h_n;\ \ \alpha_n + k_n) = 0$$

$$G(\theta_n + h_n;\ \ \alpha_n + k_n) = 0$$

4. Results and discussion

4.1. Contact angle of a plane solid surface

Some results of the contact angle of several systems (liquid–solid–air) calculated on the basis of the two models (Good Van Oss and Owens and Wendt) are illustrated in Table 1.

The Good Van Oss model can be applied for surfaces having a free energy or average and weak surface. Consequently, the Owens and Wendt model can be applied only for surface solids having a weak free energy (Teflon, polyisobutylene, polypropylene, etc.), whereas with surfaces having average energy (calcium carbonate, polymethylmethacrylate), this model has divergences in comparison with the Good Van Oss model (Fig. 3).

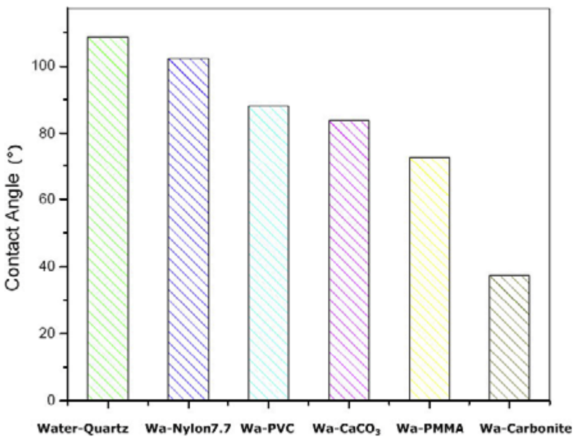

Fig.3. Calculated contact angle (Good theory) between water and planar solids.

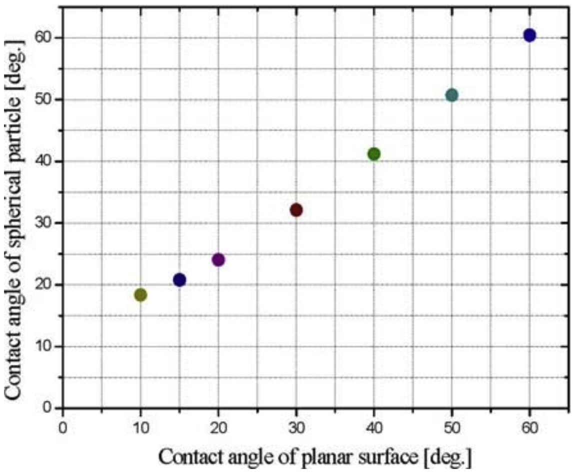

Fig. 4. Contact angle of a spherical surface vs a plane surface of a solid of diameter 20 µm and density 2.65 g/cm^3.

4.2. Contact angle of a spherical surface

The calculated angles for plane surfaces and spherical surfaces were roughly similar. Below 40°, the contact angles for the spherical particles are higher than for those plane surfaces. Then, the values of the contact angles for the plane surfaces and those of spherical surfaces tend to remain the same.

Table 1
Calculated contact angles of sessile drop for various planar systems

System–air	Contact angle	
	Good Van Oss (θ_G)	Owens and Wendt (θ_W)
Water–Teflon	164	164
Water–polypropylene	111	111
Water–polyisobutylene	110	110
Water–quartz (treated DAHCl)	109	78
Water–polyethylene	105	105
Water–Nylon 7.7	102	102
Water–polyvinylchloride	88	95
Water–parbonate	84	96
Water–coal	88	101
Water–polymethylmethacrylate	72	101
Water–carbonite	37	55
Glycerol–Teflon	102	102
Glycerol–polyethylene	87	87
Ethanol–Teflon	48	48
Toluene–polypropylene	26	26
Methanol–Teflon	51	51
Methyleneiodide–polystyrene	35	35
Cis Decalin–polyisobutylene	40	40
Formamide–talc	45	51
Ethylene glycol–Teflon	92	92

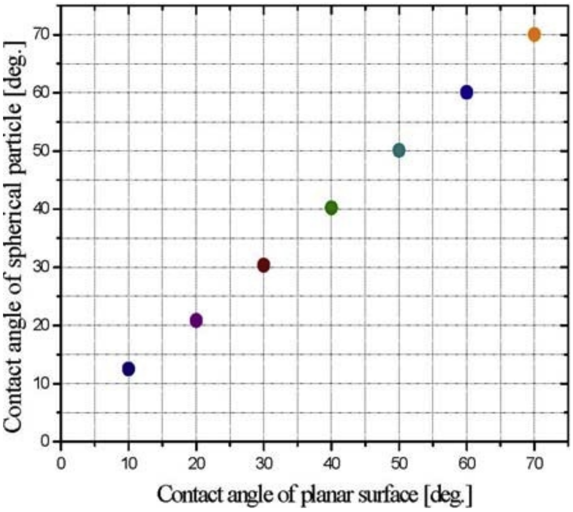

Fig. 5. Contact angle of a spherical surface vs a plane surface of a solid of diameter 120 µm and density 2.65 g/cm^3.

Table 2
Calculated contact angle showing the effect of the particle size on the contact angle

Radius of the particle (µm)	ρ_S (g/cm^3)	Angle of a plane surface (°)	Angle of a spherical surface (°)
10	2.65	10	22
10	2.65	20	27
100	2.65	10	13
100	2.65	20	21

One can notice that when the spherical particle is small, the contact angle of a spherical surface is larger than that of a plane surface (Table 2); then it remains almost constant beyond 40°. The larger is the size of the particle, the closer are the contact angles shown in Figs. 4 and 5.

5. Conclusions

A difference exists between the contact angle of small spherical particles and contact angles on planar surfaces. This difference decreased with increasing contact angle and increasing particle size. This difference can be interpreted as the interaction energy between a particle and a bubble since the attractive hydrophobic interaction increases as the hydrophobicity of the particle increases.

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